M1 Maths

S3-3 Critiquing

- critiquing data collection, data representation and conclusions drawn from data

Summary Lead-In Learn Solve Revise Answers

Summary

People often present conclusions from data in misleading ways, often because they want to convince you of a particular point of view. It is therefore important, when faced with such material, to look critically at the way the data was collected, the way it is presented and the conclusions drawn from it.

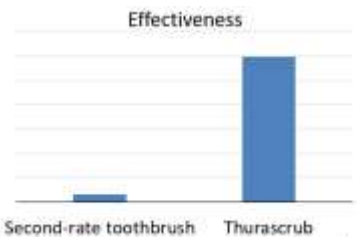
Before being convinced, you should satisfy yourself that the data was collected properly, i.e. that the sample size was adequate, that the sample was unbiased and that any questions were asked in a neutral way.

You should also be satisfied that any data presentations, e.g. graphs, are properly constructed with all the required information present.

And finally, you should check that any conclusions drawn from the data are valid.

Lead-In

In a recent survey of leading dentists, it was found that 4 out of 5 recommend Thurascrub toothbrushes.



So, if you want healthy white teeth, throw out that second-rate toothbrush and get a Thurascrub. On sale at all good chemists and supermarkets.

Would this advertisement entice you to buy a Thurascrub? Hopefully not.

Thurascrubs may not be any better than other toothbrushes (in fact they may well be worse). But the advert is designed to make you think that Thurascrubs are way better.

Of course, companies are not allowed to lie in their advertisements, but they can make them misleading – design them so as to give a false impression to gullible people.

If you look at the ad carefully, you can see that it really gives no evidence at all that Thurascrubs are good toothbrushes. Let's go through it.

Firstly, the survey. The ad does not mention the sample size. It may well have been 5 and that is not sufficient to draw reliable conclusions.

Also, it does not mention how the sample was chosen: it might well be biased. Bias could have happened in many ways:

- They could have actually asked 100 dentists and four of those recommended Thurascrub. So they chose as their sample those four plus one other.
- They could have surveyed just those who bought Thurascrubs.
- They could have asked just employees of Thurascrub.

We do not know that the question was asked in a neutral way. The question might have been:

- 'Would you recommend using a Thurascrub rather than a wire brush?'
- We are paying $200 to dentists who recommend Thurascrubs. Would you recommend them?

Then there is the term 'leading dentists'. This sounds good, but means nothing. Dentistry is a service, not a competition. It could just mean the dentists leading the move to Thurascrubs.

Then let's have a look at the graph. It is titled 'Effectiveness'. But that leads one to ask 'Effectiveness at what?' – maybe at removing the enamel from people's teeth or skin from their gums; maybe even at unplugging a constipated cat.

There is no scale on the vertical axis. We do not know how effectiveness is measured or what units it is measured in, nor whether the scale starts at zero. The scale could run from 95 to 100, meaning that, although it looks like Thurascrubs are about 10 times as effective (at whatever) than 'second-rate' toothbrushes, they might be only marginally more effective. The scale could even conceivably be in reverse with 0 at the top and 100 at the bottom.

Then of course, what is a second-rate toothbrush? The ad gives no indication of what the Thurascrub is being compared to. It could be a toothbrush that lost all its bristles in a fire.

Now for the concluding paragraph. 'If you want healthy white teeth, throw out that second-rate toothbrush and get a Thurascrub'. Everyone wants healthy white teeth,

but the ad isn't claiming that a Thurascrub will give you healthy white teeth – it's just telling you to get one.

And finally, what is a *good* chemist or supermarket? It may be that they call a shop *good*, if it sells their toothbrushes. This is a commonly used ploy to put pressure on shops to stock a product, in that customers might think the shop isn't good if they don't stock it.

Of course, the ad doesn't necessarily mean that Thurascrubs aren't any good, but one is led to think that, if they were any good, they would be able to say something more meaningful about them.

One final point worth remembering is that dentists make the most money when people's teeth decay. So using a dentist's recommendation in choosing a toothbrush may not be the best idea anyway.

Learn

To **critique** something means to list its good and bad points. The paragraphs in the Lead-In above were a critique of the advertisement for Thurascrub toothbrushes. Admittedly, it only listed bad points, but that was because there weren't too many good points.

Sometimes people collect and present data and draw conclusions from it in order to inform. Other times, people do so in order to convince someone of a particular point of view – maybe that you should buy their product or that you should vote for them.

