M1 Maths

P2-1 Compound Events

- addition and multiplication rules

Summary Learn Solve Revise Answers

Summary

To find the probability of something happening **OR** something else happening, we **ADD** their probabilities. If they are not mutually exclusive, we have to use P(A or B) = P(A) + P(B) – P(A and B).

If we want to know the probability of something happening **AND** something else happening, we **MULTIPLY** their probabilities. If they are dependent, we have to use P(A and B) = P(A) × P(B | A).

Learn

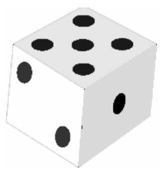
Addition Rule – Basic Version

If we roll a die and look at the number that comes up, there are six possible outcomes: 1, 2, 3, 4, 5 and 6.

We know that the probability of getting a 4 is $^1/_6$. We also know that the probability of getting a 5 is $^1/_6$. In fact, the probability of getting any of the six numbers is $^1/_6$. This is because each will happen one sixth of the times in the long run.

What is the probability of getting a 4 *or* a 5? We have to answer the question 'What fraction of times will we get a 4 or a 5 in the long run?'

This is quite easy really. On average, out of six rolls, there will be one 4 and one 5. So, we will get a 4 or a 5 two times out of every 6, or 2 sixths of the time. In other words, the probability of getting a 4 or a 5 is $^2/_6$.



In the same way, the probability of getting a 1, 2 or 3 is $^3/_6$, the probability of getting an even number is $^3/_6$, the probability of getting a number less than 5 (i.e. 1, 2, 3 or 4) is $^4/_6$.

We find the probability of these multi-outcome events by adding the probabilities of the outcomes. $^1/_6 + {}^1/_6 = {}^2/_6$, $^1/_6 + {}^1/_6 + {}^1/_6 = {}^3/_6$, and so on. This is the **addition rule**. Adding probabilities works for any situation where we want to find the probability of something happening ***or*** something else happening. We just add the probabilities of the various somethings.

In probability, OR means ADD

Suppose the probability that a matchbox lands on its end is 0.08 and the probability that it lands on its edge is 0.33. Then the probability that it lands on its end or its side is 0.08 + 0.33, i.e. 0.41.

Note that the word **event** is similar in meaning to outcome, except that an outcome is just one of the things that can result from a trial, whereas an event consists of one or more outcomes. So, when rolling a die, the outcomes might be: getting a 1, getting a 2, getting a 3, getting a 4, getting a 5 and getting a 6. Events might be: getting a 5 or a 6, getting an even number, not getting a 5, getting a 2, and so on. Getting a 2 is an outcome and an event because events consist of **one** or more outcomes.

To some extent, the distinction between outcomes and events is arbitrary: we could call getting an even number an outcome if we chose to do so.

You've met the term ***sample space*** too. The sample space for an experiment is the list of all the possible outcomes. When rolling a die, the sample space is the six outcomes 1, 2, 3, 4, 5, 6.

In the case of the die, 3 of the outcomes are even numbers, so the probability of the event 'getting an even number' is $^1/_6 + {}^1/_6 + {}^1/_6 = {}^3/_6$. This is the number of outcomes in the event divided by the number of outcomes in the sample space. Some people like to use the formula

$$Probability\ of\ an\ event = \frac{number\ of\ outcomes\ in\ the\ event}{number\ of\ outcomes\ in\ the\ sample\ space}$$

This is sometimes written as

$$Probability\ of\ an\ event = \frac{n(E)}{n(S)}$$

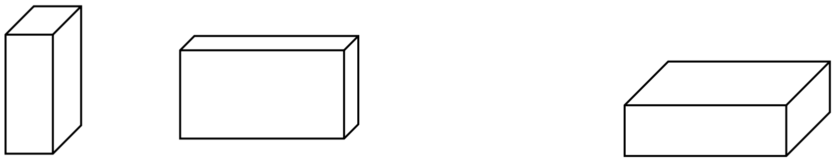
Note however, that this works **only** if all the outcomes are known to be equally likely due to symmetry. If you try to kick a soccer ball into a basketball hoop from 30 metres away, you might consider that the sample space has 2 outcomes – success and failure. Success is one of these outcomes. But that doesn't mean that the probability of success is ½. For most people it would be much less.

