M1 Maths

P1-1 Probability

- the meaning of probability
- finding approximate probabilities using data (from experiment and pre-existing) and understanding the reliability of the approximations
- estimating probabilities by guesswork
- finding exact probability using indifference, deciding when it applies
- calculating expected frequency

Summary Learn Solve Revise Answers

Summary

Probability is the fraction of times that something would happen in the long run.

Probability can be approximated using data from an experiment. An experiment consists of a number of trials. A trial has a number of possible outcomes. The set (or list) of the possible outcomes is called the sample space. An occurrence of the outcome we are interested in is called a success. The number of successes is the frequency. Dividing the frequency by the number of trials gives the relative frequency. Relative frequency is an approximation to probability, the more trials, the more reliable the approximation.

An alternative to getting data from an experiment is to use pre-existing data.

Probabilities can also be estimated by guesswork without conducting an experiment or using pre-existing data.

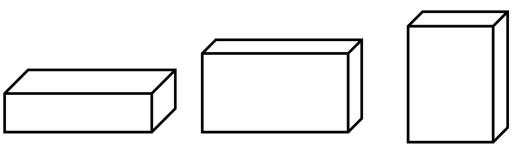
When there is no difference between the outcomes that could make one more likely than any other, we can determine the probabilities exactly using indifference.

The expected frequency of an outcome is the product of its probability and the number of trials. The most likely number of successes is generally obtained by rounding the expected frequency to the nearest whole number.

Learn

Probability

Suppose you dropped a matchbox. It could land three ways – flat, on an edge or on an end.



You would probably agree that it is more likely to land flat than on its end.

Probability is a measure of how likely something is using a number (a fraction between 0 and 1 or percentage between 0% and 100%).

The **probability** of the matchbox landing flat is the fraction of times it would land flat in the long run; in other words, the fraction of times it would land flat if we dropped it a very large number of times.

It might be that it would land flat ¾ of the times we tried it in the long run, i.e. 0.75 or 75% of the times. Then we would say that the probability that it would land flat on any drop is ¾ or 0.75 or 75%.

That gives us a good indication of how likely it is to land flat.

Probability is the fraction of times something will happen in the long run

Finding probabilities

The probability of an event can be found in a number of ways.

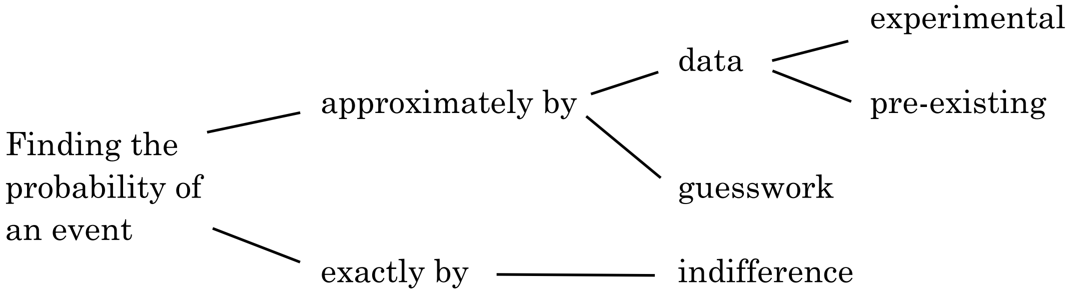
We can perform an **experiment** and collect **data**. The experiment might consist of dropping a matchbox 200 times and seeing how many times it landed on its edge.

Sometimes it won't be feasible to do an experiment, e.g. if we wanted to know the probability of dying from ebola. But in such cases data on how many people have caught it and how many have died might already exist and we can use that. Such data is called **pre-existing data**.

Sometimes, if we only want a very rough idea of the probability, we can use **guesswork**.

In situations where there is no difference between any of the outcomes that could make one more likely than any other, like finding the probability that a coin comes down heads, we can use **indifference**.

These methods are summarised in the diagram below.

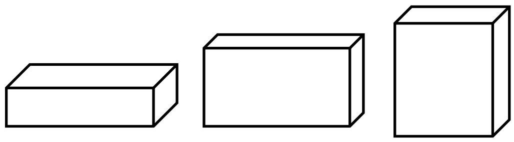


Finding Approximate Probabilities using Experimental Data

Let's say we want to find the probability that our matchbox will land on its edge. We would do an ***experiment***. An experiment in probability means trying something a number of times to see what happens. In this case, it means dropping the matchbox quite a few times to see how many times it lands on its edge.

Each time we drop the matchbox is called a **trial**. If we drop it 200 times in our experiment, then the experiment consists of 200 trials.

Each trial has three possible **outcomes** – flat, on an edge and on an end. An outcome is one of the things that can happen in a trial.



