M1 Maths

P2-3 Venn Diagrams

- using Venn diagrams to determine numbers and probabilities

Summary Learn Solve Revise Answers

Summary

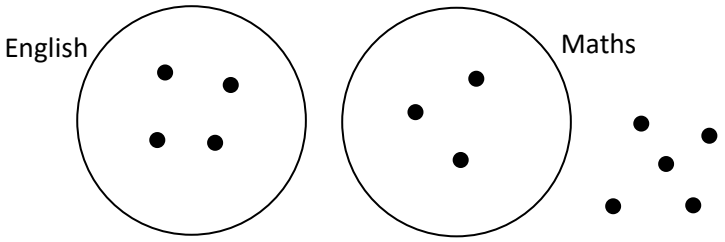
A Venn diagram shows which of two or three groups certain items or people belong to. Venn diagrams can be used to work out probabilities.

Learn

Venn Diagrams with Dots

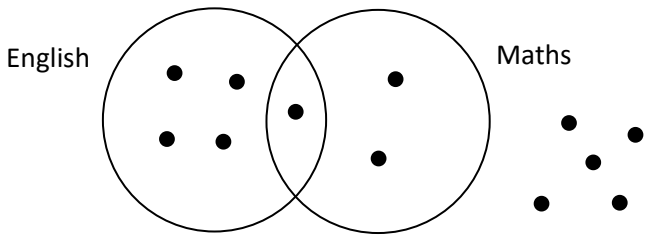
A Venn diagram is a visual way of showing which of two or three groups certain items or people belong to.

For example, a school might have 12 teachers. Suppose 4 of them teach English, 3 teach maths and 5 teach neither. We can show this visually on a Venn diagram like this:



The way the diagram works is that those who teach English are represented by dots in the English circle; those who teach Maths are represented by dots in the maths circle and those who teach neither are represented by dots in neither circle.

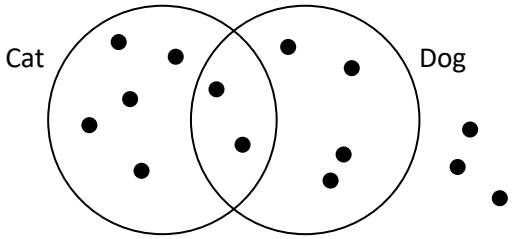
But suppose one of the Maths teachers was given an English class as well. That teacher then needs to be in both circles. Giving her two dots might be misleading because we would have 13 dots and so might think that there are 13 teachers. Instead we make the circles overlap, so that one dot can be in both circles, like this:



This Venn diagram now has 5 dots in the English circle, showing that 5 of the teachers teach English. It has 3 dots in the Maths circle, showing that 3 of the teachers teach Maths. And it has 5 teachers not in either circle, showing that 5 teachers don't teach either. And there are still just 12 dots showing that there are just 12 teachers.

Practice

Q1 This Venn diagram shows the students in Class 8C who have a cat or a dog at home.



(a) How many have a cat and a dog?
(b) How many have a cat?
(c) How many don't have a cat?
(d) How many have a cat, but not a dog?
(e) How many have a dog?
(f) How many don't have a dog?
(g) How many have a dog but not a cat?
(h) How many have neither?
(i) How many students in the class?

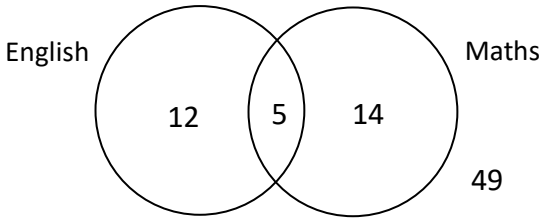
Make sure you can get all parts of Q1 right before you go on.

Venn Diagrams with Numbers

Now, using dots is ok when there are just a few people. But suppose the Venn diagram for English and Maths teachers above was for a big school where 12 teachers teach just English, 14 teach just maths, 5 teachers teach both English and Maths and 49 teachers teach neither.