Some textbooks define probability as $\frac{n(E)}{n(S)}$ without mentioning the condition that all outcomes have to be equally likely. This can be quite misleading. **Don't be misled!** The probability of an event is the fraction of times that the event would happen in the long run. For most people the probability of getting the ball through the hoop would be something more like $^1/_{1000}$.

Complements

The **complement** of an event is a useful concept. The complement of an event means all the outcomes in the sample space except those in that event. The complement of *getting a head* if you toss a coin is *getting a tail*. And the complement of *getting a tail* is *getting a head*. We say that *getting a head* and *getting a tail* are complements.

The complement of *getting a 6* on a die is *getting a number from 1 to 5*. The complement of a matchbox *landing on its end or side* is *landing flat*. An event and its complement always make up the whole sample space.



Suppose the probability of a bent coin landing *heads up* is 60%. This means that it will land *heads up* 60% of the time in the long run. This means it will land *tails up* the other 40%. So the probability of *tails up* is 40%.

Heads up and *tails up* are complements. Because they are complements, their probabilities must add to 100%. (One of them will happen every trial.) The probability of *tails up* is 100% minus the probability of *heads up* (100% – 60% = 40%) and the probability of *heads up* is 100% minus the probability of *tails up* (100% – 40% = 60%). In fact . . .

The probability of any event is
100% minus the probability of its complement.

or, of course, 1 minus the probability of its complement.

Knowing this can make some calculations easier. Later in this module you will learn how to find the probability that, if you toss 3 coins, you will get 3 heads. It is 1/8. It is a lot harder to work out the probability of 2 heads and a tail or of 2 tails and a head. But using the idea of complements, we can see that the probability of getting less than 3 heads (i.e. 0, 1 or 2 heads) is $1 - {}^{1}/_{8} = {}^{7}/_{8}$.

Practice

Q1 If you roll a normal 6-sided die, what is the probability of:

(a) getting a 4 or a 5 [Note that in probability, you are not always expected to give common fractions in simplest, though you may. Giving them in un-simplified form makes it more obvious how they were obtained. Some of the answers given in this module are un-simplified. You may need to convert your answers to check them.]

(b) getting an even number

(c) getting a number >4 {Hint: it can sometimes be worth making a list of the numbers in the event first.]

(d) not getting a 6

(e) not getting a number less than 3

(f) getting a number which is at least 3

Q2 If you pick a card from a standard pack of 52, find the probability of

(a) getting the 2 or 3 of spades

(b) getting a 2 or 3 of any suit

(c) getting a picture card (J, Q or K)

(d) getting a heart

(e) not getting a heart

(f) getting a card

Q3 A bag contains 2 white balls, 3 red balls and 5 black balls. Apart from their colour, they are identical. If you pick one at random, find the probability of:

(a) picking a red one

(b) picking a black one or a red one

(c) picking one that's not black or red

(d) picking a green one

Q4 If the probability of a matchbox landing on its end is 12% and on its side is 30%

(a) what is the probability it will land on its end or its side?

(b) what is the probability it will land flat?

(c) what is the probability that it will land on its end or side or flat?

(d) what is the probability that it will land on its end or flat?

Q5 If a thumb tack has a 55% chance of landing point down, what is the probability that it will land point up?

Multiplication Rule – Basic Version

Suppose we toss two normal coins and repeat this many times. The first coin will be heads ½ the times we do it. Of that ½ of the times, the second toss will come down heads ½ of those times. So both tosses will be heads ½ of ½ of the times we try it, i.e. ¼ of the times. ½ of ½ is ¼. In other words, ½ × ½ is ¼. The probability of getting H on the first coin and H on the second coin is ½ of ½, i.e. ½ × ½, i.e. ¼. In decimals this is 0.5 of 0.5, i.e.0.5 × 0.5, i.e. 0.25.

The probability of getting a head on the first coin **AND** a head on the second coin is ½ × ½. In fact the probability of anything happening **AND** anything else happening can be calculated by **MULTIPLYING** the probability of the first thing happening by the probability of the second thing happening. This is the **multiplication rule**.