Sample space is a term used for the list (or set) of all possible outcomes of a trial. The sample space in the matchbox experiment is: flat, edge, end.

We are interested in the outcome, edge, so we call landing on an edge a **success**. This doesn't mean that on its edge is any better than flat or on its end; it is just the word we use for the outcome we are trying to find the probability of.

We do our experiment of 200 trials and count the number of successes (the number of times the matchbox land on an edge). Let's say there are 38 successes (it lands on an edge 38 times). We say that the **frequency** is 38. The word frequency in everyday usage means how often something happens, but in probability, it just means the number of successes.

The **relative frequency** is the frequency relative to the number of trials. It is the frequency divided by the number of trials. In this case the frequency is 38 and the number of trials is 200, so the relative frequency is 38 ÷ 200, which is 0.19.

This relative frequency is an approximation to the probability – the higher the relative frequency, the higher the probability and the more likely the outcome is. But the relative frequency doesn't give us the exact probability – it is just an approximation. This is because, if we did the experiment again, we might get a relative frequency of only 0.16 or of 0.21.

What we find is that the relative frequency will vary a lot between experiments if the number of trials is small, but it will become much more consistent if the number of trials is large. If we did experiments with just 4 trials, we would find that sometimes we get 2 successes, giving us a relative frequency of 0.5, sometimes we would get 0 successes, giving a relative frequency of 0.

If we did experiments with a few hundred trials, though, we would get relative frequencies all quite close together, maybe around 0.18.

If we did experiments with a few billion trials, we would find the relative frequencies were all very close together, maybe all very close to 0.183. We would know that, if we did another experiment of a few billion trials, we would again get a relative frequency very close to 0.183.

As we do more trials, the relative frequency becomes a more and more relaiable approximation to the probability. Theoretically, to get an exact probability, we would have to do an infinite number of trials. As we never have time for that, probabilities from experiments are always approximate.

Practice

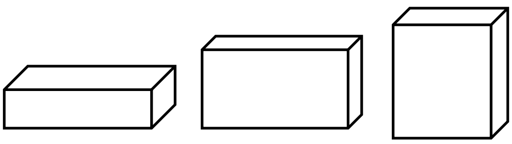
Q1 Write down the word that should replace each letter in the passage below.

Jethro wanted to find the probability that, if he stuck his tongue out at a stranger as he passed them on the street, they would stick their tongue out at him. In other words, he wanted to find the …(a)… of times it would happen in the …(b)… …(c)…

He decided to do an …(d)… with 50 …(e)… The …(f)… …(g)… consisted of just two …(h)…: that they would stick their tongue out and that they wouldn't. Sticking their tongue out would be a …(i)…

He got 11 successes out of the 50 trials, giving him a …(j)… of 11 and a …(k)… …(l)… of $^{11}/_{50}$ or 0.22 or …(m)…%. This was an approximation to the …(n)… To get a more reliable result, he would have to do another …(o)… with more …(p)…

Q2 When you drop a matchbox, it can land flat, on its side of on its end – as in this diagram.



Suppose you conducted an experiment to find the probability that it will land on its end. You drop the matchbox 100 times and find it lands on its end just 4 times.
(a) What is the number of trials?
(b) What are the possible outcomes?
(c) What is a success?
(d) What is the frequency?
(e) What is the relative frequency?
(f) What can you say about the probability?
(g) How could you get a more reliable estimate of the probability?

There are quite a few new words in this module (or at least, words used in new ways). You need to learn them. They are in red, so you can look through and test yourself to make sure you know what each one means in the context of probability.

Also, you might like to actually do an experiment to find the probability of something like a matchbox landing on its edge or your mother's best glasses breaking if you drop them on the floor. This will help to reinforce these ideas in your mind.

Finding Approximate Probabilities Using Pre-existing Data

When we do an experiment, we collect ***data***. The number of trials and the frequency of success are the data. If we drop a matchbox 100 times and find that it lands on its end 4 times, that is data. When the data is collected in an experiment, it is called ***experimental data***.

But suppose we wanted to know the probability that a person who catches bird flu will die of it.

We could do an experiment to collect experimental data. We could get 1000 people and infect them all with bird flu. Then we could see how many die. If 150 die, then the probability that a given person will die is about 15%. But, this kind of experiment is frowned upon, so we don't.

However, quite a few people have already caught bird flu and records have been kept of who died and who survived. We can use this data instead of collecting our own with an experiment. This data is called ***pre-existing data*** because it existed before we needed it. If the records show that 184 people have caught bird flu and 31 of these have died, then the probability of dying would be about $^{31}/_{184}$ or 0.17 or 17%.

So approximating probabilities can be done by using either experimental data (which we produce in an experiment) or pre-existing data (which already exists and which we just have to find).