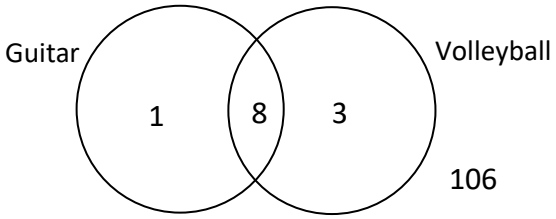
Using dots would be messy and anyone who wants to read the diagram will have to do a lot of counting. So instead, we tend to just write the numbers in each region of the diagram instead of using dots. In fact, we tend to use numbers even when the numbers are small.

So the Venn diagram for the big school would look like this:



Practice

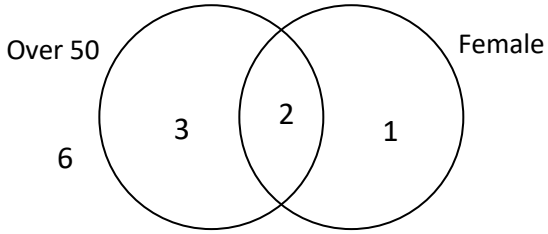
Q2 This Venn diagram shows the students that play guitar and volleyball at Cabbage Tree High School.



(a) How many play just guitar?
(b) How many play guitar?
(c) How many don't play guitar?
(d) How many play volleyball?
(e) How many play volleyball but not guitar?
(f) How many don't play volleyball?
(g) How many play guitar and volleyball?
(h) How many play neither?
(i) How many students in the school?

Drawing Venn Diagrams

We can draw Venn diagrams from information about how many people are in each group. For example, if we are told that there were 22 people in a club, that 5 of them were over 50, 13 of them were female and 6 of them were neither, we can draw the diagram like this:



Note that we have to work out that there are 2 in the overlap by seeing that $5 + 13 + 6 = 24$ and thus that 2 people must have been counted twice. Once we know this, we can work out that there must be 3 in the rest of the Over 50 circle and 11 in the rest of the female section.

Practice

Q3 Draw a Venn diagram to show the following information.
There are 17 people in the swimming club. 5 race freestyle, 4 race breast stroke, 10 race neither.

Q4 Use your Venn diagram from Q3 to find out:
(a) How many race freestyle and breast stroke.
(b) How many race freestyle but not breast stroke.
(c) How many race breast stroke but not freestyle.
(d) How many race freestyle or breast stroke or both.

Q5 Draw a Venn diagram to show the following.
Of the 30 people in Rudolf's class, 14 have a pool at home, 21 have a barbeque and 10 have both.

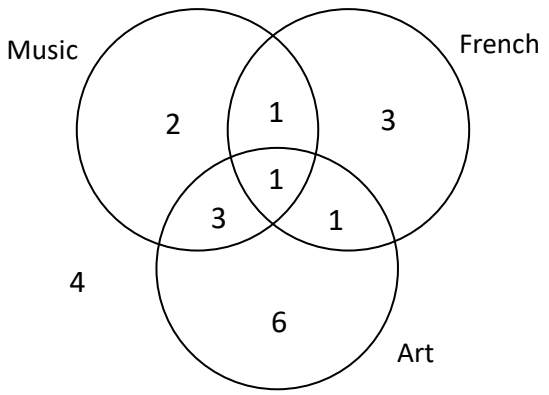
Q6 Use your Venn diagram from Q5 to find out:
(a) How many have neither.
(b) How many have a pool but not a barbeque.
(c) How many have a barbeque but not a pool.
(d) How many don't have a pool.

Q7 In a group of 12 sailors, 4 were sea sick and 2 of those were drunk as well. If only one was neither, draw a suitable Venn diagram to show this information.

Venn Diagrams with 3 Circles

Venn diagrams can be drawn with 3 circles. You may not need to be able to deal with these, but if you can master them, the 2-circle ones will seem easy.