The same applies for more than two things. We just multiply the probabilities of each of the individual otcomes. So, if you toss a coin 3 times, the probability of getting all heads is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$, which is $^1/_8$. If we toss a coin and roll a die, the probability of getting a T is $\frac{1}{2}$. The probability of getting a 5 is $^1/_6$. The probability of getting a T and a 5 (i.e. T5) is $\frac{1}{2} \times {}^1/_6$, which is $^1/_{12}$.

Suppose we pick a student at random. Suppose that the probability that a student is in Year 10 is 0.1 and that the probability that it is a girl is 0.5. Then the probability that it is a girl **and** in Year 10 is 0.5×0.1, which is 0.05.

In general, if we want the probability of something happening **AND** something else happening, we **MULTIPLY** the two probabilities.

In probability, AND means MULTIPLY

Practice

Q6 Gerty has a coin and five cards – the A, 2, 3, 4 and 5 of hearts. She tosses the coin and picks a card at random. Use multiplication to find the probability that she gets a head and the A of hearts.

Q7 Haggot has two 6-sided dice. He rolls both. Use multiplication to find the probabilities of getting:

(a) a 1 then a 4

(b) a 4 then a 1

(c) a 5 then a 2

(d) a double 6

Q8 When Sue has her baby, the probability that it will be a boy is 0.5 and the probability that it will be blonde is 30%. Use multiplication to find the probability that she has

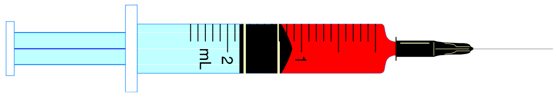
(a) a blonde girl

(b) a blonde boy

(c) a baby that is neither blonde nor male

(d) On average, one day in 5 is rainy. Find the probability that Sue's baby is born on a rainy Tuesday.

(e) One person in 8 has the O blood group. One in 12 is rhesus negative. Find the probability that Sue's baby is O–.



(f) Find the probability that Sue's baby is born on a dry Saturday with O blood.

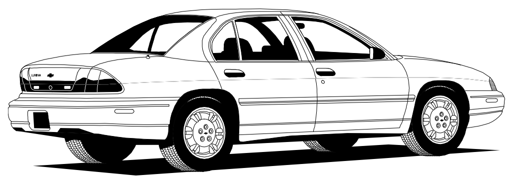
Q9 The probability of a car picked at random from the Magnadome car park being a Toyota is 22%. The probability of it being white is 17%. The probability of it being a current model is 12%. Find the probability that a car picked at random is

(a) all of these

(b) a white Toyota that is not a current model

(c) a current Toyota

(d) a current Toyota that is not white



Shorthand

There is a shorthand notation for when dealing with probabilities that is used a lot and worth knowing.

If we call an event 'A', then we can write the probability of A occurring as P(A). The probability of A not occurring is P(A').

The addition rule can therefore be stated as P(A or B) = P(A) + P(B). The idea of complements tells us that P(A') = 1 – P(A).

Addition Rule – Full Version

Now there is a complication to the addition rule when the outcomes are not mutually exclusive.

As we have seen, if we want to calculate the probability of event A happening **OR** event B happening, we **ADD** the probabilities.

P(A or B) = P(A) + P(B)

This is assuming that A and B are mutually exclusive events i.e. A can happen or B can happen, but they can't both happen in the same trial.

Suppose we are picking a card from a pack of 52 playing cards. Let event A be picking an ace and let event B be picking a picture card (J, Q or K). P(A) = $^{4}/_{52}$; P(B) = $^{12}/_{52}$.

These are mutually exclusive events. The card chosen cannot be both an ace and a picture card.

The probability of picking an ace or a picture card, P(A or B), is P(A) + P(B) which is $^{16}/_{52}$.

But, if event A is picking an ace and event B is picking a red card, then A and B are not mutually exclusive. It is possible to pick a card which is both an ace and a red card. There are two such cards – the ace of hearts and the ace of diamonds.

In this case P(A) = $^{4}/_{52}$; P(B) = $^{26}/_{52}$. So P(A) + P(B) = $^{30}/_{52}$.