Practice

Q3 Use the given pre-existing data to find approximate probabilities for the following. Give each answer as a common fraction, as a decimal fraction and as a percentage.

(a) It has snowed in Canberra 11 out of the past 32 years. What is the probability that it will snow there next year?

(b) Of 125 bacterial cultures produced in a pathology lab, 14 were resistant to penicillin. What is the probability that the next culture is resistant?

(c) 425 students at Colville High School are right-handed; 72 are left-handed. What is the probability that a student picked at random will be left-handed? [Be careful here – you have to work out the total number of students yourself.]

(d) A matchbox was dropped several times. It landed on its end 14 times, on its side 42 times and flat 165 times. What is the probability of it landing flat?

(e) Of the 127 dogs that were brought to Sam's Veterinary Practice in Perth last week, 12 had eye problems. What is the probability that a dog picked at random in Perth will have eye problems? [Trick question – be careful!]

Here are some general questions on what we have covered so far. Make sure you can answer them all before you go on.

Practice

Q4 Roger tossed a bent coin 100 times and found it landed heads up 58 times and tails up 42 times.

(a) How many trials were there in the experiment?

(b) How many possible outcomes were there for each trial?

(c) What were the outcomes?

(d) The data used was the number of trials and the number of heads. To Roger, was this experimental or pre-existing data?

(e) What is the probability of it landing heads up next toss?

(f) Is this probability exact or approximate?

(g) If approximate, how many trials would be needed to get an exact probability?

Q5 (a) What is the probability of something which you know will happen?

(b) What is the probability of something which you know will not happen?

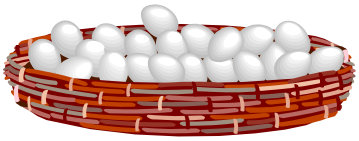
(c) What is the probability of something happening if it is equally likely to happen or not happen?

(d) What might the probability be for something that might happen, but probably won't.

Q6 Cedric dropped 200 eggs, one at a time from 1 metre onto a wooden floor to find the probability that they will break. 184 of them broke.

(a) Based on this, what is the probability that the next egg will break?

(b) Did Cedric use experimental data or pre-existing data to determine the probability?



Q7 Carol looked up some rainfall records to find the probabilities of rain on her birthday, June 17, and her boyfriend's birthday, October 9. She found that in the past 26 years it had rained 5 times on June 17 and 8 times on October 9.

(a) Based on this, what is the probability that it will rain on June 17?

(b) Also based on this, what is the probability that it will rain on October 9?

(c) Did Carol use experimental data or pre-existing data to work out the probabilities?

Q8 (a) Which would be the most reliable of the estimates of probability from Q7a and Q7b?

(b) Why?

Q9 (a) Use the Internet to find the population of Australia and the number of road deaths in Australia last year. Use your results to estimate the probability that a given person will die on the roads next year.

(b) To answer part (a) did you use experimental data or pre-existing data.

Q10 Suppose you wanted to do an experiment to find the probability that, if you toss three coins, you would get two heads and a tail.

(a) Draw up a table to record the data.

(b) How many trials do you think you might use if you wanted to get the answer to within about 1%? (Only a rough answer required.)

(c) How would you calculate the probability from your results?

Approximating Probabilities by Guesswork

If someone is shown a matchbox and asked 'What is the probability that it will land on an end if we toss it?' they could give a reasonable answer without doing an experiment or collecting any pre-existing data. They would use guesswork.

They might look at the box and notice that the ends are a lot smaller than the other faces and so think that it is not very likely to land on an end. They might guess 5%.

This would be quite a reasonable estimate. 60% wouldn't be. So guesswork can be used to get an approximation of a probability. Obviously it is not as reliable as using data (from an experiment or pre-existing), but it is a lot quicker and easier.

Imagine you were at an amusement park and one of the stalls involved rolling a $1 coin down a shute and onto a table. Imagine too that there was a circle 1 mm bigger than a $1 coin at the far end of the table and that if the coin landed inside the circle, you won $50. Would you play?

Without trying it, you could estimate that the probability of getting the coin in the circle was a lot less than $^1/_{50}$, meaning that it would probably cost you a lot more than $50 to have enough goes to win $50. So you wouldn't play.

Practice

Q11 Approximate the probability of each of the following events by guesswork.

(a) that, if you dropped a matchbox, it would land on its edge
(b) that, if you tossed 6 coins, 5 of them would come down heads
(c) that you will be sick next Tuesday
(d) that you will fall in love next year
(e) that your mother will get a new cat next year
(f) that your best friend would get the same answer to the last question as you
(g) that the sun will shine on you tomorrow
(h) that you will go to Russia some time in your life

This box contains information about probability which is probably not needed to pass tests, but which will give a fuller understanding of the subject. Reading it is optional. Beware, however, that it contains tales of cruelty to old ladies and discussion of religion and the afterlife.