The Venn diagram to the right shows the students in a class who study Music, French and Art.

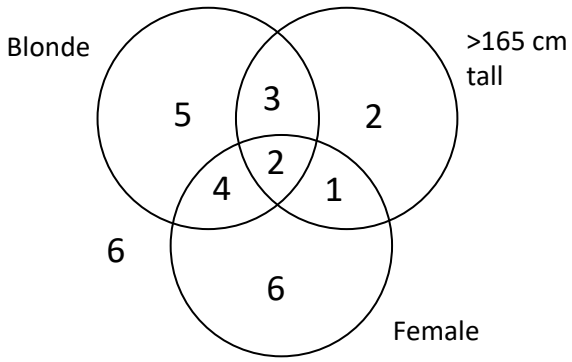


Note that, to find out how many people do Music, we add up all the numbers inside the Music circle. There are 7. Likewise, we can see that 11 do art and 6 do French. Of the students who do French, 2 do Art. One students does all three subjects. And so on.

The good news is that it's not geometrically possible to have a Venn diagram with more than 3 circles that still shows all the possible regions.

Practice

Q8 Use this 3-circle Venn diagram for Class 8A to answer the questions below.



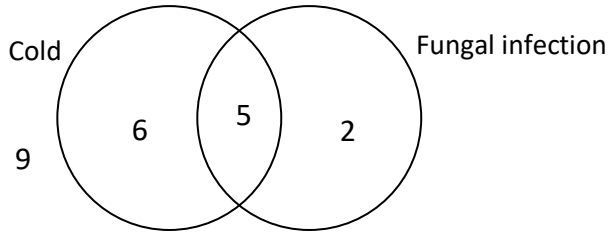
(a) How many students in the class?
(b) How many girls in the class?
(c) How many boys? (Assume all students are either a boy or a girl.)
(d) How many blonde girls are there?
(e) How many blonde girls over 165 cm?
(f) How many blonde boys over 165 cm?
(g) How many students are <165 cm?
(h) How many girls are >165 cm?
(i) How many blonde girls <165 cm?
(j) How many blonde boys < 165 cm?
(k) How many dark-haired boys <165 cm? (Assume anyone not blonde is dark-haired.)

Q9 Of the 15 people on a bus, 7 had dogs, 4 had cats and 5 had birds. 3 had both dogs and cats and one of those had birds as well. One had just cats and 2 had just birds. Present this information as a Venn diagram.

Q10 Of the 18 people in a restaurant, all but 6 are having potatoes. 6 are having beef and potatoes. 4 are having fish and potatoes and 3 are having beef and fish, one of whom is also having potatoes. More are having beef than fish. Another 3 are having none of those. Present this information on a Venn diagram.

Probability

Consider this Venn diagram of people on a tour bus who have a cold and who have a fungal infection on their feet.



If you got on the bus and sat next to someone at random, what is the probability that they would have a cold? It would be the number of people who have a cold divided by the total number of people on the bus.

From the diagram, we can see that the number with a cold is 11 and that the total number on the bus is 22. So the probability that the person you sit next to has a cold is $^{11}/_{22}$ or 50%.

Note that this is the same question as 'What fraction of the people on the bus have a cold?'

Other probabilities can be worked out in the same way. Suppose you got on the bus and sat next to someone with a cold. What is the probability that they would have a fungal infection too? Well, there are 11 people with a cold. 5 of those also have a fungal infection. So the probability would be $^{5}/_{11}$.

Practice

Q11 Use the Cold-Fungal infection Venn diagram above to answer these questions.

(a) What is the probability that a random person on the bus would have a fungal infection?

(b) What is the probability that a random person on the bus would have a cold and a fungal infection?

(c) What is the probability that a random person on the bus would have neither?

(d) What is the probability that a random person on the bus would have a cold, but not a fungal infection?

(e) If you got on the bus and sat next to someone with a cold, what is the probability that they will have a fungal infection too?

