M1 Maths

P4-1 Complex Probabilities

- using the addition and multiplication rules in combination

Summary Learn Solve Revise Answers

Summary

The addition rule and the multiplication rule can be used in combination to solve problems like finding the probability of ‘A and B happening or C and D happening’. The procedures are basically common sense.

The types of problems that can be solved are basically the ones that can be solved with two-way tables and tree diagrams. But where there are many outcomes or many stages, the table and tree diagram methods bean become very cumbersome. Just using the addition and multiplication rules can be very much quicker.

Once you master this, you will rarely need to use tables or tree diagrams.

Learn

In Modules P2-2 and P3-1 we met two-way tables and tree diagrams as methods for finding probabilities that involve the addition and multiplication rules.

But two-way tables are very limited in the situations they can handle and both two-way tables and tree diagrams can be very tedious and time consuming, particularly if there are a lot of possible outcomes or a lot of stages.

It is possible to use the addition and multiplication rules without diagrams. It is a little more difficult conceptually, but can be very much quicker. Once you master doing this, you probably won’t use two-way tables or tree diagrams much more.

Using the multiplication and addition rules together

We roll two dice, a blue one and a red one. We want to know the probability that we will get a double 5 or a double 6.

To get a double 5 we need a 5 on the blue die **AND** a 5 on the red die. The probability of each of these is $^1/_6$. So, using the multiplication rule, the probability of getting a double 5 is $^1/_6 \times {}^1/_6$ or $^1/_{36}$. Similarly, the probability of getting a double 6 is $^1/_{36}$.

Using the addition rule,

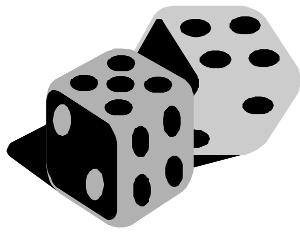
P(double 5 **OR** double 6) = P(double 5) + P(double 6) – P(double 5 **AND** double 6)

$= \frac{1}{36} + \frac{1}{36} - 0$

$= \frac{2}{36}$.

Here we have used the addition rule and the multiplication rule together. This idea should be common sense and a bit of practice will make you good at it.

Here is another example, explained a bit differently.



Suppose we wanted to find the probability of getting a total of 9 when we roll two normal 6-sided dice.

We know that we can get a total of 9 by getting

a 3 on the first die AND a 6 on the second

OR a 4 on the first die AND a 5 on the second

OR a 5 on the first die AND a 4 on the second

OR a 6 on the first die AND a 3 on the second

Using the multiplication rule, we can find the probability of the event on each of these four lines. Each is $\frac{1}{6} \times \frac{1}{6}$, which is $\frac{1}{36}$.

Then, using the addition rules, we can find the probability of getting any one of the four events. It is $\frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36}$, which is $\frac{4}{36}$ or $\frac{1}{9}$.

So the probability of getting a total of 9 is $\frac{1}{9}$.

And one more example. The Smith family and the Jones family each has 3 children. What is the probability that the Smiths have 1 boy and 2 girls and the Jones have 1 girl and 2 boys.

Let B be the event that a child is a boy; let G be the event that a child is a girl.

First, we need to work out the probability that the Smiths have 1 boy and 2 girls. This is P(BGG or GBG or GGB), which is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$.

Then we need to work out the probability that the Jones have 1 girl and 2 boys. This is P(GBB or BGB or BBG), which is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$.

Then to find the probability that the Smiths have 1 boy and 2 girls AND the Jones have 1 girl and 2 boys, we multiply these probabilities: $\frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$.

This combination of multiplying and adding allows us to solve a vast range of probability problems. Here are a few.