Note on Unrepeatable Scenarios

Sam says he has a 90% chance of passing his Year 12 Maths exam. 'Chance' is a word with similar meaning to likelihood. But the fact that he has put a number on it, means he is using it in the same way as probability. He could have said that the probability that he will pass the exam is 90%.

This is a meaningful statement. It tells us that he is quite confident, though not certain.

But, by our definition of probability, this is saying that, if he took the exam a very large number of times, he would pass it 90% of the time. This is not really what he means, because (a) he can only take it once, and (b) if he could take it lots of times, eventually he would know all the answers and would pass it every time.

Clearly Sam is using the word 'probability' in a slightly different way.

Similarly, we can talk about the probability that grandma would die if she fell out of the bedroom window. If grandma was 48 years old and the bedroom was on the ground floor, we might put the probability at 1%; but if she was 85 and the bedroom was on the third floor, we might put it at 90%.

Let's say Grandma is 85 and the bedroom is on the third floor. It is quite meaningful to say that the probability that she would die is 90%. But it is not meaningful to say that if she fell out 1000 times, she would die 900 times. This is not a repeatable scenario. We could theoretically push 1000 similar grandmothers out of the window and see how many die, but our figure of 90% is certainly not based on such an experiment.

So how do we get the figure of 90%. Obviously not in the ways that we have described above for dropping matchboxes, rolling dice etc.

No, what we do is decide that Grandma dying is ***as likely as*** something with a probability of 90%. The concept of 'as likely as' is an intuitive concept which cannot readily be defined mathematically. It can be thought of in terms of how surprised one would be if it happened. If something has a 90% probability, we wouldn't be at all surprised if it happened, but if it has just a 1% probability, we would be quite surprised. If it had a 0.000 001% probability, we would be very surprised. So saying that Grandma has a 90% chance of dying is basically saying that we wouldn't be at all surprised – in fact it is most likely that she will.

So, we can assign probabilities in unrepeatable scenarios. We do it by saying that the outcome seems ***as likely as*** an outcome of a repeatable experiment with a certain probability.

Of course, even though most people would think the probability that Grandma will die is something like 90%, different people might say 80%, 95% etc. The number we come up with does involve guesswork. So it is not exact; 92% wouldn't be wrong; nor would 75%. There is no right or wrong – just different estimates.

A more extreme example of this kind of thing is the answer to 'What is the probability that there is an afterlife?' Some religious people would say 100%. Convinced atheists might say 0%. Most people would be somewhere in between. None of them are right or wrong. They are just indications of how likely it seems to different people based on what they have experienced in life.

While such statements of probability are meaningful, mathematics deals mainly with the repeatable scenarios where we can get data from a large number of trials and thus answers that everyone would agree on.

Exact Probabilities Using Indifference

Some probabilities can be calculated without collecting any data. What's more, they can be calculated exactly. For example, we know that the probability of getting a head if we toss a normal coin is 50%. It is exactly 50%. That's not just an approximation.

When we toss a coin, there are two possible outcomes – heads up and tails up. There is no difference between the two sides of the coin which can make one more likely to come up than the other. [The two sides do have different pictures on them, but that isn't going to make it more likely to land one way than the other.] Therefore, in the long run, the coin will land equally often on both sides, i.e. 50% heads and 50% tails. So the probability of getting a head is 50%.

Likewise, when you roll a die, there is no difference between the six outcomes which could make any face more or less likely to come up than any others. Therefore the die will land on each number an equal numbers of times in the long run. Each number will come up $^1/_6$ of the times in the long run.

So:

the probability of getting a 1 is $^1/_6$;

the probability of getting a 2 is $^1/_6$;

the probability of getting a 3 is $^1/_6$;

the probability of getting a 4 is $^1/_6$;

the probability of getting a 5 is $^1/_6$;

the probability of getting a 6 is $^1/_6$.

In the above examples of a coin and a die, there is no difference between the outcomes that can make any one any more likely than any other. We say that the outcomes are indifferent (in this case meaning 'not different' rather than the more everyday use of the word to mean 'not caring'). In calculating probabilities using this idea, we say we are using **indifference**.

Clearly, we can only use indifference where the outcomes are indifferent. We can use it with a coin or a die, when picking a card from a pack etc., but we can't use it when tossing a matchbox or deciding the probability that a stranger will punch us if we pull a face at him (or her).

