

Assignment – Poker

Q1.

Tong is a variation on poker, in which players are dealt 6 cards instead of 5. The hand categories are similar to those for 5-card poker, but with some differences. They are:

- Ace high
- Pair
- Three pairs
- Crowded house (2 sets of 3-of-a-kind)
- Straight
- Half flush (3 cards the same suit)

For each hand category, calculate the probability of being dealt it. Thus rank the categories from least likely (and therefore top-ranked) to most likely (and therefore bottom-ranked). Consider a hand to contain a category even if it also contains a higher-ranked category. So, for example, a hand with a crowded house also contains a pair.

Q2.

Invent another variation on poker by removing some of the cards from the pack and changing the number of cards dealt to each player. Use six hand categories, at least two of which should be significantly different from those in tong or poker. Your game needs to be original enough that it is unlikely to be thought of by other people. Also, it needs to be sufficiently involved that the calculations are not trivial.

Give a brief explanation of the game, give the hand categories with explanations where appropriate, and calculate the probabilities associated with each category.

Run a simulation of your game by using the Excel macro provided on the school intranet, altering the appropriate parts as necessary. Simulate the dealing of 100 hands and find the relative frequency of each hand category.

- a) Present your information in a table showing, for each category, the calculated probability and the relative frequency obtained from the simulation.
- b) Email a copy of your Excel file with all the data for the 100 simulations in it to your teacher.
- c) Comment on the reasonableness of your results and whether the simulations support your calculations.



Poker: Solution: Version 1

Number of possible combinations $\binom{52}{6} = 20,358,520$ possible combinations

Hand	Combinations	Probability
Ace high	${}^{52}C_6 - {}^{48}C_6$	0.397230
	$1 - 48/52 * 47/51 * 46/50 * 45/49 * 44/48 * 43/47$	0.397230
Pair	$1 - 1 * 48/51 * 44/50 * 40/49 * 36/48 * 32/47$	0.655
Three Pair	${}^{13}C_3 * ({}^4C_2)^3 + {}^{13}C_1 * {}^4C_4 * {}^{12}C_1 * {}^4C_2$	0.003034405251
	3 different pairs . . four + pair	0.003080381089
Crowded house (2 sets of 3-of-a-kind)	$\binom{13}{2} \binom{4}{3}^2 = 1,248$	0.0000613011
Straight (of 6 cards)	$9 * ({}^4C_1)^6 = 36,864$	0.001810740663
Half flush (3 cards the same suit)	$({}^{13}C_3 * {}^{39}C_3 + {}^{13}C_4 * {}^{39}C_2 + {}^{13}C_5 * {}^{39}C_1 + {}^{13}C_6) * 4$	0.62784*
	$({}^{13}C_3 * ({}^{39}C_3 - {}^{13}C_3 * 3) + {}^{13}C_4 * {}^{39}C_2 + {}^{13}C_5 * {}^{39}C_1 + {}^{13}C_6) * 4$	0.5796

* The first answer will be accepted. The second is correct.

Solution is to include working for their calculations.

Response to include:

- Explanation of the game including ranks, suits etc.
- Table showing for each category, the calculated probability and the relative frequency obtained from the simulation
- the data produced by the simulation in an emailed spreadsheet
- Comment on the reasonableness of the results and whether the simulations support the calculations

Poker: Solution: Version 2

Number of possible combinations $\binom{52}{6} = 20,358,520$ possible combinations

Hand	Example	Combinations	Probability
High Card		6,985,044	0.3431017579
One Pair		$\binom{13}{1} \binom{4}{2} \binom{12}{4} \times 4^4 = 9,884,160$	0.4855048402
Two Pair		$\binom{13}{2} \binom{4}{2}^2 \binom{11}{2} \times 4^2 = 2,471,040$	0.1213762101
Three of a Kind		$\binom{13}{1} \binom{4}{3} \binom{12}{3} \binom{4}{1}^3 = 732,160$	0.0359633215
Three of a Kind plus a Pair		$\binom{13}{1} \binom{4}{3} \binom{12}{1} \binom{4}{2} \binom{11}{1} \times 4 = 164,736$	0.00809174733
Three Pair		$\binom{13}{3} \binom{4}{2}^3 = 61,776$	0.003034405251
Straight		$9 \times \binom{4}{1}^6 - 36 = 36,828$	0.001809168
Four of a Kind		$\binom{13}{1} \binom{4}{4} \binom{12}{2} \binom{4}{1}^2 = 13,728$	0.0006743122
Flush		$\binom{13}{6} \binom{4}{1} - 36 = 6,828$	0.000335387
Two Three of a Kind		$\binom{13}{2} \binom{4}{3}^2 = 1,248$	0.0000613011
Full House (Four of a kind + a pair)		$\binom{13}{1} \binom{4}{4} \binom{12}{1} \binom{4}{2} = 936$	0.000045975
Straight Flush		$9 \times \binom{4}{1} - 4 = 32$	0.000001571
Royal Flush		$\binom{4}{1} = 4$	0.0000001964
Total		$\binom{52}{6} = 20,358,520$	1

Solution is to also include examples of each hand, and explanation on their calculations

Response to include:

- Explanation of the game including ranks, suits etc.
 - Table similar to Q4 with calculated combinations and probabilities in ranked order
- b)
- i) Simulation of their game for 100 iterations to give relative frequency of each hand with results in a table
 - ii) Comment on the reasonableness of results
 - i.e. not all hands possible – odds etc., yet randomness of excel allows a reasonable result even though 10 hands dealt does diminish the chances as 5th card dealt is dealt from reduced pack and hence may have removed cards needed for achieving some hands