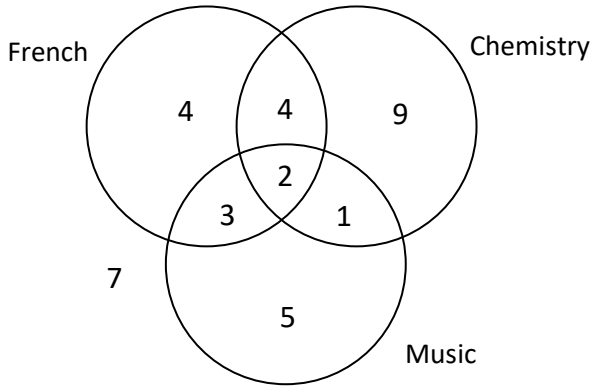
(f) If you got on the bus and sat next to someone with a fungal infection, what is the probability that they will have a cold too?

(g) If you got on the bus and sat next to someone with a cold, what is the probability that they won't have a fungal infection?

(h) If you got on the bus and sat next to someone without a cold, what is the probability that they will have a fungal infection?

(i) If you got on the bus and sat next to someone with a fungal infection, what is the probability that they won't have a cold?

Below is a Venn diagram showing the numbers of people in a class who are studying French, Chemistry and Music.



If we took one of the music students at random, what is the probability that they would be studying chemistry as well?

As you can probably tell, to work this out, we look at how many music students there are – 11. Then we look at how many **of these** study chemistry – 3. Then the probability is $^{3}/_{11}$.

Practice

Q12 Use the French – Chemistry – Music Venn diagram above to answer these questions.

(a) What is the probability that a chemistry student picked at random would be studying music?

(b) What fraction of the French students do chemistry?

(c) What fraction of the chemistry students don't do French?

(d) If you picked a student who doesn't do music, what is the probability that they would do French?

(e) If you picked a student who doesn't do French, what is the probability that they wouldn't do any of these three subjects?

(f) If you picked a student who doesn't do chemistry, what is the probability that they don't do music?

(g) If you pick a student who does music, what is the probability that they do French, but not chemistry?

(h) What fraction of those doing chemistry do French or music, but not both?

Solve

Q51 Harry has 22 pets. 11 are tortoises, 5 are guinea pigs, 7 weigh more than 500 g and this include 4 of the tortoises. 3 fit none of these categories. Present this information as a Venn diagram.

Q52 If you picked a whole number at random, what is the probability that it would be rational number?

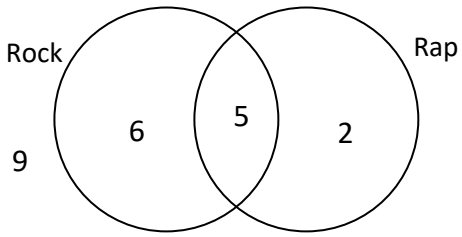
Q53 If you picked a rational number at random, what is the probability that it would be an integer?

Q54 If you picked an integer at random, what is the probability that it would be a whole number?

Revise

Revision Set 1

Q61 This Venn diagram shows how many people who live in Caylee Street like rock music and how many like Rap.



(a) How many like rock and rap?

(b) How many like rock but not rap?

(c) How many like neither?

(d) If you picked someone at random from the street, what is the probability that they would like rap?

(e) What fraction of people in the street like rap?

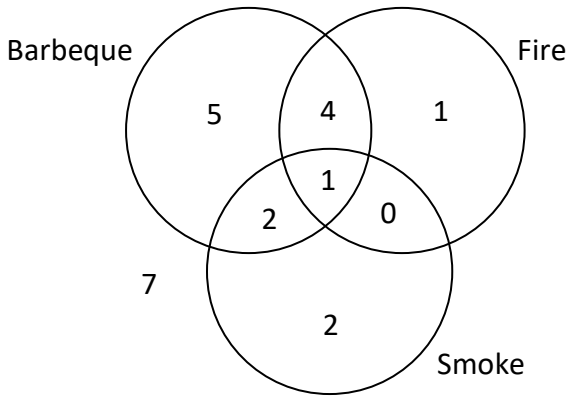
(f) If you picked someone at random from the street, what is the probability that they would like rap but not rock?