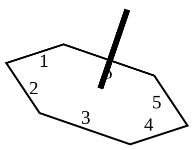
It is because the outcomes heads and tails on a coin are indifferent and because the outcomes 1 to 6 on a die are indifferent that they are used a lot in games where we need something that is fair.

Practice

Q12 For each of the following situations, say whether indifference can be used to find the probability we are after or whether we would have to collect data. If there is indifference, work out the probability wanted.

(a) A coin bent towards the heads side is tossed. We want to know the probabilities of getting a head and of getting a tail.

(b) A spinner is made by sticking a match through the middle of a regular hexagon made of card. The numbers 1 to 6 are written on the edges. The number obtained is the number on the edge that is resting on the table. We want to know the



probability of getting a 3.

(c) A bag has three marbles in it. They are the same size, weight and feel, but different colours (one is blue, one is green and one is red). We shake the bag then put in a hand without looking and pick out a marble. We want to know the probability of getting the blue one.

(d) A bag has three marbles in it. They are the same colour, but different sizes. We shake the bag then put in a hand without looking and pick out a marble. We want to know the probability of getting the biggest one.

(e) A student throws her maths book at the teacher from the other side of the classroom. The book might hit the teacher or it might miss. We want to know the probability that it will score a hit.

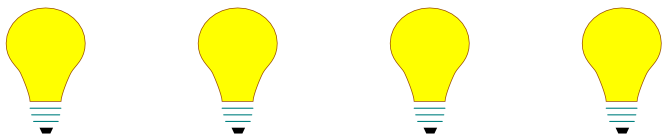
(f) A standard pack of 52 playing cards is shuffled, then someone picks a card without looking. We want to know the probability that they will get the Ace of Hearts.

(g) A three-legged stool is thrown onto the floor. We want to know the probability that it will break.

(h) A man and two women compete against one another on a television quiz show. There will be one winner. We want to know the probability that it is a woman.

(i) A Siamese cat and a Chihuahua dog are sitting outside the front door. The owner throws them a bone. We want to know the probability that the cat will end up with the bone.

(j) Mrs Bright buys four 18 watt light globes, all the same brand, from the supermarket. She puts them in the four identical light fittings out on the veranda of her house. She turns them all on together. We want to know the probability that the one at the left end lasts longest.



(k) A six-year-old boy walks across the thin ice on a frozen lake. He might fall through the ice and he might not. We want to know the probability that he does.

(l) We want to know the probability that Wednesday will be the warmest day of the second week of July next year.

(m) The Phys. Ed. teachers play the Maths teachers at cricket. We want to know the probability that the Maths teachers win.

(n) A matchbox has the numbers 1 to 6 on its six faces. We want to know the probability that, if we drop it, it will land with face 3 upwards.

Expected Frequency

As we have seen, the relative frequency and the probability are roughly equal, the more trials, the closer they are likely to be.

Because of this, we can use the relative frequency as an approximation for the probability. But we can also use the probability as an approximation for the relative frequency.

If the probability that a thumb tack lands point-up is 0.55 or 55%, then, if we drop it a few times, the relative frequency will be about 0.55 or 55%. In other words, it will land point-up about 55% of the times we drop it. Let's say we drop it 83 times. The frequency (number of times it lands point-up) will be about 55% of 83 (0.55 × 83), which is about 45.65. We call this the **expected frequency** or ***expected number of successes***.

Note that the expected frequency doesn't have to be a whole number. If we wanted the **most likely frequency** (most likely number of successes), this can generally be obtained by rounding the expected frequency to the nearest whole number. In this case, it would be 46.

So, the expected frequency is the probability multiplied by the number of trials and the most likely frequency is generally the expected frequency rounded to the nearest whole number.

If Robbie has a 45% probability of getting a ball into the basket from 6 metres and we gave him 70 shots, the expected number of baskets would be 45% of 72, which is 32.4. He would be most likely to get 32.

Practice

Q13 The probability of a matchbox landing on its side is 9.5%. If we drop it 120 times,
- (a) what is the expected number of times it would land on its side?
- (b) what is the most likely number of times?

Q14 The probability of Kristy shooting a basket from 10 m is 24%. If she has 60 shots from 10 m,
- (a) what would be the expected number of baskets?
- (b) how many is she most likely to get?

Q15
- (a) The probability of a certain bent coin landing heads up is 0.55. What is the most likely number of times it would land heads up if you toss it 20 times?
- (b) Will it land heads up this many times out of every 20 tosses? Why?

Q16 If the probability of a straight coin landing heads up is ½, is it possible to get 8 heads from 10 tosses?

Q17 63% of guinea pigs die before they are 5 years old.

(a) What is the probability that a given new-born guinea pig will live more than 5 years?

(b) Out of a batch of 57 newly born guinea pigs, how many are likely to be still alive on their 5th birthday?

