M1 Maths

P6-2 Conditional Probability

- conditional probability

Summary Learn Solve Revise Answers

Summary

Probabilities related to dependent events can be worked out using techniques of conditional probability.

Learn

Conditional Probability – Equally Likely Outcomes

In Module P2-1 (Compound Events) we looked at dependent and independent events. If two events are independent, then whether or not one of them happens doesn't affect the probability of the other happening. For instance, if you toss a coin and roll a die, the probability of getting a 5 on the die is the same whether or not the coin comes down heads.

However, if two events are dependent, then the probability of one happening depends on whether the other happens. For example, suppose you roll 2 dice and look at the total. If the red die comes down 6, then the probability of getting a total of 9 is $^1/_6$. However, if the red die comes down 2, then the probability of getting a total of 9 is 0. We say that the event 'getting a 6 on the red die' and the event 'getting a total of 9' are dependent'.

That the probability of getting a total of 9 is affected by the outcome of the red die is fairly obvious. But when two events are dependent, the dependency goes both ways. So the probability that the red die comes down 6 is different if we know that the total was 9 from what it would be if the total was 5. Fairly obviously, if the total was 5, then it is not possible that the red die came down 6. But if the total was 9, then it is possible.

We say that the probabilities of dependent events are conditional. We might want the probability that we get a total of 9, given the condition that the red die came down 6. Or we might want to know the probability that the red die came down 6, given the condition that we got a total of 9.

Working out the probability of getting a total of 9 if the red die came down 6 is quite easy. The blue die has to come down 3 and the probability of that is $^1/_6$.

The other way is a bit more involved: if the total was 9, what is the probability that the red die came down 6?

What we have to do is see how many outcomes for the red and blue dice give 9 and how many of those have a 6 on the red die. We can list the outcomes or use a table. Listing looks like this if we present the outcomes as ordered pairs (red, blue): (6, 3), (5, 4), (4, 3), (3, 6).

We can see that 1 outcome out of 4 has a 6 on the red die. So the probability that the red die came down 6, given that the total was 9 is $^1/_4$.

If we used a table, it might look like this:

Red

Blue

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

We can see that, of the 4 possible outcomes, 1 has a 6 on the red die. So the probability is $^1/_4$.

Practice

Q1 You roll a red die and a blue die. Find:

(a) the probability of getting a total of 8, given that the red die came down 4

(b) the probability of getting a total of 8, given that the red die came down 1

(c) the probability of getting a total of 5, given that the red die came down 3

(d) the probability of getting a total of 5, given that the red die came down 5

(e) the probability that the red die came down 4, given that the total was 10

(f) the probability that the red die came down 4, given that the total was 7

(g) the probability that the red die came down 4, given that the total was 5

(h) the probability that the blue die came down 4, given that the total was 3

If there are a large number of possible outcomes, then lists and tables can become very cumbersome. Suppose you throw a dart at random at a board with the numbers 1 to 100 on it and score the number it lands on; then you throw it again and add your two scores to get a total.

Suppose too that we wanted the probability that the first dart scored 44, given that the total was 79. In this case, it is easier just to calculate the numbers of outcomes involved. If the total was 79, the first dart could have scored anything from 1 to 78. One of these is 44, so the probability that the first dart scored 44 is $^1/_{78}$.

If we wanted the probability that the first dart scored 51, given that the total was 125, the first dart could have scored anything from 26 to 100, 75 possibilities. One of these is 51, so the probability that the first dart scored 51 is $^1/_{75}$.

Practice

Q2 In the game with the dart and the hundred board above, find:

