M1 Maths

N4-1 Compound Interest

- compound interest

Summary Learn Solve Revise Answers

Summary

With simple interest, you invest the principal for so many years and, each year, they give you a certain percentage of the principal as interest.

With compound interest, they add interest at the end of the first year. Then, at the end of the second year, they add interest on the principal and the first year's interest, i.e. on the whole amount you have invested, not just the principal. You get interest on the interest as well as on the principal. So each year, your balance is multiplied by $(1+r)$. After t years, the amount owed, A, is given by the formula $A = P(1+r)^t$.

Sometimes interest is calculate more than once per year. In the formula, then, r is the per annum interest rate divided by the number of times per year it is calculated and t is the number of years multiplied by the number of times per year it is calculated.

When you borrow money, the bank charges you interest, worked out the same way.

The effective interest rate if the percentage increase after a year if the interest is compounding more frequently.

The future value of money so many years into the future is what the money could grow to in that time at a given interest rate.

Learn

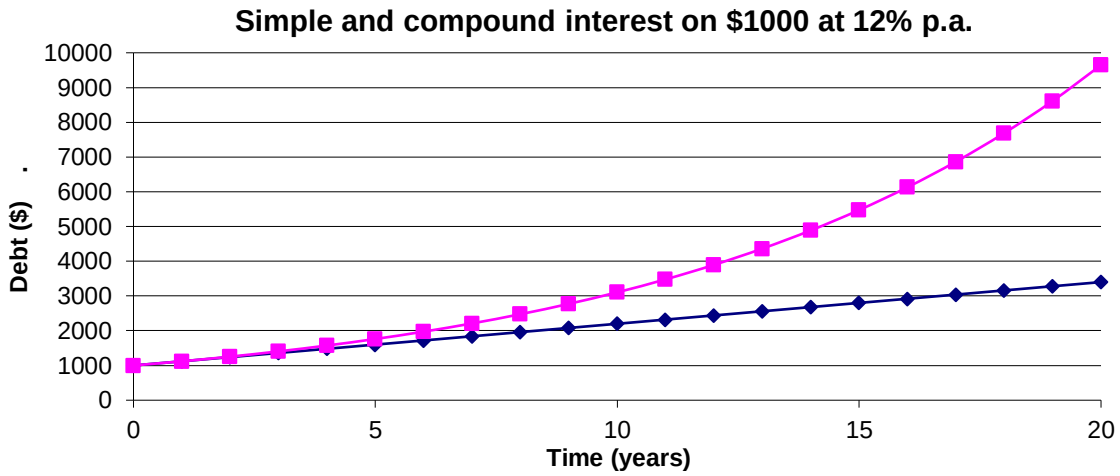
With simple interest, you only ever get or pay interest on the principal.

With compound interest, you get or pay interest on the interest as well.

Suppose you borrow \$500 at 8% p.a. compound interest for 3 years. At the end of the first year, you will be charged 8% of the \$500, i.e. \$40 interest. You will then owe \$540. At the end of the second year, you will be charged 8% of the \$540 rather than 8% of the \$500. 8% of \$540 is \$43.20. So then you will owe \$583.20. At the end of the third year, you will be charged 8% of \$583.20 which is \$46.656. This might be rounded to \$46.66. So then you will then owe \$629.86.

The simple interest for 3 years was $620.

At first, simple and compound interest are roughly the same, but after a while compound interest gets further and further ahead of simple interest. The following graph shows how a $1000 debt builds up over a period of 20 years with simple and compound interest at 12% p.a.



Practice

Q1 If you invest $4000 at 10% p.a. compound interest, how much will you have in the account at the end of 4 years?

Short cuts for compound interest

Working out compound interest over 3 or 4 years is ok, but for 25 years it would be tedious. But there is a nifty short cut. To use the short cut, you have to remember that adding 12% to an amount is the same as taking 112% of the amount. This is because you take all of the amount (100% of it) and add another 12% of it.

112% is the same as 1.12. So we just take 1.12 of the original amount. So to find out how much you would have after one year if you invested $1000 at 12% compound interest, we just work out $1000 × 1.12.

Then at the end of the next year, we just multiply by 1.12 again and so on. So after 6 years, you would have $1000 × 1.12 × 1.12 × 1.12 × 1.12 × 1.12 × 1.12 = $1973.82.