It is important, when using data presentations and conclusions drawn from them, to be critical and to check that

- the data collection was appropriate (adequate sample size, unbiased sample and neutral questions),
- the display is not misleading, i.e. that it actually says what it seems to say,
- the conclusions drawn are logical and valid.

If you don't do this, you will have a tendency to believe whatever the person presenting the data wants you to believe, be misinformed and ignorant and maybe be conned. This is possibly one of the most important lessons to take away from school maths.

Data Collection

If a survey or collected data is used to make a point, then, to decide whether the point is valid, it is important to know how the data was collected, in particular the sample

size and whether the sample might be biased and whether any questions were asked in a neutral way.

If information about the way data is collected is not provided, then you should be sceptical of any conclusions drawn. For example, the Thurascrub ad above is based on a survey, but it is not at all clear that the sample size was sufficient or that the sample was unbiased or that the questions were asked appropriately.

Note that, if this information is not provided, it doesn't necessarily mean that the data was collected badly, just that it might have been collected badly and therefore that you shouldn't rely on it or base decisions on it.

An exception would be if the information is from a reputable source which has nothing to gain by persuading you one way or another. For example, data from the Australian Bureau of Statistics is generally well collected and reliable.

Practice

Q1 For each of the following statements, comment on the data collection.

(a) According to the latest poll, 65% of Australians believe that the government's policy on climate change is not appropriate.

(b) In a survey of 250 motorists stopped at random and breathalysed on Stanley Boulevard, it was found that 6 had blood alcohol levels above the legal limit. This shows that 2.4% of the population drive while drunk.

(c) In the last general election, official figure show that only 42% of voters put the National Losers Party candidate down as their first preference.

(d) In a recent survey, 1270 men were called by phone and asked whether they had ever beaten their wife. 2 replied 'yes', indicating that 0.2% of men beat their wives.

Data Representations

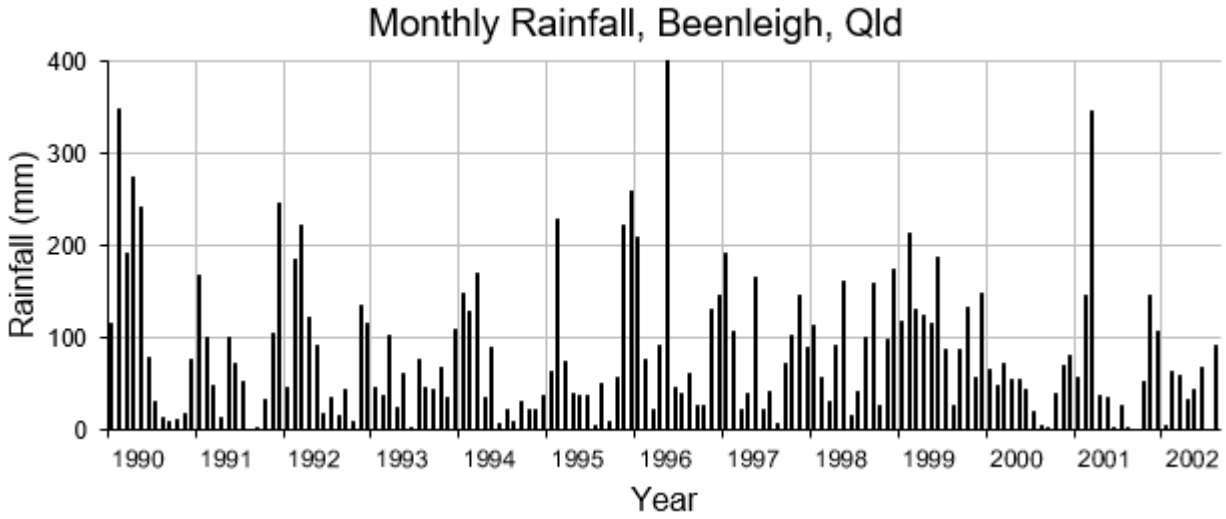
Data representations are tables, graphs etc. These can be well presented or poorly presented – or somewhere in between.