Practice

Q1 If you roll two 6-sided dice, what is the probability of getting a total of
(a) 12 (b) 11 (c) 10 (d) 7 (e) 4

Q2 If you roll 2 four-sided dice each numbered 1 to 4, find the probability of getting a total of
(a) 2 (b) 3 (c) 4

Q3 You toss three coins. Find the probability that
(a) all come down heads
(b) the first two are heads and the last is a tail
(c) two are heads and one is a tail

Q4 A bag contains 2 red balls and 3 green balls. You pick two without replacement. Find the probability that
(a) both are red
(b) you get one of each colour

Q5 You have the A to 10 of spades. You shuffle and deal yourself a hand of two cards. Find the probability that the hand contains
(a) the A and 2 (be careful – the answer isn't $^1/_{90}$ – think of the first and second cards)
(b) the A and 2 or the 8 and 9
(c) two successive numbers, e.g. 5,6 or A,2

Q6 You have the A to 10 of hearts. You shuffle and deal yourself a hand of three cards. Find the probability that the hand contains
(a) the A, 2 and 3 (again, be careful)
(b) the A, 2 and 3 or the 8, 9 and 10
(c) three successive numbers, e.g. 5,6,7 or A,2,3

Those practice questions were fairly straightforward and the method to use in each was fairly obvious. With many problems, though, it is not so obvious how to go about solving them. Many can be thought of and worked out in more than one way. Often, calculating the probability of the complement of the event we are interested in is quicker and easy than calculating the probability of the event directly.

Sometimes, the required event can occur in a number of ways which need to be counted before their probabilities are added.

Here is an example. You have a pack of 52 cards and you deal yourself 3. What is the probability that exactly 2 of them are aces?

Let A be the event that a card is an ace; let N be the event that it is not.

The sequence could be AAN, ANA or NAA.

P(AAN) = $^4/_{52} \times {}^3/_{51} \times {}^{48}/_{50} = {}^{24}/_{5525}$

$P(ANA) = {}^{4}/_{52} \times {}^{48}/_{51} \times {}^{3}/_{50} = {}^{24}/_{5525}$

$P(NAA) = {}^{48}/_{52} \times {}^{4}/_{51} \times {}^{3}/_{50} = {}^{24}/_{5525}$

P(exactly 2 aces) = $P(AAN) + P(ANA) + P(NAA) = {}^{24}/_{5525} \times 3 = {}^{72}/_{5525}$

Practice

Q7 You roll two dice, a blue one and a red one. Find the probability that:

(a) the blue gives a 6
(b) the blue one doesn't give a 6
(c) both give a 6
(d) both give a 6 or both give a 5
(e) both give a 5 or 6
(f) you get a double 1, double 2 or double 3
(g) both give an even number
(h) the blue one gives an even number and the red one gives a 4
(i) the blue one gives a 1 and the red one doesn't
(j) one gives a 5 and the other gives a 6
(k) they give a total of 11
(l) they give a total of 10

Q8 A bag contains 5 black balls, 4 white ones and a red one. A ball is taken and returned, then another ball is taken. Find the probability that:

(a) the first is red and the second white
(b) both are black
(c) neither is black
(d) one is black and one is white
(e) they are different colours

Q9 A bag contains 2 black balls, 2 white balls, 2 red balls, 2 blue balls and 2 green balls. Two balls are picked without replacement. What is the probability that they will be different colours? [Hint: working out the complement first would be easier.]

Q10 A bag contains 5 black balls and 4 white ones and a red one. Two balls are taken out without replacement. Find the probability that:

(a) the first is red and the second black
(b) both are white
(c) neither is white
(d) one is black and one is red
(e) they are the same colour

Q11 A pack of 52 cards is shuffled and two cards are dealt. Find the probability that:

(a) both are aces
(b) both are diamonds
(c) the first is a diamond, the second a club
(d) one is a diamond, the other a club
(e) one is a diamond and one a club or they are both diamonds
(f) both are aces or both are hearts
(g) both are the same suit
(h) they are different suits
(i) both are aces or they are different suits
(j) one is an ace or 2 and the other is a picture card (J, Q or K)
(k) they are the same suit or the same number (count A, K, Q, J as numbers)
(l) they are adjacent numbers between 3 and 7 inclusive

Q12 A couple have three children. Assuming that P(boy) = P(girl) = ½, Find the probability that:

(a) the first is a boy
(b) the second is a girl, given that the first is a girl
(c) the first two are different sexes
(d) the first two are different sexes and the third is a girl
(e) two are girls, the other is a boy
(f) there are more girls than boys

