M1 Maths

P6-1 Sets

- set terminology and notation

Summary Learn Solve Revise Answers

Summary

In mathematics, a set is a collection of distinct objects called the elements of the set. There is a terminology and symbolism associated with sets.

In probability, outcomes, events and sample spaces can be thought of in terms of sets.

Learn

Set Terminology

In mathematics, a **set** is a collection of distinct objects. By distinct, we mean that you can count them – or at least start to count them – there may be too many to count them all. You can have a set of people, a set of cakes, a set of numbers etc.

But you can't have a set of water or a set of space because you can't count water or space. You could, however, have a set of water molecules.

The numbers 1, 2, 3, 4, 5 and 6 are a set. We say that 1, 2, 3, 4, 5 and 6 are **elements** of the set. A set can be given a name. We might call this set T. Then 2 is an element of T. We can write this in shorthand as $2 \in T$. The symbol $\in$ looks a bit like an E for element. It is pronounced 'is an element of'.

If we wish to list the elements in a set, we can do it by listing the elements inside curly brackets like this: $T = \{1, 2, 3, 4, 5, 6\}$.

T has a finite number of elements – 6 to be precise. It is a finite set. The number of elements in a set is called its **cardinality**. So T has cardinality 6. A set can have an infinite number of elements. It is then an infinite set. The set of whole numbers is an infinite set. $W = \{0, 1, 2, 3, \ldots\}$. A set can have just one element, e.g. $\{8\}$. It can also have no elements. It is then called the empty set which is written in shorthand as $\varnothing$. $\varnothing = \{\,\}$. Its cardinality is 0.

Sets of Teachers

Everyone knows that a group of lions is called a pride. A pride of lions is a set.

Most people also know that a group of crows is a murder. Fewer know that a group of teachers is a quiz and even fewer know the collective nouns for groups of specific types of teacher. Here are some examples:

A matrix of maths teachers
A muscle of Phys Ed teachers
A symphony of music teachers
A eureka of science teachers
An infestation of biology teachers
An eruption of geology teachers
A legion of Latin teachers
An economy of business teachers
A sentence of English teachers
A palette of art teachers
A cast of drama teachers
A force of physics teachers
A compound of chemistry teachers
A constellation of astronomy teachers
A roast of cooking teachers
A ponder of philosophy teachers

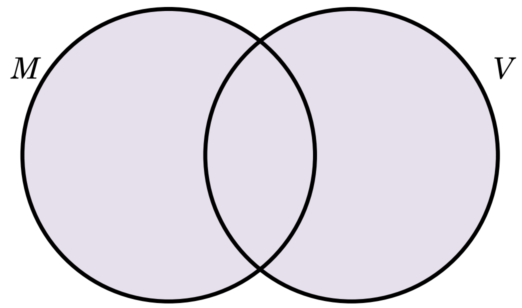
One set, B, is a **subset** of another set, S, if all the elements of B are also elements of S. For example, the set B of boys in class 9D is a subset of the set S of students in class 9D. This can be written as $B \subseteq S$ (a bit like $6 \leq 8$).

Even if there were no girls in the class, we can still say $B \subseteq S$ because all the elements of B are also elements of S. We could also say $B = S$ because they contain exactly the same elements. If there are some girls in the class and the set of boys is smaller than the set of students, then we can also say that B is a **proper subset** of S, written $B \subset S$ (compare $<$).

Suppose we have a set M of all the students who play a musical instrument in 9D and a set V of all the students who play volleyball in 9D. If we put all these students together, they would form another set, which we might call R. R is said to be the **union** of M and V: it consists of all the students who are in either or both of M and V. The shorthand for the union of M and V is $M \cup V$ (the symbol looks a bit like a U for union). We say $R = M \cup V$, pronounced 'M union V'.

On a Venn diagram, the union of M and V would be the shaded area below.

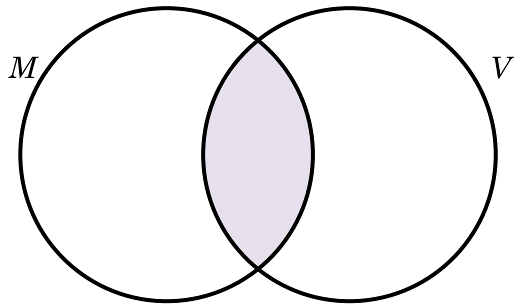
$M \cup V$



Some of the students in R will probably be in both M and V. The set of those which are in both is called the **intersection** of M and V, written $M \cap V$ and pronounced M intersection V.

On a Venn diagram, $M \cap V$ would be the shaded area in the diagram below. It is called the intersection because it is the intersection of the two circles.

$M \cap V$



Practice

Q1 Let A be the set {dog, cat, pig}

Let B be the set {dog, cat, pig, goat}

Let C be the set {goat, crow, cow}

Let D be the set {dog, cat, pig, goat, crow, cow}

Write T or F (true or false) for the following statements:

(a) dog $\in A$ (b) cat $\in C$ (c) $C \subseteq B$

(d) $A \subseteq B$ (e) $A \subset B$ (f) $A \cup C = D$

(g) $B \cup C = D$ (h) $A \cap C = B$ (i) $B \cap C = \{\text{goat}\}$

(j) $A \cap C = \varnothing$ (k) $A \in B$ (l) $C \in$ cow

(m) $A \cap B = A$ (n) $A \cup B = D$ (o) $A \cup B = B$

Check your answers. If you get any wrong and can't see why, go back and read the definitions carefully. Be careful to note whether each symbol in the question represents a set or an element. A set has a name or is a list of elements in curly brackets.