Data representations can be good or bad in terms of

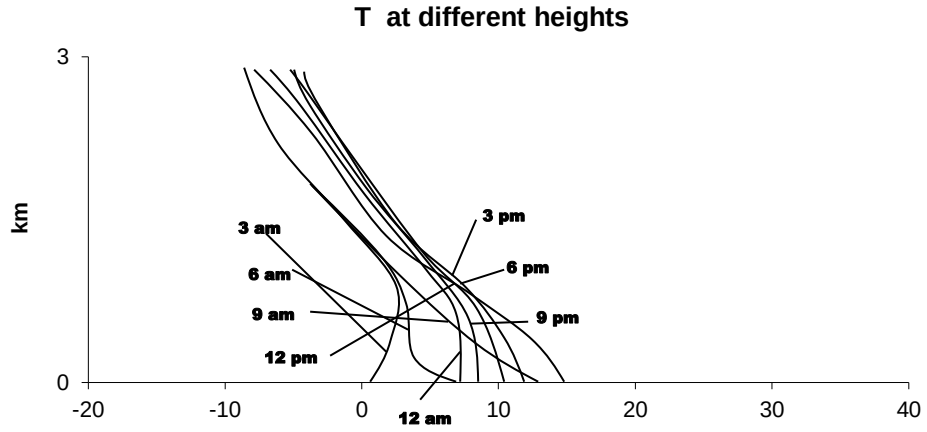
- how clear and easy to read they are (title, axis labels, units etc. are necessary for clarity);
- whether they give the right impression or a misleading impression.

Clarity

Data representations can be clear and easy to read like this:



or unclear and difficult to read like this:



This second graph is difficult to read for a number of reasons. See if you can list some. Then read on.

Here are some comments on the clarity.

1. The graph has a title, though it should have been written in full. It is unclear what T is. It was actually meant to be air temperature. There is also no indication of where or when this data is for. Admittedly, this could have been explained in accompanying text, but a good graph should be able to be read without reading the text.

2. The horizontal axis has no label. It is actually degrees Centigrade.

3. One might be able to guess that the vertical axis is height, though it is not labelled as such. The units are given, however, and this helps, though the scale is of little use.

4. It is not obvious what the different curves are, though one might guess that each is a graph of temperature against height above sea level at a different time of day.

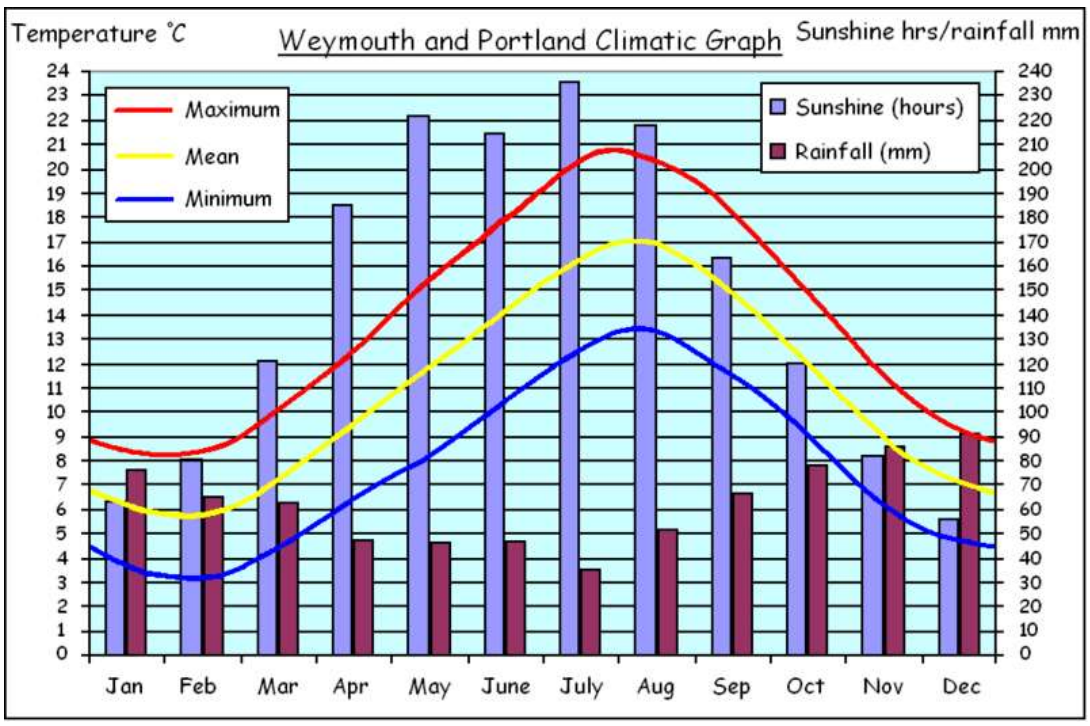
5. The time of day is indicated, though it is hard to see which time refers to which curve. This is partly because the pointer lines are the same thickness and colour as the curves, partly because they cross each other, and partly because some point to places where the curves are very close together, making it hard to see which curve it being pointed to.

