M1 Maths

Learning by Thinking

N4-1 Compound Interest

- compound interest

Learn Answers

This LbT (Learning by Thinking) module is an alternative to the 'Learn' section of the normal module. It is designed to lead the student to work out the maths themselves by solving problems. Thus it contains only minimal explanations. The rationale behind the approach can be read **here**.

Learn

With simple interest, you only ever get or pay interest on the principal.

With compound interest, you get or pay interest on the interest as well.

Suppose you borrow \$500 at 8% p.a. compound interest for 3 years. At the end of the first year, you will be charged 8% of the \$500, i.e. \$40, interest. You will then owe \$540. At the end of the second year, you will be charged 8% of the \$540 rather than 8% of the \$500. 8% of \$540 is \$43.20. So then you will owe \$583.20. At the end of the third year, you will be charged 8% of \$583.20 which is \$46.656. So then you will then owe \$629.856.

With simple interest for 3 years you would have owed \$620.

Q1 Let's say you invest \$1000 at 12% p.a. simple interest in the S Bank and \$1000 at 12% p.a. compound interest in the C Bank. On the same graph, but in two different colours, plot how much you would have in each bank at the end of each of the first 20 years.

When working out the amount in the C bank at the end of each year, you can see how much is there at the end of the previous year, work out 12% of it and add that on. But hopefully you remember the short-cut method for adding 12% to an amount by multiplying by 1.12. That makes the calculation a lot quicker.

Q2 Suppose you invested \$2000 at 8% p.a. compound interest for 50 years. How much would you have at the end of the 50 years?

You might have done that calculation by multiplying by 1.08 50 times. But hopefully you found the much quicker method of just multiplying by 1.08^{50}. Use the short-cut method on the next questions.

Q3 How much would you have at the end if you invested
(a) \$500 for 27 years at 7% p.a. compound interest?
(b) \$2700 for 9 years at 2.5% p.a. compound interest?
(c) \$45.70 for 77 years at 11% p.a. compound interest?

Q4 How much would you have to pay back at the end if you borrowed
(a) \$5000 for 27 years at 4% p.a. compound interest?
(b) \$200 for 12 years at 4.75% p.a. compound interest?
(c) \$2115 for 30 years at 9% p.a. compound interest?
(d) In the last question, how much of what you pay back is interest?

Formula

You will have noticed that the method for working out simple interest works out the interest. If you want to know the total amount at the end, you have to add the interest to the principal.

The method for working out compound interest, however, works the other way. It works out the amount at the end. If you want to know the interest, you have to subtract the principal from the final amount as you did in Q4(g).

Accordingly, the usual formula for compound interest works out the final amount.

Q5 If the principal is P, the interest rate **as a decimal** is r, the term (number of years) is t and the final amount is A, find the formula for A under compound interest.

As with simple interest, the formula is useful mainly when working backwards to find P, r or t. Actually, you don't know enough algebra yet to work out r or t, so, at this stage, you can only use the formula for working out P (or A if you want to).

Q6 (a) How much would you need to invest at 5% p.a. compound interest to have \$800 at the end of 10 years?
(b) Susie borrowed \$2000 at 8% p.a. compound interest. How much would she have to pay back 5 years later?
(c) How much interest would Susie have paid?
(d) Brian borrowed some money to buy shares. He paid 9% compound interest. When he paid it back 6 years later, he paid \$251 565. How much did he

borrow?

(e) Edith put some money in an account earning 10% p.a. compound interest when she was 16. She forgot about it until the bank contacted her when she was 79 and told her she had $16 210.60 in the account. How much did she put in?

(f) Theo borrowed $15 000 for 4 years at 7.7% p.a. compound interest. How much interest would he have to pay?

Different compounding periods

So far we have dealt with interest that is calculated once a year, but most banks etc. work it out more frequently. Once a month is very common.

Let's say a bank pays compound interest at 6% p.a. compounding monthly. (*Compounding monthly* just means *worked out monthly*.) They add $^1/_{12}$ of the annual rate, i.e. 0.5%, to your account each month.

If the bank pays interest quarterly (i.e. every 3 months), then we say it is *compounding quarterly* and they add a quarter of the annual interest percentage each quarter. *Compounding half-yearly* means interest is paid every 6 months. *Compounding annually* means it is paid every year.

Q7 (a) Sarah-Jane Edwards invested $1200 at 6% p.a. interest compounding monthly. How much would she have in the account after 2 years?

(b) Bob invested $6000 at 7.2% p.a. compounding quarterly. He left it there for 18 months. How much did he have then?

(c) Gary has the choice of investing his $200 at 7.4% p.a. compounding annually or 7.3% compounding monthly. If it is to be invested for 2 years, work out how much he would end up with either way.