But P(A or B) = $^{28}/_{52}$. This is because there are 26 red cards and 2 other aces, i.e. 28 cards which are an ace or red.

In just adding the probability of an ace and the probability of a red card, we have counted the probability of the ace of hearts and the ace of diamonds twice each making our answer $^{2}/_{52}$ too high. To correct for this, we need to subtract the probability that the card chosen is both an ace and a red card, i.e. P(A and B).

So in this case, P(A or B) = P(A) + P(B) – P(A and B).

P(A) + P(B) – P(A and B) = $^{4}/_{52}$ + $^{26}/_{52}$ – $^{2}/_{52}$ = $^{28}/_{52}$. And this is the correct probability.

All Events

So, if we write the addition rule as P(A or B) = P(A) + P(B) – P(A and B), it allows for events which are not mutually exclusive. For mutually exclusive events, P(A and B) = 0, so the new rule still applies. So for all situations, the addition rule can be written

P(A or B) = P(A) + P(B) – P(A and B)

Practice

Q10 A pack of 52 cards is shuffled and a card is picked. Find the probability that:

(a) it is the ace of diamonds
(b) it is an ace
(c) it is an ace or a 2
(d) it is a picture card
(e) it is a 2, 5 or 8
(f) it is a club
(g) it is a club or a spade
(h) it is a black card
(i) it is an ace or a club

(j) it is an ace or a black card
(k) it is a picture card or a red card
(l) it is a black card or a club

Q11 A quarter of the days in Melbourne are sunny. Find the probability that Sam's baby is born on:
(a) a Friday
(b) a cloudy day
(c) a Friday or a cloudy day
(d) A cloudy day on a weekend
(e) a day that starts with S or is sunny
(f) a day that ends in Y or is sunny
(g) a day that is Monday, Tuesday, Wednesday or sunny

Multiplication Rule – Full Version

We have seen that, if we want to calculate the probability of event A happening **AND** event B happening, we **MULTIPLY** the probabilities. P(A and B) = P(A) × P(B).

For example, if you have a bag with 2 black balls and 3 white balls and you draw one out, look at it, put it back then draw again, the probability that the first is white and the second is white will be $^{3}/_{5} \times {}^{3}/_{5} = {}^{9}/_{25}$.

This is the case if the probabilities in the first drawing and those in the second drawing are ***independent***, i.e. neither affects the other.

But suppose we drew one out, put it on the table, then drew a second one out. Now the probabilities in the second draw will depend on what happened in the first draw. We say that the probabilities are ***dependent***.

The probability that the first is white is still $^{3}/_{5}$. The probability that the second is white will be $^{2}/_{4}$ if the first was white (because there will be 4 balls left, 2 of which are white), but $^{3}/_{4}$ if the first was black (because there will be 4 balls left, 3 of which are white). To find the probability that both are white, we still multiply the probabilities, but we have to multiply the probability that the first is white by the probability that the second is white, given that the first was white. This is $^{3}/_{5} \times {}^{2}/_{4}$, which is $^{6}/_{20}$ or $^{3}/_{10}$.

We might call the probability that the first is white P(A) and the probability that the second is white, given that the first was white, P(B|A), then the probability that both are white is P(A) × P(B|A). [| means 'given that'.]

P(A and B) = P(A) × P(B|A)

This is the full version of the multiplication rule. It applies even in cases where the probabilities are independent. In that case P(B|A) is just equal to P(B), so P(A) × P(B|A) = P(A) × P(B).

This is quite mathematically formal and the symbols would be hard to understand and remember. But common sense will generally give the correct answer in these sorts of situations without thinking about the formal rule. Nonetheless, it is worth being aware of the formal rule.

To summarise, when using the multiplication rule with dependent events, we work out the probability of the first event as normal, but then we have to work out the probability of the second one assuming that the first one happened in the way we are interested in. Then we can multiply. Practise this idea on the following questions.