(a) the probability that the first dart scored 41, given that the total was 60

(b) the probability that the first dart scored 41, given that the total was 130

(c) the probability that the second dart scored 98, given that the total was 104

(d) the probability that the second dart scored 92, given that the total was 188

(e) the probability that the first dart scored 32, given that the total was 168

Conditional Probability – Unequally Likely Outcomes

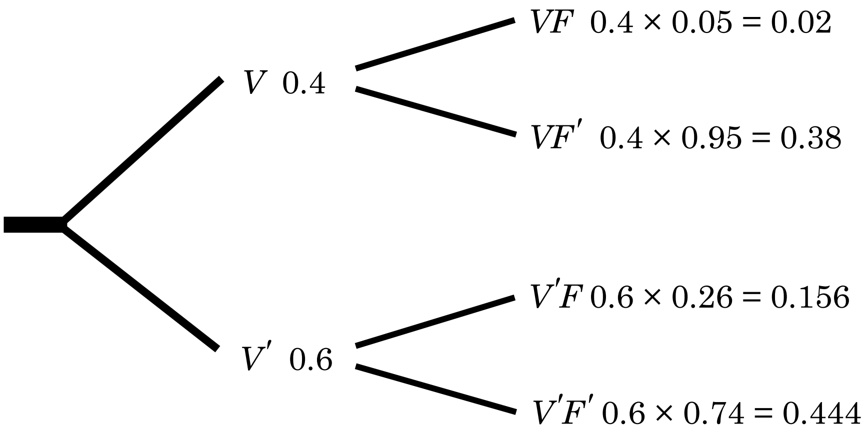
In the above conditional probability examples, all the outcomes in the experiment were equally likely. For example, the probability of getting any score with the dart is the same as any other score. A different approach is used where outcomes are not equally likely.

Let's say that, without vaccination, the probability of getting the flu in any particular year is 26%. In other words, 26% of unvaccinated people will get the flu. Let's also say that those who are vaccinated have only a 5% chance. Let's also say that 40% of people get vaccinated.

Suppose someone gets the flu. What is the probability that they were vaccinated?

One way to answer this is with a tree diagram. The first branching is whether people are vaccinated or not; the second branching is whether they get the flu or not.

The tree diagram will look like this. [V means *vaccinated*, V' means *not vaccinated*. F means *got the flu*, F' means *didn't get the flu*.]



The proportion who get the flu is 0.02 + 0.0156 = 0.0176. Of those, 0.02 were vaccinated. The proportion of those getting the flu who were vaccinated is therefore 0.02 ÷ 0.176 = 11.4%.

So the probability that a person who gets the flu was vaccinated is 11.4%.

Practice

Q3 Suppose that 55% of people get the flu vaccination. Suppose that a vaccinated person has a 4% chance of getting the flu, while an unvaccinated person has a 20% chance.

(a) If Edith gets the flu, what is the probability that she was vaccinated

(b) If Ezra doesn't get the flu, what is the probability that he was vaccinated?

Q4 Suppose that 30% of those in a refugee camp have no food. Also a person with no food is 20% likely to steal some, while a person with food is only 4% likely.

(a) If someone steals food, what is the probability that they had none?

(b) If someone doesn't steal food, what is the probability that they had none?

There is a technique called Bayes' Theorem which can be used instead of tree diagrams for the same types of problems. It can make the job slightly quicker, but it is harder to get one's head around. For most people, the tree diagrams are easiest.

Bayes theorem states $P(A\,|\,B) = \frac{P(B|A)}{P(B)} \times P\,(A)$, where $A\,|\,B$ means A, given B.

In the context of flu vaccination, this would be $P(V\,|\,F) = \frac{P(F|V)}{P(F)} \times P(V)$.

Solve

Q51 If it rains one day, the probability that it will rain the next day is 0.6. If it doesn't rain one day, the probability that it will rain the next is 0.2. If it rains Wednesday,

(a) what is the probability that it rained Tuesday?

(b) what is the probability that it rained Monday?

Revise

Revision Set 1

Q61 The score on two dice is 8. What is the probability that the red die gave 3?

Q62 Rupert is equally likely to get any whole number of marks on a test from 0 to 40. His total for the Maths and Swahili tests was 61. What is the probability that he got 40 on the maths test?

Q63 In jails in the 19^{th} Century, 30% of those put into solitary confinement went mad. 8% of the others were mad. If 20% of prisoners had been in solitary confinement, what is the probability that a given loony prisoner had been in solitary?

Answers

Q1	(a) $^1/_6$	(b) 0	(c) $^1/_6$	(d) 0	
	(e) $^1/_3$	(f) $^1/_6$	(g) ¼	(h) 0	
Q2	(a) $^1/_{95}$	(b) $^1/_{70}$	(c) $^1/_{96}$	(d) $^1/_{12}$	(e) $^1/_{32}$
Q3	(a) 0.196	(b) 0.595			
Q4	(a) 0.682	(b) 0.263			
Q51	(a) $^3/_4$	(b) $^9/_{13}$			
Q61	$^1/_5$				
Q62	$^1/_{20}$				
Q63	48%				