Q18 Ball bearings are mass produced by a machine. A sample of 60 ball bearings from the machine is chosen at random for quality testing. 4 were found to be sub-standard. How many sub-standard ball bearings are there most likely to be in a production run of 20 000?

Some Common Misconceptions in Probability

Memory

Bruce has a normal 20 c coin which, in the long run, comes down equal numbers of heads and tails. He decides to toss it 6 times. The first 5 times, the coin comes down heads. Is the probability of getting a head on the sixth toss

A: 0.5 B: less than 0.5 or C: more than 0.5

A lot of people will answer B. They quote the law of averages, saying that the coin must come down equal numbers of heads and tails, so it has to come down more tails now to make up for the excess of heads in the first 5 tosses.

This is not true: the answer is A. The coin has no memory of what happened in previous tosses and the probability is always 0.5

The relative frequency will even out in the long run. But this doesn't happen by the coin making up for the first 6 tosses later; it happens by the effect of those first 6 tosses becoming less and less significant as we do more and more tosses. In fact, random walk theory tells us that, if we tossed a coin an infinite number of times and kept track of the relative frequency, the relative frequency would change between <0.5 and >0.5 an infinite number of times. This would still be true even if we cheated and made the coin come down heads the first 100 times.

Choice of outcomes

When we toss a coin, it will come down heads 50% of the times and tails 50%. But some will say that they are not the only two possible outcomes: it could fall off the table or be scooped up in mid-flight by a passing alien space ship.

This is true. But what we are saying is that, of the times the coin produces a head or a tail, 50% will be heads and 50% tails.

In a probability experiment, we decide on the outcomes we are interested in and ignore others. In a coin tossing experiment, we might decide that that our outcomes are: land on the table; land on the floor; land in someone's lap; be scooped up by a space ship. And then we might find that the probability of landing on the table is 62% and so on.

Which Way Round the Coin Starts

When we toss a coin, some people think that the probability of getting a head will be different if the coin is heads-up before we toss it from if it is tails-up before we toss it.

The idea of tossing a coin is that it spins dozens of times in the air and no one has the skill to make it spin 37½ times to come down tails rather than 37 times to come down heads. So it makes no difference. If you aren't convinced, this is an easy idea to test experimentally. Toss a coin from the heads-up position 1000 times and record the number of heads and the number of tails, then do the same starting with the coin tails-up.

Some people can get the coin to land more one way then the other if it only spins two or three times in the air. But most people, seeing that, will consider it cheating and demand it be tossed again. There is also a way of tossing a coin such that it looks as though it is spinning, but it is in fact spinning about a vertical axis and thus staying the same face up. This is hard, though, and not many people can do it.

Practice

Q19 Bruce has a normal 20c coin which, in the long run, comes down equal numbers of heads and tails. He decides to toss it 6 times. The first 5 times, the coin comes down heads. Is the probability of getting a head on the sixth toss
A: 0.5 B: less than 0.5 or C: more than 0.5 ?

Solve

Q51 It rained on 12th October in 21% of the past 80 years. If it rains on 11th October this year, will the probability of rain on the 12th still be 21%?

Q52 The probability that a matchbox will land on it end is 6%; the probability that it will land on its side is 21%. What is the probability that it will land flat?

Q53 When you roll a die, what is the probability of getting a number greater than 3?

Q54 If you toss a 10c coin and a 20c coin, what is the probability that
(a) both will come down heads?
(b) both will come down tails?
(c) one will come down heads, the other tails?

Revision Set 1

Q61 What is probability?

Q62 When Hillary, a darts player, aims for the bullseye, she might get it or she might not. She wanted to know the probability that she would get it, so she tried 50 times and got it 11 times.

(a) Did she use experimental data or pre-exiting data?
(b) What was the number of trials?
(c) What were the possible outcomes (the sample space)?
(d) What was a success?
(e) What was the frequency?
(f) What was the relative frequency?
(g) What can she say about the probability?
(h) How could she get a more reliable estimate of the probability?

Q63 Belle walked down the street and offered 50 passers-by a 20c coin to find the probability that they will take it. 47 took it.

(a) Based on this, what is the probability that the next person will take one?
(b) Did Belle use experimental data or pre-existing data to determine the probability?

Q64 Tom found a book on cats in the library. It said that, based on the few thousand cats the author had studied, 20% of cats live past 20 years of age.

(a) Based on this, what is the probability that Tom's new cat will die before it is 20?
(b) Did Tom use experimental data or pre-existing data to work out the probability?
(c) Would Belle's or Tom's estimates of probability be the more reliable?
(d) Why?

Q65 What is the probability of something which you know will happen every time you try it?

