M1 Maths

S3-5 Data Distributions

- distribution shape
- normal distributions
- skewed distributions
- unimodal and bimodal distributions

Summary   Learn   Solve   Revise   Answers

Summary

A bar graph will show how data is distributed across the possible values. The distribution can be described by its shape.

A normal distribution is bell shaped and symmetrical. Many distributions are approximately normal.

Distributions which are similar to a normal distribution but asymmetric with one tail longer than the other are said to be skewed – positively skewed if the right tail is longer, negatively skewed if the left tail is longer.

Distributions with a single peak are said to be unimodal; those with two peaks of roughly the same height and a significant trough between them are said to be bimodal.

Learn

Data Distribution Shape

This bar graph shows the heights of all 5-year olds in Drillingham.
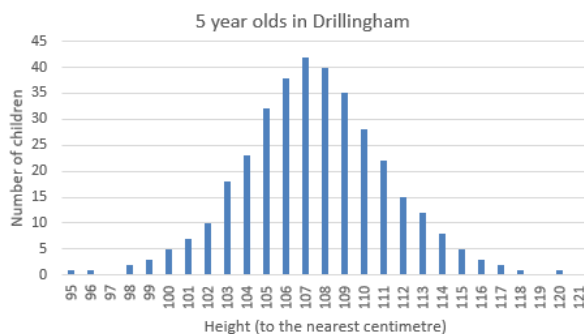
It shows how the 5 year olds are distributed across the range of heights. Most are around 105-110 cm, with some lower and some higher. In other words it shows the

distribution of the 5 year olds across the range of heights. This is a data distribution.

The shape of the distribution is a bit like a bell with a peak in the centre and tails going off to both sides. This shape of distribution is often called a bell curve. Bell curves are common with data which clusters around a mean value but with some variation above and below.

Normal Distributions

In fact bell-shaped distributions are so common that they are called normal distributions.

Actually a normal distribution is a bell-shaped distribution but with a very specific mathematically defined shape (See Module P6-5 – Continuous Random Variables). For data distributions there is always some slight deviation from the exact normal distribution shape, so it is better to describe the shape as approximately normal.

The graph above is a bar graph, but note that dot plots, histograms and stem-and-leaf plots also show the shape of a distribution.

Skewed Distributions

Normal distributions are symmetrical. Some distribution are asymmetrical. We say that they are skewed. This graph shows the time taken to run 100 m for the people who live in Little Brentwood.
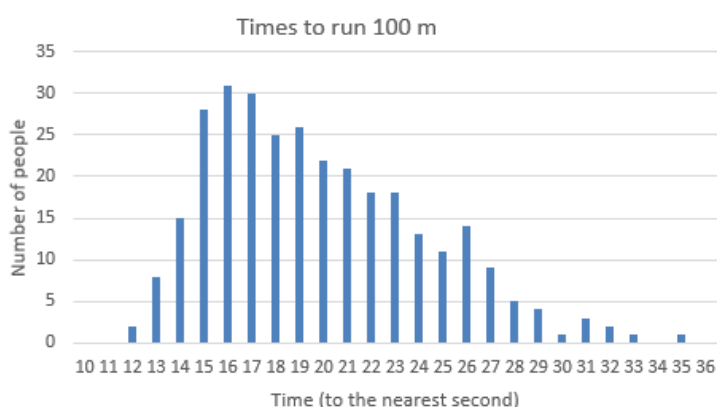

It is skewed. It has a longer tail to the right than to the left. The right is the positive direction on the horizontal axis so we say the distribution is positively skewed. If the longer tail was on the left, we would say it was negatively skewed.

So distributions can be symmetrical, positively skewed or negatively skewed.

Bimodal Distributions

Of course, not all symmetrical distribution are approximately normal. This distribution of ages of people playing football at Dagwood Park is fairly symmetrical, but nothing like normal.
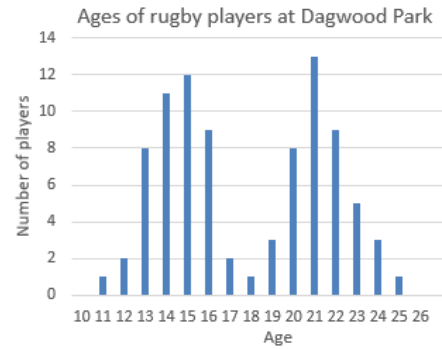
There were several games on, some for adults and some for juniors.