Practice

Q12 A bag contains 3 blue balls and a red ball. A ball is taken out, looked at and replaced. Then the bag is shaken and another ball is taken out. This is called taking two balls with replacement. Find the probability that:

(a) both balls are blue
(b) both balls are red
(c) both balls are green
(d) the first ball is blue
(e) the second ball is blue
(f) the second ball is blue, given that the first ball is blue
(g) the second ball is blue, given that the first ball is red
(h) the first ball is blue, given that the second ball is red

Q13 A bag contains 3 blue balls and a red ball. A ball is taken out, looked at and put on the table. Then a second ball is taken out. This is called selecting two balls without replacement. Find the probability that:

(a) both balls are blue
(b) both balls are red
(c) both balls are green
(d) the first ball is blue
(e) the second ball is blue
(f) the second ball is blue, given that the first ball is blue
(g) the second ball is blue, given that the first ball is red
(h) the first ball is blue, given that the second ball is red

Q14 A bag contains 3 blue balls and a red ball. Two balls are taken out at the same time. This is effectively the same as selecting two balls without replacement. Think about this if it isn't obvious. One of the balls will be touched first, even if by just a nanosecond. Once that ball is touched, it is chosen and the second ball must be a different one, chosen from the rest of the balls as if the first one wasn't there. It can't be the same ball again. Find the probability that:

(a) both balls are blue

(b) both balls are red

(c) both balls are green

Q15 A pack of 52 cards is shuffled. Then a hand of three cards is dealt. Find the probability that:

(a) the first card is an ace

(b) the first two cards are aces

(c) all three cards are aces

(d) the first card is an ace, the second a king and the third a queen

(e) the first card is an ace and the other two are kings

(f) the third card is an ace, given that the first two were aces

(g) the second and third cards are both aces, given that the first was an ace

(h) the second card was an ace, given that the first wasn't

(i) the second card is an ace given that the first was a spade

(j) the second card is a club, given that the third card was an ace

Q16 A bag contains 4 black balls and 6 white balls. A ball is taken at random then replaced. Then the balls are mixed and again a ball is taken at random. Find the probability that

(a) the first ball is black and the second white

(b) both balls are white

Q17 The same as the last question except that the first ball is not replaced before the second one is chosen. Find the probability that

(a) the first ball is black and the second white

(b) both balls are white

Q18 A standard pack of 52 cards is shuffled, then two cards are taken in succession without replacement. Find the probability that

(a) both are hearts

(b) the first is a heart, the second a spade

(c) both are aces

Q19 From a pack of 52 cards, a hand of four cards is dealt. Find the probability that

(a) all four cards are aces

(b) all are hearts

(c) the first three are hearts

(d) the first three are hearts and the last is a diamond

Q20 A pack of cards is shuffled and laid out in a line. Find the probability that
(a) the first six cards are all spades.
(b) the first and last cards are Kings

Solve

Q51 It rains on 20% of days. But, if it rained the previous day, the probability of it raining is 60%. Find the probability that it will rain Monday and Tuesday next week, but not Wednesday

Q52 You toss two coins. Both are bent such that it gives heads 60% of the time. What is the probability that both are heads or both are tails?

Q53 If you toss 4 coins, what is the probability that you will get 3 heads and a tail?

Revise

Revision Set 1

Q61 You pick a card from a normal pack of 52. Find the probability of getting
(a) the 2 or 3 of diamonds
(b) a red 8 or a red 9
(c) an ace or a heart
(d) a picture card or a black card

Q62 If the probability of a matchbox landing on its end is 8% and on its side is 22%
(a) what is the probability it will land on its end or its side?
(b) what is the probability it will land flat?
(c) what is the probability that it will land on its end or side or flat?

