M1 Maths

P3-1 Tree Diagrams

- using tree diagrams to determine probabilities

Summary Learn Solve Revise Answers

Summary

Tree diagrams are a visual way of solving problems which involve the addition and multiplication rules.

Unlike two-way tables, they can be used where there are more than two stages and where outcomes are not all equally likely.

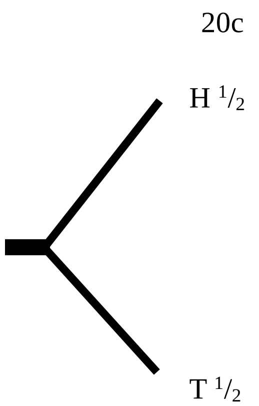
Learn

In Module P2-2 we met two-way tables, which are a visual way of finding probabilities in situations which would otherwise involve the addition and multiplications rules. The limitations of two-way tables are that they are limited to two-stage events and all the outcomes have to be equally likely.

Tree diagrams are another way of solving problems that involve the addition and multiplication rules. But tree diagrams can be used on any number of stages and for unequally likely outcomes.

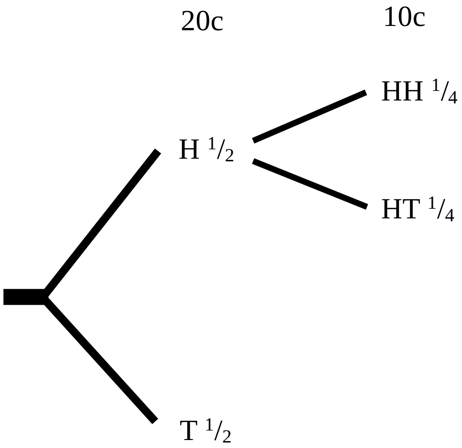
Now, all tree diagrams can theoretically be solved using the addition and multiplication rules without actually drawing a tree diagram, but you will quite likely be asked to draw a tree diagram in a test, so it is still worth learning how to do it.

Suppose we toss a 20c coin a 10c coin and a 5c coin. The 20c coin can come down heads or tails. The probability of each is ½. We can represent this with a diagram like so:

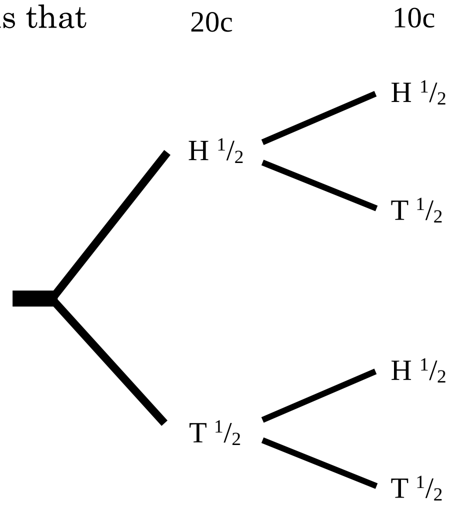


If the 20c coin comes down heads, the 10c coin can come down heads or tails, each with a probability of ½. Likewise if the 20c coin comes down tails.

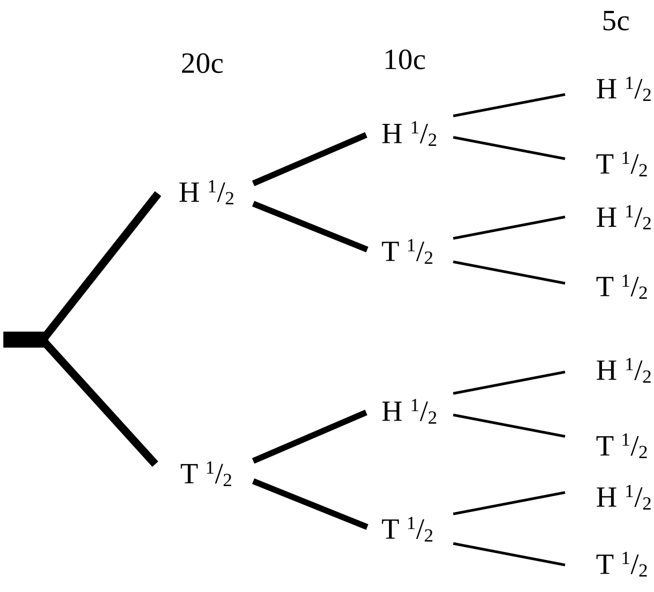
We can continue our diagram like this:



Likewise for the half of the trials that the 20c coin comes down tails.