Q2 Let N be the set of natural numbers. So $N = \{1, 2, 3, 4, \ldots\}$

Let W be the set of whole numbers. So $W = \{0, 1, 2, 3, 4, \ldots\}$

Let Q be the set of rational numbers and let R be the set of real numbers. (If you can't remember what these are, refer to Module N2-1.)

Write T or F (true or false) for the following statements:

(a) $4 \in N$ (b) $-1 \in R$ (c) $2.5 \in W$

(d) $N \subseteq W$ (e) $N \subset W$ (f) $N = W$

(g) $N \in W$ (h) $W = N \cup 0$ (i) $W = N \cup \{0\}$

(j) $W = N \cup \varnothing$ (k) $Q = Q \cup \varnothing$ (l) $N \cap R = N$

(m) $Q \subset R$ (n) $0.3 \subset R$ (o) $2 \subset 3$

Again, check your answers. If you get any wrong, think about them carefully until you can see why.

Complement of a Set

If V is the set of volleyballers in 9D, we sometimes call the set of non-volleyballers V' (pronounced *V dash*) or $\bar{V}$. We will use V'. V' is said to be the **complement** of V. Similarly we can indicate the complement of any set with a dash.

But the complement of a set doesn't generally mean everything else in the universe. V' is meant to be the set of non-volleyballers in 9D. So, when defining the complement of a set, it must be in the context of some **universal set**. In this case, the universal set is the students in 9D. What's in the universal set depends on the context and is generally taken to be all elements relevant to the situation under consideration. All the students in 9D are relevant here, but the students in 4A probably aren't. Nor is Cuthbert's weasel or Grandma's kitchen sink.

The universal set is generally denoted U.

When rolling a die, we might define A as $\{5, 6\}$. In that case, we would probably take A' to be $\{1, 2, 3, 4\}$. We would be assuming that $U = \{1, 2, 3, 4, 5, 6\}$.

When dealing with numbers, we might call the even natural numbers E. Then $E = \{2, 4, 6, 8, \ldots\}$. E' would then be the odd natural numbers, $E' = \{1, 3, 5, 7, \ldots\}$ and U would be all the natural numbers: $U = \{1, 2, 3, 4, 5, \ldots\}$

Probability in terms of Sets

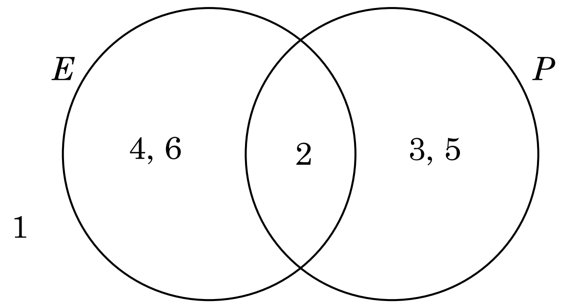
If you roll a die, you might be interested in 2 outcomes: it rolls off the table; it stays on the table.

More often, though, we are interested in the number that comes up. In this case, there would be 6 outcomes: 1, 2, 3, 4, 5, 6. These outcomes form a set called the sample space. The sample space if often denoted by the Greek letter ε (epsilon – a Greek e). So we can say $\varepsilon = \{1, 2, 3, 4, 5, 6\}$.

We might also be interested in the event ‘getting an even number’. We could call the set of outcomes in that event E. So $E = \{2, 4, 6\}$. The complement of E, E' is all the other outcomes in the sample space. So $E' = \{1, 3, 5\}$. Note that we generally take the sample space as our universal set, U.

We might also be interested in the event of getting a prime number. Let’s call this set of outcomes P. $P = \{2, 3, 5\}$.

Then the set of numbers which are even and prime is $E \cap P$, which is {2}; and the set of numbers which are even and/or prime is $E \cup P$, which is {2, 3, 4, 5, 6}.



Set Theory

The mathematics of sets is called set theory. We have just given a very brief introduction here. However, set theory is one of the most fundamental ideas in mathematics and is a major part of the fundamental logical foundation of all mathematical knowledge.

Solve

Q51 If $A \subseteq B$, $B \subseteq C$ and $C \subseteq A$, is it impossible, possible or certain that $A \cup B \cup C = A \cap B \cap C$? Why?

Q52 If A, B and C are sets and $A \in B$ and $B \in C$, is it impossible, possible or certain that $A \in C$? Why?

Revise

Revision Set 1

Q61 Let N be the set of natural numbers, W be the set of whole numbers, Z be the set of integers and R be the set of real numbers. Write T or F (true or false) for the following statements:

(a) $1.4 \in R$	(b) $N \in Z$	(c) $Z \subseteq W$
(d) $N \subset Z$	(e) $W = N \cup \{0\}$	(f) $W = N \cup \varnothing$
(g) $N \cap Z = N$	(h) $(5, 5.6, 0\} \subset Z$	(i) $\{4,5\} \subseteq R$

Answers

Q1	(a) T	(b) F	(c) F	(d) T	(e) T
	(f) T	(g) T	(h) F	(i) T	(j) T
	(k) F	(l) F	(m) T	(n) F	(o) T
Q2	(a) T	(b) T	(c) F	(d) T	(e) T
	(f) F	(g) F	(h) F	(i) T	(j) F
	(k) T	(l) T	(m) T	(n) F	(o) F

Q51 Certain. The three sets must have exactly the same elements

Q52 Possible. Set C could contain set B as one element and set A as another.

Q61	(a) T	(b) F	(c) F
	(d) T	(e) T	(f) F
	(g) T	(h) F	(i) T