Q66 For each of the following situations, say whether indifference can be used to find the probability we are after or whether we would have to collect data. If there is indifference, work out the probability wanted.

(a) A rat is offered two pieces of apple, one at the left end of the cage and one at the right end. We want to know the probability that it will go for the one at the left end.
(b) A girl selects a letter as follows. She opens a book at a random page, and sticks a pin into the page without looking. She then looks to see what letter is closest to the pin hole. We are interested in the probability that it is an e.

Q67 The probability of a certain matchbox landing on its end is 4%.

(a) What is the expected number of times it would land on its end if you dropped it 1660 times?
(b) What is the most likely number of times?
(c) Is it possible for the matchbox to land on its end 6% of the 1660 times it is dropped?

Revision Set 2

Q71 What is probability?

Q72 Mr Knuckles wanted to know the probability that he would hit a wrong note when playing 'Happy Birthday' on his ukelele. So he played it 25 times and found he hit a wrong note on 21 of those.
(a) Did he use experimental data or pre-exiting data?
(b) What was the number of trials?
(c) What was the sample space?
(d) What was a success?
(e) What was the frequency?
(f) What was the relative frequency?
(g) What can he say about the probability?
(h) How could he get a more reliable estimate of the probability?

Q73 Abbey walked down the street and stuck her tongue out at 500 passers-by to find the probability that they return the gesture. 185 did.
(a) Based on this, what is the probability that the next person will?
(b) Did Abbey use experimental data or pre-existing data to determine the probability?

Q74 Jeff found a book on dogs in the library. It said that, based on the 240 dogs the author had studied, 8% of dogs live past 15 years of age.
(a) Based on this, what is the probability that Jeff's new dog will live to be 15?
(b) Did Jeff use experimental data or pre-existing data to work out the probability?
(c) Would Jeff's or Abbey's estimate of probability be the most reliable?
(d) Why?

Q75 What is the probability of something which you know will never happen?

Q76 For each of the following situations, say whether indifference can be used to find the probability we are after or whether we would have to collect data. If there is indifference, work out the probability wanted.
(a) A three-legged stool is thrown onto the floor. It can land on its feet, on its side, or upside down. We want to know the probability that it will land upside down.
(b) A tin of dog meat is used instead of tossing a coin at the start of a cricket game. The tin is tossed. It is agreed that if it lands up the right way, team A wins the toss; if it lands upside down, team B wins the toss; if it lands on its

side, it won't be counted but will be tossed again. We want to know the probability that team A wins the toss.

Q77 The probability of a pair of coins both landing Heads is 25%.

(a) What is the expected number of times this would happen if you tossed them 47 times?

(b) What is the most likely number of times?

(c) Is it possible for the matchbox to land on its end only 6% of the 2 000 times it is dropped?

Revision Set 3

Q81 What is probability?

Q82 Foster wanted to know the probability that the sky would be clear at sunrise in May. So every morning one May he got up and checked. He found it was clear on 8 days.

(a) Did he use experimental data or pre-exiting data?

(b) What was the number of trials?

(c) What was the sample space?

(d) What was a success?

(e) What was the frequency?

(f) What was the relative frequency?

(g) What can he say about the probability?

(h) How could he get a more reliable estimate of the probability?

Q83 Suzi found 32 YouTube videos of people singing 'Happy Birthday to You'. 21 were female, the rest were male.

(a) Based on this, what is the probability that a random person singing 'Happy Birthday to You' on YouTube will be male?

(b) Did Suzi use experimental or pre-existing data to determine the probability?

Q84 Blurgle did some research on toothpaste use. Of the 400 people he surveyed, 85 used Splodge.

(a) Based on this, what is the probability that a random person will use Splodge?

(b) Did Blurgle use experimental or pre-existing data to work out the probability?

(c) Would Suzi's or Blurgle's estimates of probability be the more reliable?

(d) Why?

Q85 What is the probability of something happening if it is equally likely to happen or not happen?

Q86 For each of the following situations, say whether indifference can be used to find the probability we are after or whether we would have to collect data. If there is indifference, work out the probability wanted.

(a) A girl selects a letter as follows. She writes every letter 4 times in different random places on a sheet of paper, then sticks a pin into the page without looking. She then looks to see what letter is closest to the pin hole. We are interested in the probability that it is an f.

(b) Seven coins are thrown into a bowl. We want to know the probability that no coin will end up lying on top of another.

Q87 The probability of a die coming up with a 5 is $^1/_6$.

(a) What is the expected number of 5s if you roll a die 200 times?

(b) What is the most likely number of 5s?

(c) Is it possible to get 192 5s?