Q13 The Smiths and the Jones each have three children. Find the probability that:

(a) one family has all boys, the other all girls
(b) both families have one boy and two girls
(c) both families have the same number of boys
(d) the Smiths have 2 or 3 girls and the Jones have 1 girl
(e) between them they have 3 boys and 3 girls
(f) the Jones have one more boy than the Smiths
(g) the Smiths have more girls than the Jones

Q14 You toss 4 coins, a 5c, a 10c, a 20c and a 50c. Find the probability that:

(a) none come down heads
(b) just the 5c and 10c come down heads
(c) the 5c and 10c come down heads and the others tails or the 5c and 50c come down heads and the others tails
(d) just one comes down heads
(e) all four come down heads
(f) exactly three come down heads
(g) exactly two come down heads.

Q15 You have the A to 10 of hearts. You shuffle and deal yourself a hand of three cards. Find the probability that the hand contains

(a) the A, 2 and 3

(b) the A, 2 and 3 or the 8, 9 and 10

(c) three successive numbers, e.g. 5,6,7 or A,2,3

Q16 A 5-card poker hand is dealt from a pack of 52 cards. What is the probability that:

(a) it will contain 4 aces

(b) it will contain 4 of a kind (e.g. 4 kings, 4 sevens etc.)

(c) all the cards will be spades

(d) all the cards will be the same suit

Q17 Jenny has a 60% chance of beating Mandril in a game of snap. What is the probability that Jenny will win a competition consisting of 3 games?

Q18 If you toss 9 coins, what is the probability that

(a) you will get no heads

(b) you will get one or more heads

(c) you will get at least one tail

(d) you will get more heads than tails

Solve

Q51 A peasant is found guilty of spitting on the king's palace and is sentenced to death. But the king, being a sporting fellow, gives the peasant a chance. He gives the peasant 2 cloth bags, 8 black beads and 8 white beads. He tells the peasant to distribute the beads between the bags how he likes as long as there is at least one bead in each bag. Then the king will pick a bag at random and pick a bead at random from that bag. If the bead is white, the peasant goes free, if it is black, the peasant dies. How should the peasant arrange the beads to give himself the best chance of surviving and, if he does this, what is the probability that he will survive?

Q52 In a mini-lotto game, players have to choose 3 numbers from 1 to 10. Then 10 balls numbered 1 to 10 are mixed and 3 are chosen. You win if your 3 numbers are the same as the 3 chosen (the order doesn't matter). What is the probability of winning?

Q53 In Gold Lotto there are 45 balls and 6 are selected. In a single game you choose 6 numbers. To win (first division), you have to have all 6 correct (the order doesn't matter).

(a) If you pick 6 numbers, what is the probability that you will get the right six?

(b) If you guess 10 different sets of 6 numbers, what is the probability one of your guesses will be right?

(c) If you choose 7 numbers, what is the probability that they will include the 6 correct ones?

Q54 Each packet of Crunchy-Bug breakfast cereal contains, in addition to the edible bugs, a large luminous plastic insect. There are 6 types – a cockroach, a beetle, a dragonfly, a moth, a silverfish and a blowfly. What would be the average number of packets you would need to buy before you got the whole set?

Q55 In a quiz show, if the contestant answers a set of questions correctly, she gets to choose from 3 boxes. One contains a token for a car, the other two are empty. She makes her selection, then, before opening the box, the quizmaster opens one of the other boxes to show that it is empty. He then gives the contestant the chance to change her mind and pick a different box. Should the contestant change her mind, should she keep with her original choice or does it make no difference to the chance of winning? Explain.

