M1 Maths

N5-1 Simplifying Surds

- simplifying expressions containing surds

Summary Learn Solve Revise Answers

Summary

To be simplified to standard form, an expression containing surds must:

1. have only natural numbers inside radicals
2. rationalise the denominators of fractions – i.e. have no surds in the denominator
3. have the smallest possible number of surds in the expression
4. have the smallest possible natural numbers inside the radicals

in this order of priority.

There are standard techniques for meeting each of these requirements.

Learn

Introduction

A surd is an irrational root. So $\sqrt{2}$ is a surd, and so is $\sqrt[5]{11.4}$.

However, $\sqrt{9}$ is not a surd, nor is $\sqrt{2¼}$. Both are roots, but both are rational – they are 3 and 1½ respectively.

The $\sqrt{}$ sign is called a radical.

You have met surds before. What we will learn here is how to simplify expressions containing surds to standard form. This technique is a leftover from the days before calculators when working out $\frac{\sqrt{2}}{2}$ as a decimal was much easier than working out $\frac{1}{\sqrt{2}}$ (because the latter involved dividing 1 by 1.4142, whereas the former involved dividing 1.4142 by 2).

It can be argued that simplifying surds is an obsolete technique and doesn't need to be learnt nowadays. But there are a couple of reasons why learning it can be beneficial. The first is that a lot of mathematicians consider answer like $\sqrt{20}$ less professional than an answers like $2\sqrt{5}$. The second is that a technique of rationalising the denominator used in surd simplification is very similar to a technique needed in dealing with complex numbers which you may study later.

Of course, if you're not worried about being a professional mathematician and you don't plan to study complex numbers, then those aren't good reasons. However, if you are going to be tested on simplifying surds, then that might be a good reason to learn how to do it.

Requirements

To be simplified to standard form, an expression containing surds must:

1. have only natural numbers inside radicals
2. rationalise the denominators of fractions – i.e. have no surds in the denominator
3. have the smallest possible number of surds in the expression
4. have the smallest possible natural numbers inside the radicals

This is the order of priority. You don't, for example, leave a larger number of surds so that you can have smaller numbers inside the radicals. It is also generally the order in which you do the simplification.

You should also do any other simplifications that are possible at the end, like cancelling fractions of collecting like terms, e.g. $3\sqrt{6} + 5\sqrt{6} = 8\sqrt{6}$.

Techniques

To satisfy Requirement 1

Get rid of any fractions inside radicals using the identity $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$. For example write $\sqrt{\frac{11}{2}}$ as $\frac{\sqrt{11}}{\sqrt{2}}$.

To satisfy Requirement 2

If the denominator of a fraction is a simple surd or a number multiplied by a surd, you can fix it by multiplying the top and bottom of that fraction by the surd. For instance, if we have $\frac{5}{2\sqrt{3}}$, we multiply top and bottom by $\sqrt{3}$ to get $\frac{5\sqrt{3}}{2\times3}$, which is $\frac{5\sqrt{3}}{6}$.

If the denominator of a fraction has two terms – a number and a surd or two surds, you can fix that by multiplying the top and bottom by the conjugate of the denominator. The conjugate is the same expression but with the sign of one of the terms changed. For example the conjugate of $4 + \sqrt{5}$ is $4 - \sqrt{5}$; the conjugate of $\sqrt{3} - \sqrt{2}$ is $\sqrt{3} + \sqrt{2}$.

So we rewrite $\frac{5}{3-\sqrt{7}}$ as $\frac{5(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})}$. This might look like we've made it worse until we realise that the bottom is then the difference of two squares and can be written as $3^2 - (\sqrt{7})^2$ which is 9 – 7 i.e. 2. So we have $\frac{5}{3-\sqrt{7}} = \frac{5(3+\sqrt{7})}{2}$.

To satisfy Requirement 3

We minimise the number of radicals by using the identity $\sqrt{a}\,\sqrt{b} = \sqrt{ab}$.

So, if we have $\sqrt{10}\,\sqrt{6}$ we can rewrite it as $\sqrt{60}$.

To satisfy Requirement 4

We minimise the numbers inside the radicals using the same identity, but in reverse. We look for factors which are perfect squares inside the radicals. For example 60 has a factor of 4, so we can write $\sqrt{60}$ as $\sqrt{4 \times 15}$. This is then $\sqrt{4}\,\sqrt{15}$, which is $2\sqrt{15}$.