You need to be able to point out the features that make data representations clear or unclear.

Practice

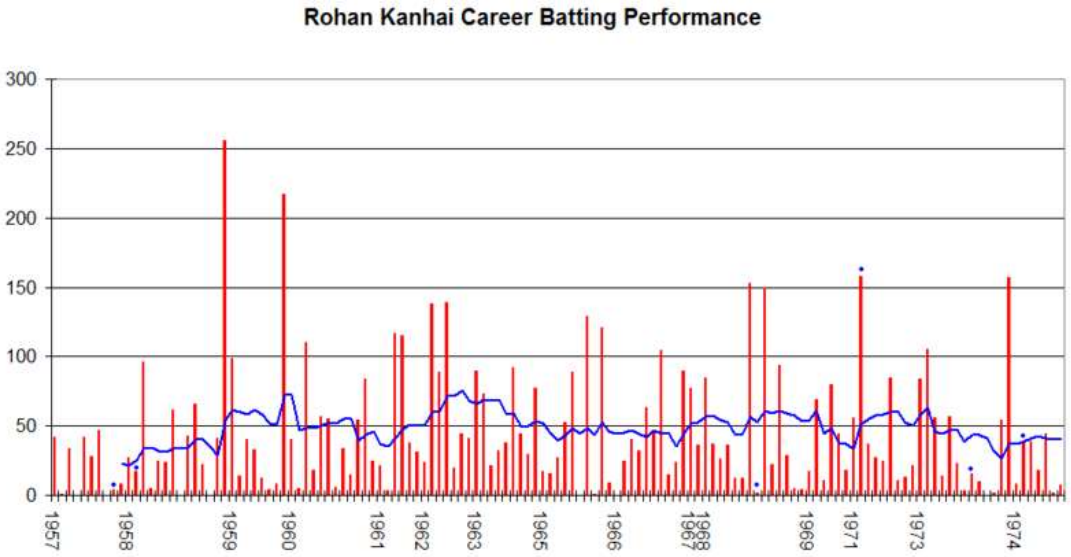
Q2 Comment on the clarity of each of the following graphs.

(a)



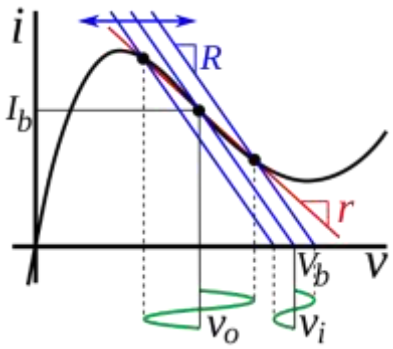
https://commons.wikimedia.org/wiki/File:Weymouth_Climatic_Graph.PNG

(b)



https://commons.wikimedia.org/wiki/File:Rohan_Kanhai_graph.png

(c)

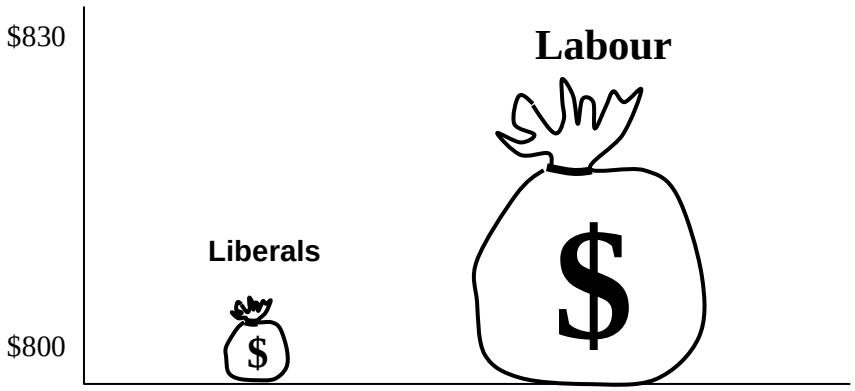


https://commons.wikimedia.org/wiki/File:Tunnel_diode_amplifier_graph.svg

Impression

Sometimes a graph can give a false or misleading impression if one does not look at it carefully. Sometimes this is because the graph was not well thought out. But sometimes it is a deliberate ploy to mislead. There are various ways to make a graph misleading. Here are a couple.