This distribution is said to be bimodal. The mode shows up on a bar graph as the tallest bar. Although, strictly speaking, the mode of this distribution is 21, there is another quite different age with nearly as many players – 15. We say a distribution is bimodal if it has two peaks with a significant trough between them, even if the peaks are a bit different in height.

By contrast, the first two distributions above were both unimodal.

We don't generally refer to bimodal distributions as positively or negatively skewed, just symmetrical or asymmetrical.

Practice

Q1   For each of these distributions say whether it is
   (i)   approximately normal
   (ii) symmetrical, asymmetrical, positively skewed or negatively skewed
   (iii) unimodal or bimodal

(a)

(b)

(c)
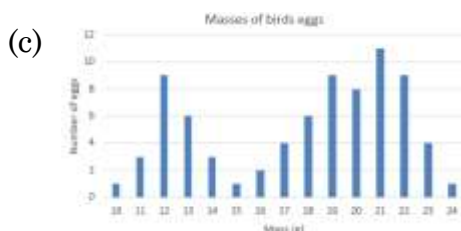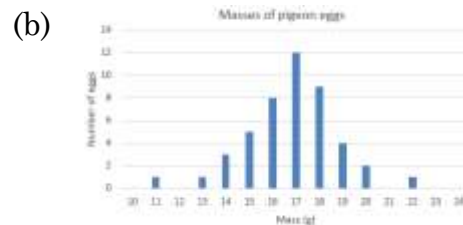
Q2   Sketch distributions which are
   (a) approximately normal
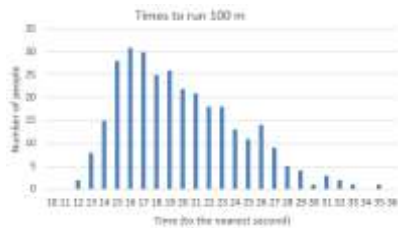   (b) positively skewed
   (c) bimodal and symmetric

Solve

Q51 Ernest counted how many rolls of a die it took for him to get a 6. He did this experiment 120 times. Sketch a likely distribution for his results. Describe its shape in terms of normality, skewness and number of modes.
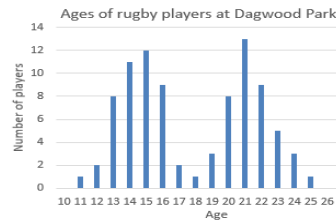
Revise

Revision Set 1

Q61 For each of these distributions say whether it is
  (i) approximately normal
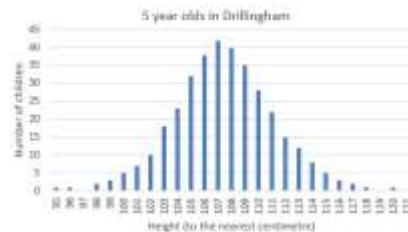 (ii) symmetrical, asymmetrical, positively skewed or negatively skewed
 (iii) unimodal or bimodal

(a)

(b)

(c)

Q62 Sketch distributions which are:
  (a) approximately normal
  (b) negatively skewed
  (c) bimodal and asymmetric

Answers

Q1   (a) not approximately normal, positively skewed, unimodal
     (b) not approximately normal, symmetrical, bimodal
     (c) not approximately normal, asymmetrical, bimodal

Q2   (a) Like L1(c)
     (b) Like the first graph on Page 2
     (c) Like the second graph on Page 2

Q51
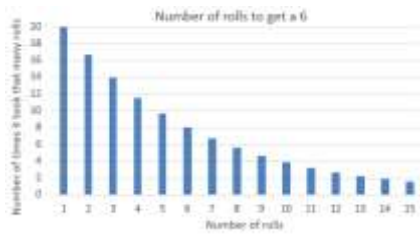
Not very normal, very positively skewed, unimodal.

Q61   (a) not approximately normal, negatively skewed, unimodal
        (b) approximately normal, symmetrical, unimodal
        (c) approximately normal, symmetrical, unimodal

Q62   (a) Like the graph on Page 1
        (b) Like P1(a)
        (c) Like P1(c)