Q63 You spin a spinner that has equal chance of coming up 1, 2 or 3, then another that has equal chance of coming up 1, 2, 3 or 4.
(a) Find the probability of getting a 1 on the first spinner and a 4 on the second
(b) Find the probability of getting a 1 on both

Q64 A bag contains 2 black balls and 4 white balls. You pick one at random, replace it, then pick again. Find the probability that
(a) you get a black one on the first pick and a white one on the second
(b) you get two black ones

Q65 A bag contains 2 black balls and 4 white balls. You pick one at random, put it on the table, then pick again. Find the probability that

(a) you get a black one on the first pick and a white one on the second

(b) you get two black ones

Q66 You deal a hand of three cards from a pack of 52. What is the probability that

(a) all three are spades

(b) the first is a spade and the other two are hearts

Answers

Q1	(a) $^{2}/_{6}$	(b) ½	(c) $^{2}/_{6}$	(d) $^{5}/_{6}$	(e) $^{4}/_{6}$	(f) $^{2}/_{3}$
Q2	(a) $^{2}/_{52}$	(b) $^{8}/_{52}$	(c) $^{12}/_{52}$	(d) ¼	(e) ¾	(f) 1
Q3	(a) $^{3}/_{10}$	(b) $^{8}/_{10}$	(c) $^{1}/_{5}$	(d) 0		
Q4	(a) 42%	(b) 58%	(c) 1	(d) 70%		
Q5	(a) 45%					
Q6	$^{1}/_{10}$					
Q7	(a) $^{1}/_{36}$	(b) $^{1}/_{36}$	(c) $^{1}/_{36}$	(d) $^{1}/_{36}$		
Q8	(a) 0.15	(b) 0.15	(c) 0.35	(d) $^{1}/_{35}$	(e) $^{1}/_{96}$	(f) $^{1}/_{7}$0
Q9	(a) 0.004488	(b) 0.032912	(c) 0.0264	(d) 0.021912		
Q10	(a) $^{1}/_{52}$	(b) $^{4}/_{52}$	(c) $^{8}/_{52}$	(d) $^{3}/_{13}$	(e) $^{12}/_{52}$	(f) ¼
	(g) ½	(h) $^{26}/_{52}$	(i) $^{16}/_{52}$	(j) $^{28}/_{52}$	(k) $^{32}/_{52}$	(l) $^{26}/_{52}$
Q11	(a) $^{1}/_{7}$	(b) ¾	(c) $^{11}/_{14}$	(d) $^{3}/_{14}$	(e) $^{13}/_{28}$	(f) $^{22}/_{28}$
	(g) $^{4}/7$					
Q12	(a) $^{9}/_{16}$	(b) $^{1}/_{16}$	(c) 0	(d) ¾	(e) ¾	(f) ¾
	(g) ¾	(h) ¾				
Q13	(a) ½	(b) 0	(c) 0	(d) ¾	(e) ¾	(f) $^{2}/_{3}$
	(g) 1	(h) 1				
Q14	(a) ½	(b) 0	(c) 0			
Q15	(a) $^{1}/_{13}$	(b) $^{3}/_{663}$	(c) $^{24}/_{132600}$	(d) $^{64}/_{132600}$	(e) $^{48}/_{132600}$	(f) $^{1}/_{25}$
	(g) $^{6}/_{2550}$	(h) $^{4}/_{51}$	(i) $^{1}/_{13}$	(j) ¼		
Q16	(a) $^{24}/_{100}$	(b) $^{36}/_{100}$				
Q17	(a) $^{24}/_{90}$	(b) $^{1}/_{3}$				
Q18	(a) $^{156}/_{2652}$	(b) $^{169}/_{2652}$	(c) $^{12}/_{2652}$			
Q19	(a) $^{24}/_{6497400}$	(b) $^{17160}/_{6497400}$	(c) $^{1716}/_{132600}$	(d) $^{22308}/_{6497400}$		
Q20	(a) $^{1235520}/_{14658134400}$ or 0.0000843 approx			(b) $^{156}/_{2652}$		
Q51	4.8%	Q52 52%	Q53 ¼			
Q61	(a) $^{1}/_{26}$	(b) $^{1}/_{13}$	(c) $^{16}/_{52}$ (d) $^{19}/_{52}$			
Q62	(a) 30%	(b) 70%	(c) 1			
Q63	(a) $^{1}/_{12}$	(b) $^{1}/_{12}$				
Q64	(a) $^{2}/_{9}$	(b) $^{1}/_{9}$				
Q65	(a) $^{4}/_{15}$	(b) $^{1}/_{15}$				
Q66	(a) $^{1716}/_{132600}$	(b) $^{2028}/_{132600}$				