So, the simplification of $\frac{3-\sqrt{\frac{18}{5}}}{\sqrt{15}}$ might look like this:

$$\frac{3-\sqrt{\frac{18}{5}}}{\sqrt{15}}$$

$$= \frac{3-\frac{\sqrt{18}}{\sqrt{5}}}{\sqrt{15}}$$

$$= \frac{\sqrt{15}(3-\frac{\sqrt{18}}{\sqrt{5}})}{15}$$

$$= \frac{3\sqrt{15}-\frac{\sqrt{15}\sqrt{18}}{\sqrt{5}}}{15}$$

$$= \frac{3\sqrt{15}-\frac{\sqrt{270}}{\sqrt{5}}}{15}$$

$$= \frac{3\sqrt{15}-\sqrt{54}}{15}$$

$$= \frac{3\sqrt{15}-\sqrt{9}\sqrt{6}}{15}$$

$$= \frac{3\sqrt{15}-3\sqrt{6}}{15}$$

$$= \frac{\sqrt{15}-\sqrt{6}}{5}$$

Practice

Q1 Simplify the following.

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{5}{2+\sqrt{3}}$ (c) $\frac{8\sqrt{5}}{\sqrt{5}-\sqrt{6}}$ (d) $\sqrt{\frac{6}{10}}$

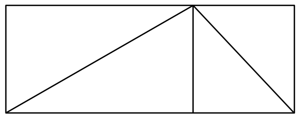
(e) $\sqrt{80}$ (f) $\sqrt{6}\sqrt{8}\sqrt{12}$ (g) $\frac{\sqrt{2}\sqrt{15}}{\sqrt{10}}$ (h) $\frac{\sqrt{20}}{\sqrt{15}\sqrt{24}}$

(i) $\sqrt{20}+3\sqrt{5}$ (j) $\frac{4+\sqrt{3}}{3-2\sqrt{2}}$ (k) $\frac{\sqrt{6}-2\sqrt{10}+\sqrt{8}}{\sqrt{10}-\sqrt{6}}$ (l) $\sqrt{45}+\sqrt{320}$

Solve

Q51 Simplify $\frac{1}{2+\sqrt{45}+\sqrt{80}}$

Q52 This large rectangle is made up of two smaller rectangles with height 2 m and diagonal lengths 5 m and 3 m. Find the aspect ratio (height ÷ width) for the large rectangle. Give the answer in standard simplified surd form.



Q53 Rewrite the complex number $\frac{2}{2+3i}$ where $i=\sqrt{-1}$ such that the denominator is real (i.e. does not contain i).

Revise

Revision Set 1

Q61 Simplify the following.

(a) $\frac{1+\sqrt{2}}{\sqrt{6}}$ (b) $\frac{8}{2-\sqrt{8}}$ (c) $\frac{4-\sqrt{8}}{\sqrt{2}-\sqrt{10}}$

(d) $\sqrt{150}-2\sqrt{24}$ (e) $\frac{\sqrt{14}}{\sqrt{28}\sqrt{8}}$ (f) $\frac{4+\sqrt{3}-\sqrt{2}}{3-2\sqrt{2}}$

Answers

Q1 (a) $\frac{\sqrt{2}}{2}$ (b) $10-5\sqrt{3}$ (c) $-40-8\sqrt{30}$ (d) $\frac{\sqrt{15}}{5}$

(e) $4\sqrt{5}$ (f) 24 (g) $\sqrt{3}$ (h) $\frac{1}{\sqrt{18}}$

(i) $5\sqrt{5}$ (j) $12+8\sqrt{2}+3\sqrt{3}+2\sqrt{6}$

(k) $4\sqrt{3}+4\sqrt{5}-2\sqrt{15}-14$ (l) $11\sqrt{5}$

Q51 $\frac{7\sqrt{5}-2}{241}$ Q52 $\frac{\sqrt{21}-\sqrt{5}}{8}$ Q53 $\frac{4-6i}{13}$

Q61 (a) $\frac{\sqrt{6}+\sqrt{12}}{6}$ (b) $-2-2\sqrt{8}$ (c) $\frac{1+\sqrt{5}-\sqrt{2}-\sqrt{10}}{2}$

(d) $\sqrt{6}$ (e) ¼ (f) $8+5\sqrt{2}+3\sqrt{3}+2\sqrt{6}$