Revise

Revision Set 1

Q61 Show that the probability of getting an 8 when you roll two 6-sided dice is $^5/_{36}$

Q62 A bag contains 5 black balls and 2 white balls. Two balls are taken without replacement. Find the probability that
(a) both are black
(b) they are different colours

Q63 10 cards are numbered 1 to 10. They are shuffled and three are dealt. Find the probability that
(a) two are even numbers, one is odd
(b) their sum is 6

Q64 A bag contains 4 black balls, 3 white balls and a red ball. Three are taken without replacement. Find the probability that
(a) they are all the same colour
(b) they are three different colours

Answers

Q1	(a) $^1/_{36}$	(b) $^2/_{36}$	(c) $^3/_{36}$	(d) $^6/_{36}$	(e) $^3/_{36}$	
Q2	(a) $^1/_{16}$	(b) $^2/_{16}$	(c) $^3/_{16}$			
Q3	(a) $^1/_8$	(b) $^1/_8$	(c) $^3/_8$			
Q4	(a) $^1/_{10}$	(b) $^3/_5$				
Q5	(a) $^1/_{45}$	(b) $^2/_{45}$	(c) $^1/_5$			
Q6	(a) $^6/_{720}$	(b) $^{12}/_{720}$	(c) $^{48}/_{720}$			
Q7	(a) $^1/_6$	(b) $^5/_6$	(c) $^1/_{36}$	(d) $^2/_{36}$	(e) $^1/_9$	(f) $^3/_{36}$
	(g) ¼	(h) $^1/_{12}$	(i) $^5/_{36}$	(j) $^2/_{36}$	(k) $^2/_{36}$	(l) $^3/_{36}$
Q8	(a) $^{20}/_{100}$	(b) ¼	(c) ¼	(d) $^4/_{10}$	(e) $^{58}/_{100}$	

Q9	$^{8}/_{9}$							
Q10	(a) $^{4}/_{90}$	(b) $^{12}/_{90}$	(c) $^{1}/_{3}$	(d) $^{1}/_{9}$	(e) $^{32}/_{90}$			
Q11	(a) $^{12}/_{2652}$	(b) $^{156}/_{2652}$	(c) $^{169}/_{2652}$	(d) $^{338}/_{2652}$	(e) $^{494}/_{2652}$	(f) $^{182}/_{2652}$		
	(g) $^{13}/_{51}$	(h) $^{39}/_{51}$	(i) $^{39}/_{51}$	(j) $^{24}/_{663}$	(k) $^{15}/_{51}$	(l) $^{128}/_{2652}$		
Q12	(a) ½	(b) ½	(c) ½	(d) ¼	(e) $^{3}/_{8}$	(f) ½		
Q13	(a) $^{2}/_{64}$	(b) $^{9}/_{64}$	(c) $^{20}/_{64}$	(d) $^{3}/_{16}$	(e) $^{20}/_{64}$	(f) $^{15}/_{64}$		
	(g) $^{21}/_{64}$							
Q14	(a) $^{1}/_{16}$	(b) $^{1}/_{16}$	(c) $^{2}/_{16}$	(d) $^{4}/_{16}$	(e) $^{1}/_{16}$	(f) $^{4}/_{16}$		
	(g) $^{6}/_{16}$							
Q15	(a) $^{6}/_{720}$	(b) $^{12}/_{720}$	(c) $^{48}/_{720}$					
Q16	(a) $^{120}/_{311875200}$		(b) $^{1560}/_{311876200}$		(c) $^{154440}/_{311875200}$			
	(d) $^{617760}/_{311875200}$							
Q17	0.648							
Q18	(a) $^{1}/_{512}$	(b) $^{511}/_{512}$	(c) $^{511}/_{512}$	(d) ½				

Q51	Bag 1: 1 white bead. Bag 2: all the rest. $^{23}/_{30}$
Q52	$^{6}/_{720}$
Q53	(a) $^{720}/_{5864443200}$ (b) $^{720}/_{586444320}$ (c) $^{5040}/_{5864443200}$
Q54	64.8
Q55	She should change her mind. Not changing her mind will give her a $^{1}/_{3}$ chance of winning (she would win in $^{1}/_{3}$ of the games). Therefore, changing will give her a $^{2}/_{3}$ chance (she would win in the other $^{2}/_{3}$ of the games). This is the Monty Hall Problem. Google it if you aren't convinced.

Q62	(a) $^{20}/_{42}$	(b) $^{20}/_{42}$
Q63	(a) $^{3}/_{8}$	(b) $^{10}/_{1000}$
Q64	(a) $^{30}/_{336}$	(b) $^{72}/_{336}$