Assignment - Fluttering

Q1. Take 10 cards, the A-10 of a suit. Shuffle the cards.

a) How many different orders could the cards end up in?

b) How many different hands of 4 cards is it possible to deal from the pack of 10? (Note: A, 4, 7, 2 is not a different hand from 7, A, 4, 2.)

c) If you shuffled the 10 cards and dealt someone 5 of them, what is the probability that their hand would include the A, 2 and 3?

Q2.

Fluttering is a shuffling technique in which you split the pack into two equal stacks, take one stack in each hand, then interleave the two stacks. Few people can do a perfect flutter, but you can simulate one by counting out the top half of the pack, then taking cards alternately from the top and bottom halves to form the shuffled pack.

Try this with your 10 cards. Start with the cards in order, ace at the top. After fluttering, they should end up (from top to bottom): A, 6, 2, 7, 3, 8, 4, 9, 5, 10.

If you flutter a pack repeatedly, the cards will eventually come back to the original order. Explain why this must be so. Your explanation must be valid for any even number of cards.

Q3. How many flutters will it take for 10 cards to come back to the original order? Find how many riffles for 2, 4, 6 and 8 cards, then for 52. Perform the flutters on paper or with a computer simulation rather than with the actual cards and hand in the working with your answer. Hand-written working is fine.

Q4. Using your working from Q3, find a formula that will allow you to work out how many flutters it would take for any even number of cards without actually doing the fluttering (or simulating it). HINT: You will need to follow where the cards go from flutter to flutter. Pay particular attention to the card that starts second from the top.
You must explain how you got from the working of Q3 to your formula. State any assumptions you make along the way.

Q5. Use your formula to find how many flutters it would take for a pack of 26 cards to return to the original order. Show working.

Fluttering Solutions

1. a) 10! = 3,628,800
   b) ${}^{10}C_4 = 210$
   c) Probability = ${}^{7}C_5 / {}^{10}C_5 = \frac{1}{12}$
2. a) Proof

3. Need to show fluttering

No. of Cards	No. of flutters taken
10	6
2	1
4	2
6	4
8	3
52	8

4. Explanation must include:
   - Top and bottom cards never move
   - The order of the cards
   - The position of cards as they move and in terms of position number in powers of 2
   - Position given by 2^n with n the number of riffles
   - Once reaches end of deck and cycles back through the formula changes to $p \equiv 2^n (mod\ c-1) with\ c\ the\ number\ of\ cards$
   - To be in the original order need p=1

5. Solve $2^n \equiv 1 (mod\ c-1)$ for n=20
   Need to show steps to get the solution