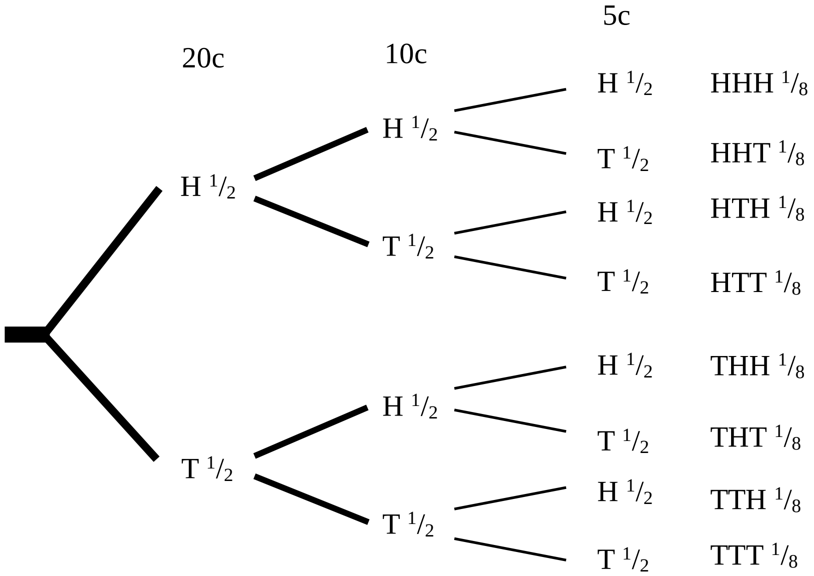


If the 20c and 10c coins both come down heads, then the 5c coin can come down heads or tails, each with a probability of ½ . Likewise for the other possibilities for the 20c and 10c coins. We can continue our diagram like this.



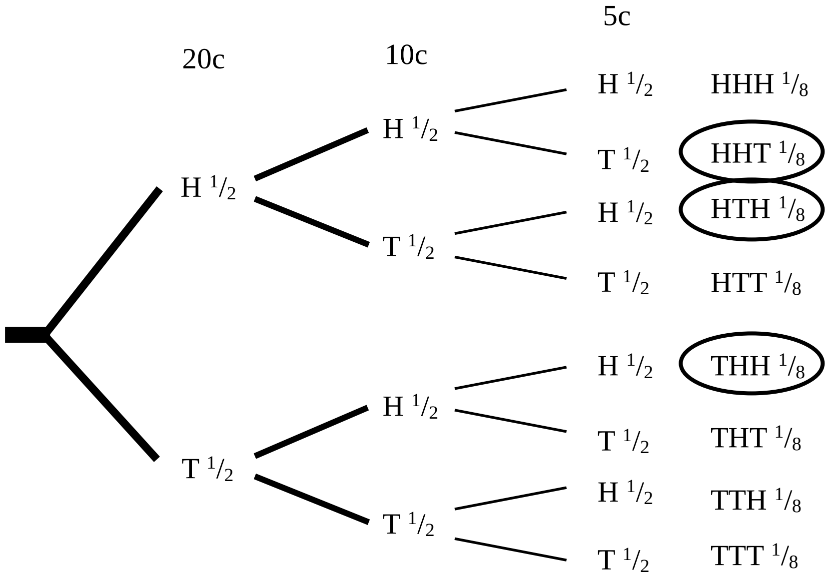
This looks like a tree on its side with its root on the left. For each branch, we can then write in the combination of heads and tails and the probability. For instance, the top branch is a head on the 20c coin, a head on the 10c coin and a head on the 5c coin. We can write this as HHH. The probability of this can be worked out by the multiplication rule. It is the probability of getting head on the 20c **and** a head on the 10c **and** a head on the 5c, i.e. $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.

We can do the same for all the branches to get this:



We now have a list of all the possible outcomes for the three coins along with their probabilities.

Now suppose we need to know the probability of getting two heads and a tail. We can circle all the outcomes which are to heads and a tail like this:



We can see that to get two heads and a tail, the outcome must be HHT **or** HTH **or** THH. The probability of this is $^1/_8 + ^1/_8 + ^1/_8 = ^3/_8$.

Practice

Q1 Donna has a coin and four cards – the A, 2, 3 and 4 of clubs. She tosses the coin and picks a card at random. Use a tree diagram to find the probability that she gets a head and the 3 of clubs.