This is quicker than adding on the money each year. But it would still be a bit tedious for 25 years (having to punch in × 1.12 twenty five times). Fortunately, there is a short cut to the short cut. Instead of doing 1000 × 1.12 × 1.12 × 1.12 × 1.12 . . . . . , we can do 1000×1.12^{25}. With a calculator, this can be done in about 10 seconds.

Practice

Q2 (a) Janice invested \$5000 at 5% p.a. compound interest. How much would she have 4 years later?

(b) How much would she have after 15 years?

(c) Brian borrowed \$150 000 to buy shares. He paid 8% compound interest. How much would he have to pay if he paid it back 6 years later?

(d) Edith put \$100 in an account earning 12% p.a. compound interest when she was 18. She forgot about it until the bank contacted her when she was 84. How much would she have in the account then?

(e) Theo borrowed \$15 000 for 4 years at 7.7% compound interest. How much did he have to pay back?

(f) Mary invested \$600 for 10 years at 7% p.a. compound interest. How much interest would she have got?

Note that in Q2 (f), you had to work out the interest, not the amount she had at the end. To do that, you would have to work out the amount she had at the end and subtract the principal.

Formula

You will have noticed that the method for working out simple interest works out the interest. If you want to know the total amount at the end, you have to add the interest to the principal. The method for working out compound interest, however, works the other way. It works out the amount at the end. If you want to know the interest, you have to subtract the principal from the final amount as you did in Q2 (f).

Accordingly, though the formula for simple interest works out the interest, the formula for compound interest works out the final amount.

If the principal is P, the interest rate **as a decimal** is r, the term (number of years) is t and the final amount is A, then

$$A = P\,(1+ r)^t$$

Compare this with what you have been doing to make sure it makes sense. That way,

if you forget the formula, you can work it out again.

As with simple interest, the formula is useful mainly when working backwards to find P, r or t. Actually, you don't know enough algebra yet to work out r or t, so, at this stage, you can only use the formula for working out P (or A if you want to).

For example, how much would you need to invest at 4% p.a. compound interest to have \$1000 at the end of 5 years?

Here, $A = 1000$, $P = ?$, $r = 0.04$ and $t = 5$.

$$A = P(1+r)^t$$

$$1000 = P \times 1.04^5$$

$$1000 = P \times 1.21665$$

$\div 1.21665 \quad \div 1.21665$

$$821.93 = P$$

So you would have to invest \$821.93.

Practice

Q3 (a) How much would you need to invest at 5% p.a. compound interest to have \$800 at the end of 10 years?

(b) Susie borrowed \$2000 at 8% p.a. compound interest. How much would she have to pay back 5 years later?

(c) How much interest would Susie have paid?

(d) Brian borrowed some money to buy shares. He paid 9% compound interest. When he paid it back 6 years later, he paid \$251 565. How much did he borrow?

(e) Sophie put some money in an account earning 10% p.a. compound interest when she was 16. She forgot about it until the bank contacted her when she was 79 and told her she had \$16 210.60 in the account. How much did she put in?

(f) Theo borrowed \$15 000 for 4 years at 7.7% compound interest. How much interest would he have to pay?

Different compounding periods

So far we have dealt with interest that is calculated once a year, but most banks etc. work it out more frequently. Once a month is very common.

Let's say a bank pays compound interest at 6% p.a. compounding monthly. (*Compounding monthly* just means *worked out monthly*.) Let's say that you deposit $5000 for 4 years. How much would you have at the end?

First we work out the monthly interest rate. The annual interest rate is 6%. The monthly rate is just $^1/_{12}$ of the annual rate. In this case it is 0.5%.

So each month the deposit is multiplied by 100.5%, i.e. by 1.005.

This happens every month for 4 years, i.e. 48 times. So we multiply by 1.005 48 times.

$\$5000 \times 1.005^{48} = \$6352.45$

If the bank paid interest quarterly (i.e. every 3 months), then we say it is *compounding quarterly* and the final amount would be $\$5000 \times 1.015^{16} = \$6344.93$. If it is every year, then we say it is *compounding annually*.

If we use the formula, $A = P(1+r)^t$, we have to remember that the term, t, is now the number of times that interest is paid, in the above examples 48 and 16 (this is called the number of rests), and that r is the rate per rest, in the above examples 0.005 and 0.015.

Practice

Q4 (a) Sarah-Jane Edwards invested $1200 at 6% p.a. interest compounding monthly. How much would she have in the account after 2 years?

