M1 Maths

P6-8 Confidence Intervals for Means

- confidence intervals for means

Summary Learn Solve Revise Answers

Summary

Often we need to estimate the mean of some property a population by surveying a sample of the population. The most likely mean for the population will be the same as the mean found in the sample.

The range in which the population mean has a 95% probability of falling is called the 95% confidence interval. To find the confidence interval we divide the standard deviation of the sample by the square root of the sample size. Then we enter this as the standard deviation in the Inverse Normal function along with the sample mean, area of (1 minus half the confidence level required), and calculate with left and right tails.

Learn

Suppose we wanted to know the mean height of a certain breed of tomato plants 20 days after planting. We plant a sample of 160 seeds and measure them 20 days later.

Suppose that we find that the mean height is 47.2 cm and that the standard deviation is 4.8 cm.

Our best estimate for the mean is of course 47.2 cm. But, again, the mean height for all possible tomatoes of that breed might be a bit different from that, maybe 46 cm or 49.2 cm. We can find the confidence interval, the range of heights for which the probability that the true mean lies within that range is say 95%. In this case the confidence interval would be from 46.6 cm to 47.8 cm.

This is how we do it.

Firstly, we need to know that when many random samples of size n are chosen from a large population with standard deviation σ the means of the samples are distributed with population $\sigma/\sqrt{n}$. [The larger the samples, the less variation there will be in the sample means. This should make sense if you think about it.]

So the sample means will have a mean of 47.2 and a standard deviation of $4.8 \div \sqrt{260}$ = 0.298.

If the population has a normal distribution, the sample means will also be distributed normally. For reasonably large samples, the distribution of sample means will be approximately normal, even if the population distribution isn't normal. So we assume that the distribution of sample means is normal.

We then use the Inverse Normal distribution function on the calculator with

Tail	:Left
Area	:0.025
σ	:0.298
μ	:47.2

and we get the result 46.6, which is the lower limit of the confidence interval. Then we repeat with the right tail to get 47.8, which is the upper limit.

So the 95% confidence interval is 464.6 to 47.8 and there is a 95% probability that the population mean lies within this range.

Practice

Q1 A researcher wants to know the mean height of the adult males on Wotapi Island. She selects 122 men at random and measures them, finding that their mean height is 174.3 cm with a standard deviation of 5.1 cm. Find the likely mean height of the men on the island and the 95% confidence interval.

Q2 35 sample were taken of the ore being loaded onto a ship. The mean copper content was 12.47% and the standard deviation was 1.18%. Find the most likely copper content of the shipment and the 90% confidence interval.

Q3 An organisation wants to know the average quarterly electricity bill for Australian households. They survey 300 households chosen at random and finds that their bills average $432.85 with a standard deviation of $44.58. Find the likely mean bill for the nation and the 99% confidence interval.

Solve

Q51

Revise

Revision Set 1

Q61 Bottles of soft drink are labelled as containing 600 mL. 80 bottles are sampled and found to have a mean of 598.4 mL and a standard deviation of 7.2 mL. Find the likely mean contents of that type of bottle and the 90% confidence interval.

Comment on the result.

Answers

Q1 174.3 cm, 173.4 to 175.2 cm
Q2 12.47%, 12.14% to 12.80%
Q3 $432.85, $426.37 to $439.33

Q51

Q61 598.4 mL, 597.1 mL to 599.7 mL.
It seems very likely that the bottles are under-filled on average.