Average Weekly Wages under the Liberals and under Labour



When you read the graph carefully, you see that average weekly wages under the Liberals were about $810. Under Labour they were about $825.

But the first impression when looking at the graph is that people earned about 10 times as much under Labour.

Another more subtle point is that the Labour government may well have been a few years later than the Liberal government and inflation may have meant that you could buy less with the $825 anyway.

Height of the bean plant

First impressions are that the plant grew much faster from the time the fertilizer as applied. But this is just because the height scale changes after 150 cm.

The actual height each two days can be read off as:

Day	0	2	4	6	8	10	12	14
Height (cm)	70	80	100	140	150	158	165	177

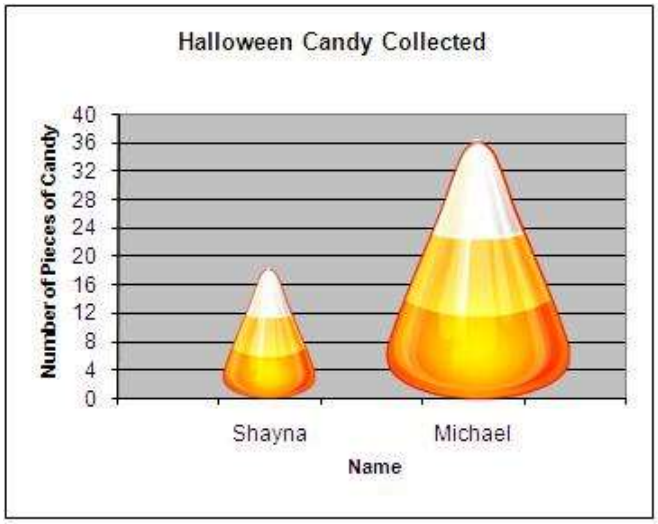
From the table, you can see that, if anything, the plant grew slower after the fertilizer was applied at Day 8.

To spot whether a graph is misleading in any way, you just need to look at it carefully to see what it is actually saying.

Practice

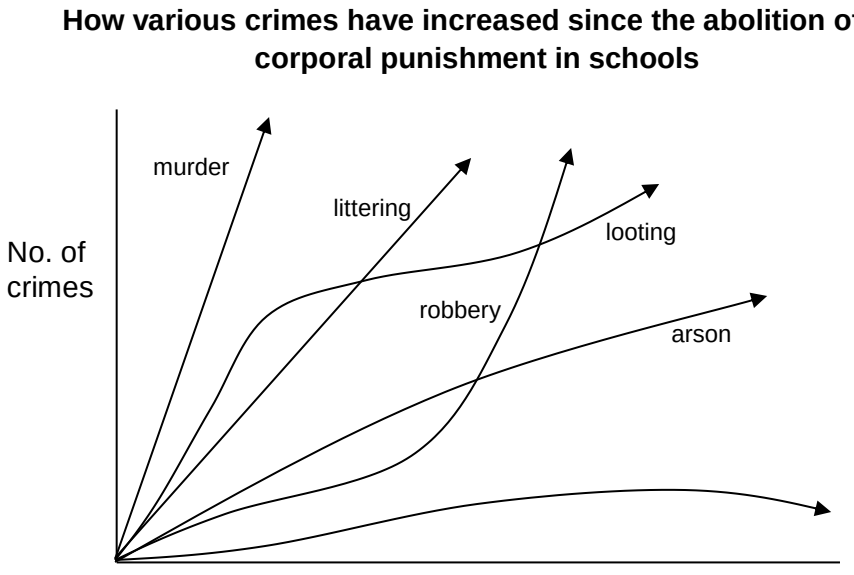
Q3 Comment on the extent to which the following graphs are designed to give the correct impression.