Q2 Griffin has two 4-sided dice, each numbered 1 to 4. He rolls both. Draw a tree diagram and use it to find the probabilities of getting:

(a) a 1 then a 4

(b) a 1 and a 4 (in either order)

(c) a total of 5

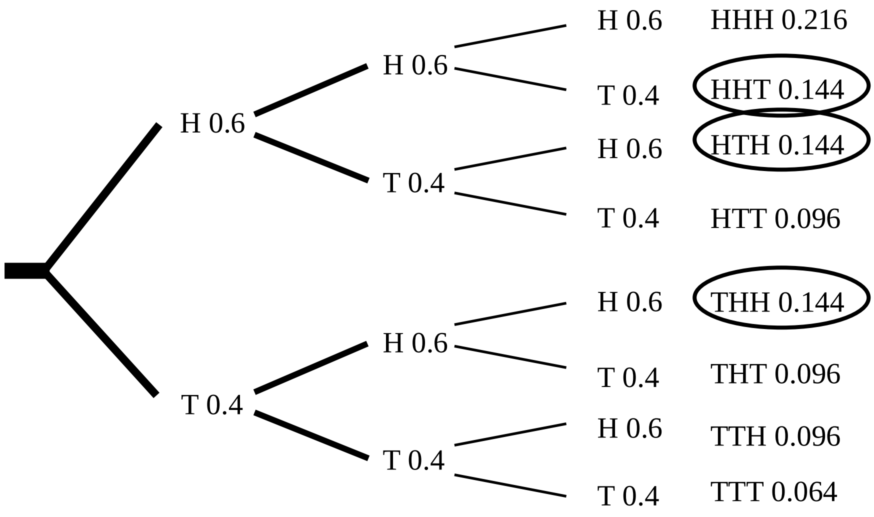
(d) a total of 8

(e) a total of 1

Unequal Probabilities

In the example above, all the outcomes at each stage had the same probability. We can use a tree diagram even if this is not the case.

Suppose we have a bent coin with P(H) = 0.6 and P(T) = 0.4. Suppose we toss it three times. The tree diagram would look like this:



The probability of getting two heads and a tail is then 0.144 + 0.144 + 0.144 = 0.432

Practice

Q3 Absolom has a coin which he accidentally bent. The probability of getting a head with it is now 0.7. Use a tree diagram to find the probability that, if he tosses it twice, he will get a head and a tail.

Q4 Absolom from the last question decides to toss his coin 3 times. Find the probability that he will get two tails and head.

Q5 Siffy has a bag with 5 red balls and 3 green balls. She pulls one out, looks at it, then puts it back, shakes the bag and pulls out another one. Use a tree diagram to find the probability that she will get one of each colour.

Q6 Hayley has a bag with the 3 red balls and 2 green balls. He draws a balls out, looks at it, then puts it back, shakes the bag and draws out another one, then does the same again, so he takes 3 balls, replacing each before taking the next one. Because he replaces the ball each time, the bag contains the same 5 balls before each draw and the probabilities stay the same. Use a tree diagram to find the probability that she will get 2 red balls and a green ball.

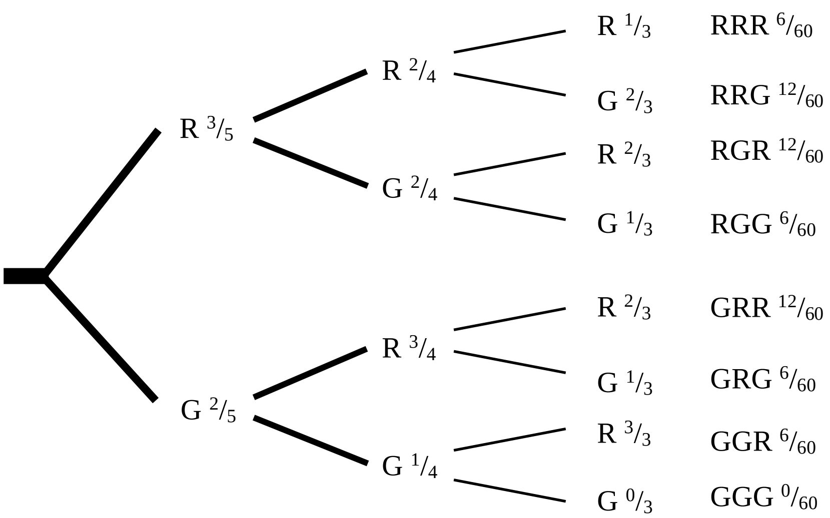
Without Replacement

Hayley, in the last question, replaced each ball before drawing the next. But what if she hadn't? For the first draw P(red) = $^{3}/_{5}$ and P(green) = $^{2}/_{5}$.

If she drew a red one, then there would be 2 red and 2 green left in the bag, so the probabilities for the next draw would be P(red)= ½ and P(green) = ½.

On the other hand, if her first ball was green, then there would be 3 red and 1 green left in the bag, so the probabilities for the next draw would be P(red= ¾) and P(green) = ¼.

The tree diagram would look like this:



So, if we want the probability of drawing a red ball and two green balls, we would add the probabilities of RGG, GRG and GGR to get $^6/_{60} + {}^6/_{60} + {}^6/_{60} = {}^{18}/_{60}$. Note than P(GGG) is 0 because you can't get 3 green balls from a bag with only 2 in it if you don't replace them.