(b) Bob invested $6000 at 7.2% p.a. compounding quarterly. He left it there for 18 months. How much did he have then?

(c) Marley invested some money at 8% p.a. compounding monthly for 7 years. At the end, he had $3844.31. How much did he invest?

(d) How much would the Dodgetown Football Club have to invest now at 7% p.a. compounding half-yearly if they wanted the account to contain $100 000 in 3 years time?

(e) Gary has the choice of investing his $200 at 7.4% p.a. compounding annually or 7.3% compounding monthly. If it is to be invested for 2 years, work out how much he would end up with either way.

Effective Interest Rate

If you invest $1000 at 8% p.a. compounding monthly, after a year you will have

$$\$1000 \times (1 + {}^{0.08}/_{12})^{12} = \$1083.00$$

This is 8.3% more than you started with. So interest at 8% p.a. compounding monthly is equivalent to 8.3% compounding annually. We say that, for interest compounding monthly at 8.3%, the effective interest rate is 8.3%. The effective interest rate is the percentage increase over a year for an investment or loan that compounds more frequently.

Practice

Q5 Find the effective interest rate in each of the following cases:

(a) $1000 invested at 12% p.a. compounding monthly

(b) Money invested at 8% compounding quarterly? (Note the answer will be the same for any amount of money, so do the calculation for $1.)

(c) Money invested at 4.6% p.a. compounding half-yearly?

(d) Money invested at 11.2% compounding daily.

Future Value

If you have $1000 now and can invest it at 6% p.a. interest compounding annually, then in 5 years time you could have $1338.23. This is called the future value of that $1000 in 5 years time. Of course the future value of money at a given time in the future depends upon the interest rate available.

Practice

Q6 Find the following future values:

(a) $1000 in 4 years time if you can get 9% p.a. compounding yearly

(b) $5000 in 12 years time if you can get 5.5% p.a. compounding monthly

(c) $750 in 35 years time if you can get 8.9% p.a. compounding quarterly

(d) $12 000 in one year's time if you can get 3.25% compounding monthly

Solve

Q51 Jonno borrowed $5000 at 9.6% p.a. interest to buy a car. A month later, the bank adds 0.8% of the loan balance to the loan as interest, but Jonno pays off $100. This makes his new balance $4940. The same happens each month. What will the balance be after he makes his sixth monthly payment? [You can work this out the long way or use a spreadsheet.]

Q52 The mass of a bacterial colony increases by 40% each hour. If its initial mass is 0.6 g, what will be its mass after 48 hours?

Q53 The value of a car falls by 20% each year. When it is 11 years old, it is worth $2200. How much was it worth when it was new? How much when it was 3 years old?

Revise

Revision Set 1

Q61 Gerty invested $500 for 7 years at 6.5% compounding annually. How much did she end up with?

Q62 How much would Gerty have ended up with if the interest had compounded monthly instead?

Q63 How long would it take for $400 to accumulate $300 in interest at 6% p.a. simple interest?

Q64 Jarrod invested $2000 on simple interest. After 6 years his account had grown to $7500. What was the interest rate?

Q65 How much would I need to invest now at 7.8% p.a. interest compounding monthly to end up with $8000 15 years from now?

Q66 Find the effective rate if money is invested at 8.75% compounding monthly.

Q67 Find the future value of $5600 in 8 years' time if you could get 5.5% p.a. compounding quarterly.

Answers

Q1 $5856.40

Q2 (a) $6 077.53 (b) $10 394.64 (c) $238 031.15
(d) $177 169.72 (e) $20 181.53 (f) $580.29

Q3 (a) $491.13 (b) $2938.66 (c) $938.66
(d) $150 000 (e) $40 (f) $5181.53

Q4 (a) $1352.60 (b) $6677.87 (c) $2200
(d) $81 350.06 (e) $230.70, $231.34

Q5 (a) 12.68% (b) 8.24% (c) 4.65% (d) 11.85%

Q6 (a) $1411.58 (b) $14 490.55 (c) $16 332. 70 (d) $12 395.86

Q51 $4632.72 Q52 6.2 tonnes Q53 $25 611.37, $13 113.02

Q61 $776.99 Q62 $787.12 Q63 12.5 years Q64 45.83%
Q65 $2492.35 Q66 9.11% Q67 $8669.14