(a)

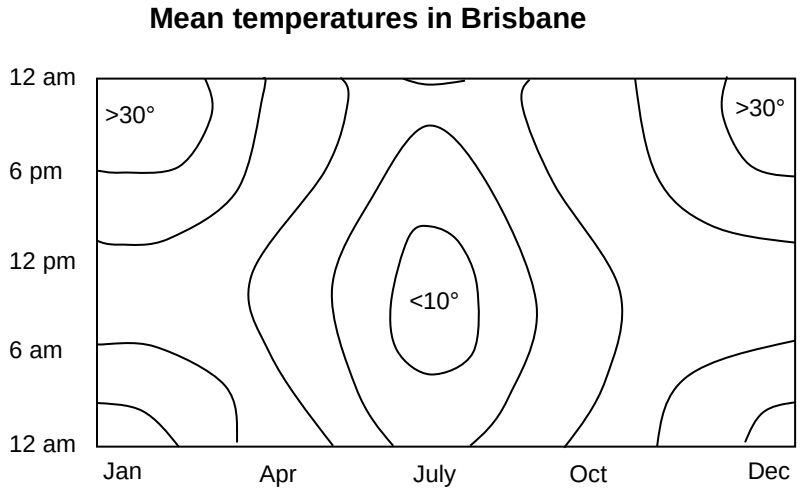


en.wikipedia.org

(b)



(c)



Conclusions Drawn from Data

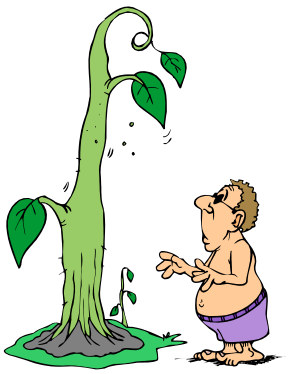
Conclusions drawn from data can be valid or invalid.

Conclusions drawn from data that isn't collected properly (e.g. too small a sample, biased sample or non-neutral questions) will be invalid. They could be correct, but you will not be able to tell whether they are or aren't from the data.

It is possible to draw false conclusions from a graph if you don't look at it carefully. Look for ways in which it might mislead, either by design or by careless preparation.

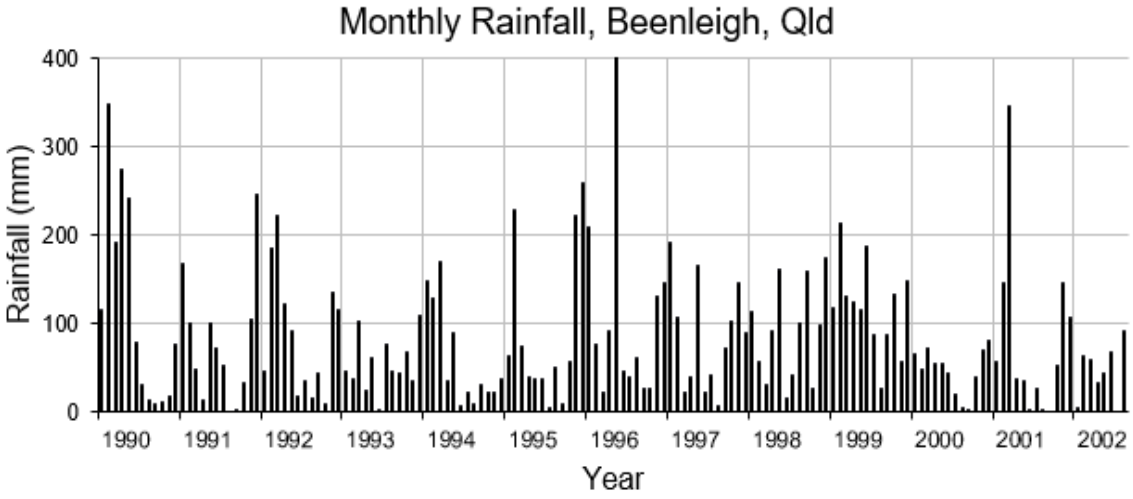
For example, if you concluded from the Grew-some graph above that the fertiliser makes plants grow faster, that would be an invalid conclusion. If you concluded that it made little difference or that it slowed them down slightly, that would be a valid conclusion.

Be particularly sceptical about claims made by someone with a financial interest in getting you to believe them. Check their conclusions carefully.



Practice

Q4 Based on this graph, say whether each of the following conclusions is valid and why.



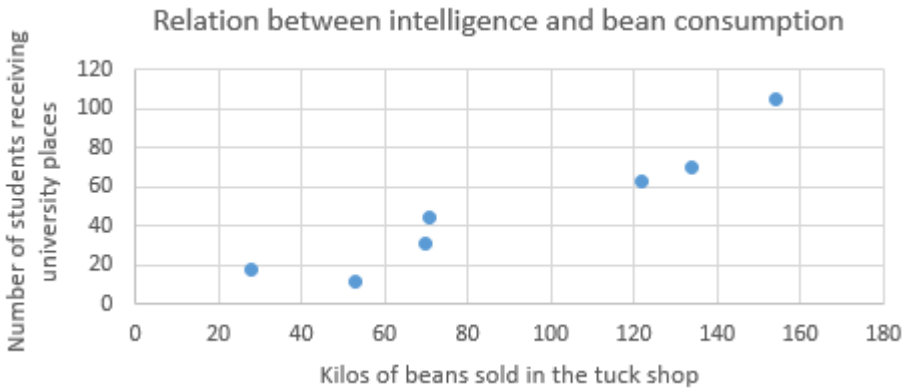
(a) In January 1998, about 110 mm of rain fell in Beenleigh.