We talk about drawing balls with replacement if the balls are replaced after each one is drawn. We talk about drawing without replacement if they aren't.

Practice

Q7 Meg had the 4 red balls and 2 green balls in a bag. She took out a ball, looked at it and put it on the table. Then she drew another one and put it on the table. In other words, she drew 2 balls without replacement. Use a tree diagram to find the probability that she will get

(a) 2 red balls

(b) 2 green balls

(c) one of each colour.

Q8 Claudia had a bag with 4 red balls and 2 green balls. She drew 3 balls without replacement. Draw a tree diagram and use it to find the probability that she gets

(a) 3 red balls

(b) 3 green balls

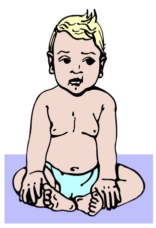
(c) one red ball and 2 green balls.

Q9 Tom has a bag with 2 black balls and a white ball. He draws 2 **with** replacement. Draw a tree diagram and find the probability that he gets one of each colour.

Q10 Sarah has a bag with 6 blue balls and 4 yellow balls. She puts in her hand and pulls out two balls. Draw a tree diagram and find the probability that she gets a blue one and a yellow one. [Note that taking two at once is the same as taking two without replacement. If this isn't obvious, consider that she can't take the same ball twice: once she touches the first ball that she will take, there are only 9 options left for the second ball.]

Q11 Famina has 5 black cards numbered 1 to 5 and 5 white cards numbered 1 to 5. She picks one of each at random. She can get a total from 2 to 10. Use a tree diagram to find the probability of each total. Put your results in a table.

Q12 When Katie has her baby, the probability that it will be a boy is 0.5 and the probability that it will be blonde is 0.2. Use a tree diagram to find the probability that she has



(a) a blonde girl.

(b) a girl that's not blonde

(c) a blonde boy

Q13 There is a 20% chance that it will rain next Monday. If it does rain Monday, there is a 50% chance it will rain Tuesday. If it doesn't rain Monday, there is a 10% chance it will rain Tuesday. What is the probability that it will rain:

(a) Monday and Tuesday

(b) Tuesday but not Monday

(c) just one of those two days

(d) Tuesday

Solve

Q51 A couple decide to keep having children until they have more girls than boys. What is the probability that they will have more than 3 children?

Q52 The start of a game board looks like this:

Start	1	2	3	4	5	6	7

Players start on 'Start', then roll a single die and move the corresponding number of squares. On their next turn, they roll again and move again, and so on.

Use a tree diagram to find the probability that they will land on square 4 on their way along the board.

Revise

Revision Set 1

Q61 Use a tree diagram to show that the probability of getting an 8 when you roll two 6-sided dice is $^5/_{36}$

Q62 A bag contains 5 black balls and 2 white balls. Two balls are taken without replacement. Use a tree diagram to find the probability that

(a) both are black

(b) they are different colours

Q63 10 cards are numbered 1 to 10. They are shuffled and three are dealt. Use a tree diagram to find the probability that

(a) two are even numbers, one is odd

(b) their sum is 6

Q64 A bag contains 4 black balls, 3 white balls and a red ball. Three are taken without replacement. Use a tree diagram to find the probability that

(a) they are all the same colour

(b) they are three different colours

Answers

Q1 $^{1}/_{8}$

Q2 (a) $^{1}/_{16}$ (b) $^{2}/_{16}$ (c) $^{4}/_{16}$ (d) $^{1}/_{16}$ (e) 0

Q3 0.42

Q4 0.189

Q5 $^{30}/_{64}$

Q6 $^{54}/_{125}$

Q7 (a) $^{12}/_{30}$ (b) $^{2}/_{30}$ (c) $^{16}/_{30}$

Q8 (a) $^{24}/_{120}$ (b) 0 (c) $^{4}/_{120}$

Q9 $^{4}/_{6}$

Q10 $^{60}/_{90}$

Q11

2	3	4	5	6	7	8	9	10
$^{1}/_{25}$	$^{2}/_{25}$	$^{3}/_{25}$	$^{4}/_{25}$	$^{5}/_{25}$	$^{4}/_{25}$	$^{3}/_{25}$	$2/_{25}$	$^{1}/_{25}$

Q12 (a) 0.1 (b) 0.4 (c) 0.1

Q13 (a) 10% (b) 8% (c) 18% (d) 18%

Q51 $^{5}/_{8}$

Q52 $^{343}/_{1296}$

Q62 (a) $^{20}/_{42}$ (b) $^{20}/_{42}$

Q63 (a) $^{3}/_{8}$ (b) $^{10}/_{1000}$

Q64 (a) $^{30}/_{336}$ (b) $^{72}/_{336}$