Answers

Q1	(a) fraction	(b) long	(c) run	(d) experiment
	(e) trials	(f) sample	(g) space	(h) outcomes
	(i) success	(j) frequency	(k) relative	(l) frequency
	(m) 22	(n) probability	(o) experiment	(p) trials

Q2 (a) 100

(b) Landing on its end; landing on its side; landing flat

(c) Landing on its end

(d) 4

(e) 0.04

(f) It is roughly 0.04

(g) Do more trials

Q3 (a) $^{11}/_{32}$, 0.34, 34%

(b) $^{14}/_{125}$, 0.11, 11%

(c) $^{72}/_{497}$, 0.14, 14%

(d) $^{165}/_{221}$, 0.75, 75%

(e) You can't tell because the percentage of dog who visit the vet with eye problems will be more than the percentage of dogs in Perth with eye problems.

Q4	(a) 100	(b) 2	(c) heads up, tails up	
	(d) experimental	(e) 0.58	(f) approximate	(g) infinity

Q5 (a) 100% or 1

(b) 0% or 0

(c) 50% or 0.5 or ½

(d) Anything more than 0 and less than 50%

Q6 (a) About 92% (b) Experimental data

Q7 (a) $^5/_{26}$ or 0.192 or 19.2% (b) $^8/_{26}$ or 0.308 or 30.8% (c) pre-exiting data

Q8 (a) 6a (b) Because there were 100 trials compared to 26 for the others

Q9 (a) Should be somewhere around 0.005% (b) pre-existing data

Q10 (a)

Outcome	Tally	Total	Probability
Two heads and a tail			
Other			

(b) Probably somewhere between 200 and 500

(c) Divide the number of times you got two heads and a tail by the total number of trials.

Q12 (a) We would need data.

(b) We could use indifference. The probability would be $^1/_6$.

(c) We could use indifference. The probability would be $^1/_3$.

(d) We would need data.

(e) We would need data.

(f) We could use indifference. The probability would be $^1/_{52}$.
(g) We could use indifference. The probability would be ½.
(h) If we assume that the quiz if fair, we could use indifference and the probability would be $^2/_3$. If the questions were all about hotted-up cars or all about chick flicks, we would need data.
(i) We would need data.
(j) We could use indifference. The probability would be ¼.
(k) We would need data.
(l) We could use indifference. The probability would be $^1/_7$
(m) We would need data. (Everyone knows maths teachers are better at cricket than Phys. Ed. teachers.)
(n) We would need data.

Q13 (a) 11.4 (b) 11
Q14 (a) 14.4 (b) 14
Q15 (a) 11 (b) No, because you only guarantee 0.55 of the tosses for an infinite number of trials.
Q16 Yes. (It won't happen very often, bit it will happen occasionally.)
Q17 (a) 37% (b) 36
Q18 1333
Q19 A. The probability for any toss is always 0.5. The coin has no memory of what has happened on previous tosses.

Q51 No, because rainy days tend to come in clusters. If it rains on 11th of October, it will be more likely to rain on 12th October than if it doesn't. The probability is only 21% if you have no information about the previous days.
Q52 73%
Q53 ½
Q54 (a) ¼ (b) ¼ (c) ½

Q61 Probability is the fractions of times something would happen in the long run.
Q62 (a) experimental (b) 50 (c) bullseye, not bullseye (d) bullseye
(e) 11 (f) 0.22 (g) Roughly 0.22 (h) do more trials
Q63 (a) $^{47}/_{50}$ (b) experimental data
Q64 (a) 80% (b) pre-existing data (c) Tom's
(d) Because there were more trials
Q65 1
Q66 (a) can use indifference, 0.5 (b) we would need data
Q67 (a) 66.4 (b) 66 (c) yes

Q71 Probability is the fractions of times something would happen in the long run.
Q72 (a) experimental (b) 25 (c) hit a wrong note, play all the notes right
(d) hit a wrong note (e) 21 (f) 0.84 (g) Roughly 0.84
(h) play it more times
Q73 (a) 37% (b) experimental data
Q74 (a) 8% (b) pre-existing data (c) Abbey's (d) Because there was more data
Q75 0
Q76 (a) we would need data (b) can use indifference, ½
Q77 (a) 11.75 (b) 12 (b) yes, but extremely unlikely

Q81 Probability is the fractions of times something would happen in the long run.
Q82 (a) experimental (b) 31 (c) clear, not clear (d) clear
(e) 8 (f) $^8/_{31}$ (g) Roughly $^8/_{31}$
(h) do the same again in subsequent years
Q83 (a) $^{11}/_{32}$ (b) experimental data
Q84 (a) 8% (b) experimental data (c) Blurgle's (d) Because she used more data
Q85 0.5
Q86 (a) can use indifference (b) we would need data
Q87 (a) 33.333… (b) 33 (c) yes, but extremely unlikely