(b) The wettest month in the 1990s in Beenleigh was in 1996.

(c) It rained in Beenleigh on 23rd May 1996.

(d) It would have rained in Beenleigh in May 2003.

Q5 Based on this graph of schools in the Avonmore District, say whether each of the following conclusions is valid and why.



(a) Schools that sold more beans tended to have more students get into university.

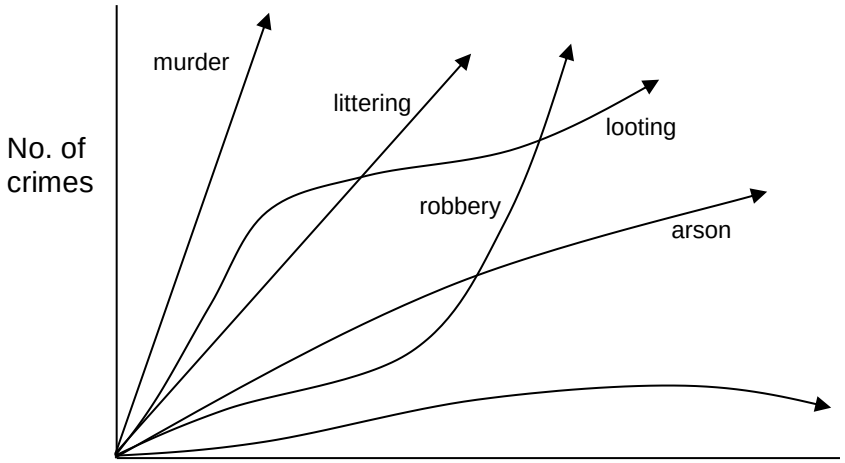
(b) Eating beans increases one's intelligence.

(c) Intelligent people tend to eat more beans.

(d) Eating beans doesn't affect one's intelligence.

Q6 Based on this graph for the Muggsville Police District, say whether each of the following conclusions is valid and why.

How various crimes have increased since the abolition of corporal punishment in schools



(a) The number of murders has increased since the abolition of corporal punishment in schools.

(b) The number of murders has more than doubled since the abolition of corporal punishment in schools.

(c) There are fewer cases of arson than of murder.

(d) The amount of litter on the ground will have increased since the abolition of corporal punishment in schools

Solve

Q51 If it were true that all dandelion flowers were yellow and 3 cm in diameter, and if Daisy found a yellow flower 3cm in diameter, could she validly conclude that it was a dandelion?

Q52 If it were true that all cats are furry and all furry things are cats, would it then be true that no rabbits are furry?

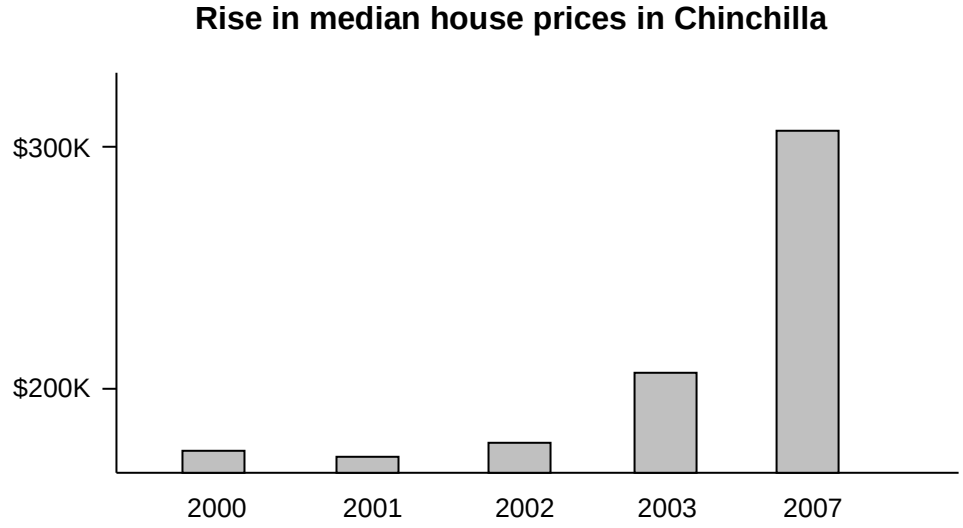
Revise

Revision Set 1

Q61 For the following graph, comment on

(a) its clarity and

(b) how well it gives the correct impression.