(g) If you picked someone at random from the street who likes rock, what is the probability that they would also like rap?

(h) If you picked someone at random from the street who doesn't like rock, what is the probability that they would like rap?

(i) If you picked someone at random from the street who doesn't like rap, what is the probability that they wouldn't like rock either?

Q62 This Venn diagram is of the people who live in Arundel Street. It shows how many have a barbeque, how many have an indoor fire and how many smoke.



(a) How many have an indoor fire?

(b) How many have a barbeque but don't smoke?

(c) How many smoke, but don't have an indoor fire or a barbeque?

(d) If you picked someone from the street at random, what if the probability that they would smoke?

(e) If you picked a smoker from the street, what is the probability that they would have a fire?

(f) If you picked a smoker with a fire, what is the probability that they would have a barbeque?

Q63 Draw a Venn diagram to show the following information. 5 of my 12 friends own a cat and 8 own a dog. 3 own neither.

Q64 Use your Venn diagram from the last question to answer the following

(a) How many own both?

(b) If you picked a dog owner at random, what would be the probability that they would own a cat?

Q65 Draw a Venn diagram to show the following information: Of the 32 people in a life boat, 12 are sea sick, 5 have a head ache and 11 are children. There are 2 sea sick children with headaches. 5 of the children are well and 6 of them are sea sick. There are 3 sea sick adults without headaches.

Q66 Use your Venn diagram from Q73 (check it's right first) to answer these questions.

(a) What fraction of the children have headaches?

(b) What fraction of the adults are well?

(c) If you picked a sea sick passenger at random, what is the probability that they would have a head ache?

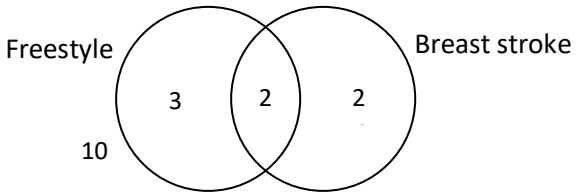
(d) I you picked an adult at random, what is the probability that they would not be sea sick?

Answers

Q1 (a) 2 (b) 7 (c) 7 (d) 5 (e) 6 (f) 8 (g) 4 (h) 3 (i) 14

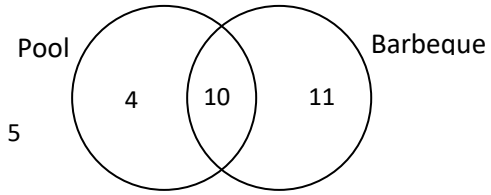
Q2 (a) 1 (b) 9 (c) 109 (d) 11 (e) 3 (f) 107 (g) 8 (h) 106 (i) 118