(c) Is it valid to conclude that prices more than doubled between 2003 and 2007?

Q62 An advertisement claimed that 'More people shop at Cottonballs'.

Comment on this statement, assuming this statement is based on some sort of survey.

Answers

Q1 (a) There is no indication of sample size, sample selection method or the question asked. So the statement is not very reliable.

(b) The sample size is stated and is probably sufficiently large; the selection method is clear. Instead of a question, there is a breathalyser test which is quite standard. So the data collection is fine. However, the conclusion might be dubious for various reasons.

(c) A general election is a census, so sampling isn't an issue. The result is probably totally reliable.

(d) The sample size is given and is quite adequate. The selection method is not clear. The question might well lead people not to give truthful answers, so the conclusion is probably not valid.

Q2 (a) The graph contains a lot of data, but if one takes the time to work out how to read it, all the necessary information is there and it is quite clear.

(b) This is a graph showing the batting performance of a famous cricketer. This may not be clear to someone who doesn't follow cricket and wasn't around in the 1960s and 70s. The

horizontal axis has no title, but one would probably assume (correctly) that it was years. The scale seems irregular, but one might conclude (validly) that each graduation represents one innings. The vertical axis has no title. It is the number of runs in each innings. The blue curve is a mystery.

(c) This graph is totally unclear. It gives no idea what it's about. Presumably this is explained in the accompanying text.

Q3 (a) According to the scale, Michael collected twice as much candy as Shayna. But representing their collections by what are clearly meant to be 3D shapes gives the impression that he collected 8 times as much.

(b) This graph is very unclear and quite possibly deliberately so in order to be misleading. In particular, there is no title on the horizontal axis and no scale on either axis.

(c) This graph is clear and does not give any misleading impression.

Q4 (a) This is valid as it is shown quite unambiguously on the graph.

(b) This is valid. The graph shows that May 1996 was the only month with 400 mm or more of rain.

(c) Not valid. Though it rained a lot that month and might well have rained that day, the graph does not show that.

(d) Not valid. The graph gives no information for 2003. As it did rain every May from 1990 to 2002, it quite probably would have rained in May 2003, but it may not have.

Q5 (a) This is valid.

(b) This is not valid. There is a correlation between the variable (one tends to be higher if the other is higher. But correlation does not necessarily mean that change in the independent variable cause changes in the dependent variable. More likely, the schools with more students sold more beans and got more students into university.

(c) Similarly, correlation does not necessarily show that changes in the dependent variable cause changes in the independent variable.

(d) Though this is what one might guess, the graph does give any evidence for this conclusion, thus the conclusion is not valid.

Q6 (a) This is quite probably valid, assuming that the horizontal axis is time since corporal punishment was abolished and the scale on the vertical axis increases upwards.

(b) This is not valid as there is no scale on the vertical axis to indicate the numbers.

(c) This is not valid. The graph suggests only that arson has increased less than murder.

(d) This is not valid. Although the suggestion is that littering has increased, it is possible that more people are now employed to pick the litter up.

Q51 No. Buttercups or other flowers could also be yellow and 3 cm in diameter.

Q52 This would be true. Unless rabbits were a type of cat, in which case, all rabbits would be furry.

Q61 (a) The graph is quite clear as long as we assume that the horizontal axis is the year and the vertical axis is the median house price.

(b) The graph gives the somewhat misleading impression of a very large rise between 2003 and 2007. This is achieved by having an irregular scale on the horizontal axis, jumping 1year, then 1 year, then 1 year, then 4 years, and also by having the vertical axis start at about the median prices for the earlier years.

(c) This conclusion is not valid. The scale shows that the rise was from $210 000 to $310 000.

Q62 The first question is 'More than what?' It could be that more people shop at Cottonballs than stick pins in their eyes. Even if the statement was based on a survey of shoppers' preferences, it gives no indication of how the survey was carried out – sample size, sample selection, questions etc.