Q3



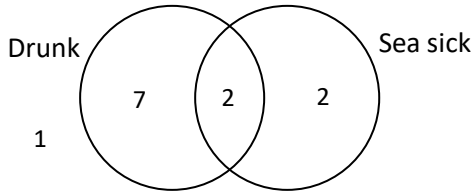
Q4 (a) 2 (b) 3 (c) 2 (d) 7

Q5



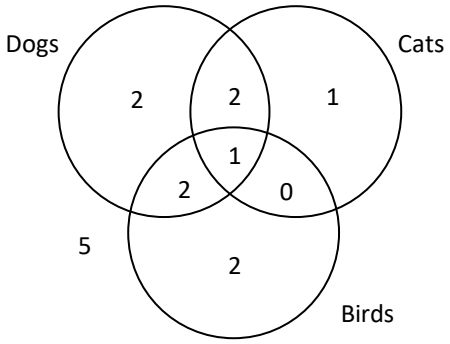
Q6 (a) 5 (b) 4 (c) 11 (d) 16

Q7

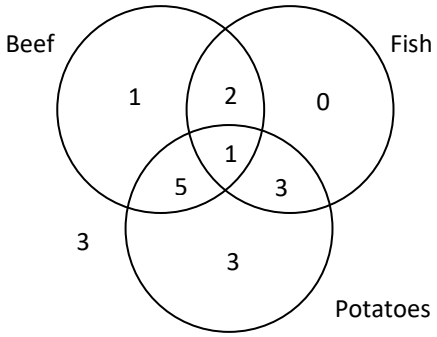


Q8 (a) 29 (b) 13 (c) 16 (d) 6 (e) 2 (f) 3 (g) 21 (h) 3 (i) 4 (j) 5 (k) 6

Q9



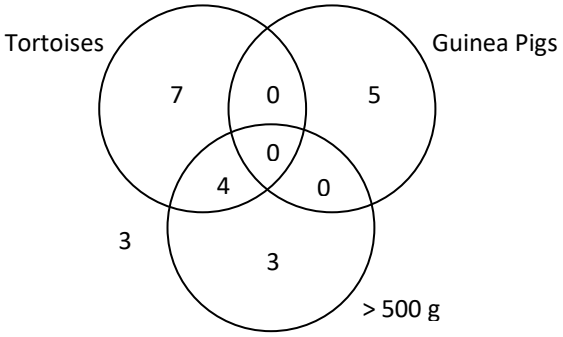
Q10



Q11 (a) $^{7}/_{22}$ (b) $^{5}/_{22}$ (c) $^{9}/_{22}$ (d) $^{6}/_{22}$ (e) $^{5}/_{11}$ (f) $^{5}/_{7}$ (g) $^{6}/_{11}$ (h) $^{2}/_{11}$ (i) $^{2}/_{7}$

Q12 (a) $^{3}/_{16}$ (b) $^{6}/_{13}$ (c) $^{10}/_{16}$ (d) $^{8}/_{24}$ (e) $^{7}/_{22}$ (f) $^{11}/_{19}$ (g) $^{3}/_{11}$ (h) $^{5}/_{16}$

Q51



Q52 1. All whole numbers are rational

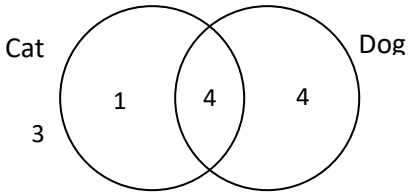
Q53 0. There are an infinite number of rational numbers between each pair of integers.

Q54 ½ The 0 will become insignificant among the infinite number of integers and whole numbers.

Q61 (a) 5 (b) 6 (c) 9 (d) $^{7}/_{22}$ (e) $^{7}/_{22}$ (f) $^{2}/_{22}$ (g) $^{5}/_{11}$ (h) $^{2}/_{11}$ (i) $^{9}/_{15}$

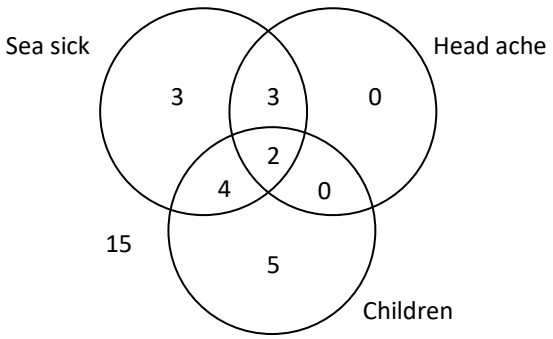
Q62 (a) 6 (b) 9 (c) 2 (d) $^{5}/_{22}$ (e) $^{6}/_{22}$ (f) 1

Q63



Q64 (a) 2 (b) $^{4}/_{8}$

Q65



Q66 (a) $^{2}/_{11}$ (b) $^{15}/_{21}$ (c) $^{5}/_{12}$ (d) $^{15}/_{21}$