Q8 (a) Marley invested some money at 8% p.a. compounding monthly for 7 years. At the end, he had $3 844.31. How much did he invest?

(b) How much would the Dodgetown Football Club have to invest now at 7% p.a. compounding half-yearly if they wanted the account to contain $100 000 in 3 years' time?

Note: If we use the formula, $A = P(1+r)^t$, we have to remember that the term, t, is now the number of times that interest is paid, in Q8 (a) 84 and in Q8 (b) 6 (this is called the number of rests), and that r is the rate per rest, in Q8 (a) 0.00666… and in Q8 (b) 0.035.

Q9 (a) How much can I borrow now at 7% p.a. compounding monthly if I will be able to pay back $25 000 in 7½ years' time?

(b) Roger put some money in the bank 5¾ years ago at 6% p.a. compounding quarterly. How now has $9 235.80. How much did he put in?

Effective Interest Rate

Q10 (a) If you invested $1000 at 8% p.a. compounding annually, how much would you have at the end of the year?

(b) If you invest the same $1000 at 8% p.a. compounding monthly, how much would you have at the end of the year?

(c) By what percentage has your money increased in part (b)?

Your answer to part (c) is called the ***effective interest rate*** for 8% p.a. compounding monthly. It is the percentage increase in your money over a full year. To distinguish it, the 8% is called the ***nominal interest rate***. The effective rate is always slightly higher than the nominal rate.

Q11 Find the effective interest rate in each of the following cases:

(a) $1000 invested at 12% p.a. compounding monthly

(b) $50 000 invested at 12% p.a. compounding monthly

(c) Money invested at 12% p.a. compounding monthly (Note the answer will be the same for any amount of money, so do the calculation for $1.)

(d) Money invested at 8% compounding quarterly

(e) Money invested at 4.6% p.a. compounding half-yearly?

(f) 11.2% p.a. compounding daily.

Future Value

If you have $1000 now and can invest it at 6% p.a. interest compounding annually, then in 5 years' time you could have $1338.23. This is called the future value of that $1000 in 5 years' time. Of course the future value of money at a given time in the future depends upon the interest rate available.

Q12 Find the following future values:

(a) $1000 in 4 years' time if you can get 9% p.a. compounding yearly

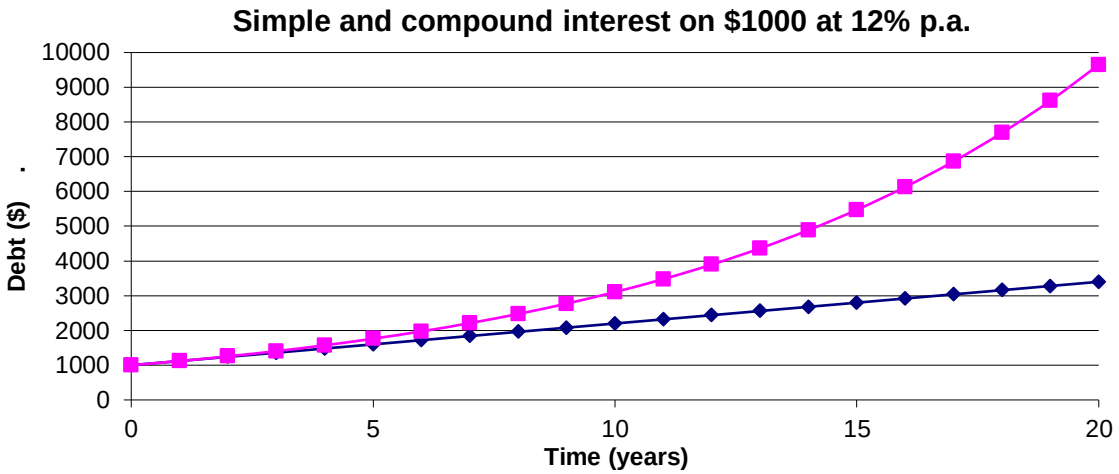
(b) $5000 in 12 years' time if you can get 5.5% p.a. compounding monthly

(c) $750 in 35 years' time if you can get 8.9% p.a. compounding quarterly

(d) $12 000 in one year's time if you can get 3.25% p.a. compounding monthly

Answers

Q1



Q2 $93 803.26

Q3 (a) $3 106.93 (b) $3371.93 (c) $141 184.03

Q4 (a) $14 416.84 (b) $349.04 (c) $28 061.14 (d) $25 946.14

Q5 $A = P(1+r)^t$

Q6 (a) $491.13 (b) $2938.66 (c) $938.66
(d) $150 000 (e) $40 (f) $5181.53

Q7 (a) $1352.60 (b) $6677.87 (c) $230.70, $231.34

Q8 (a) $2200 (b) $81 350.06

Q9 (a) $14 811.46 (b) $6557.76

Q10 (a) $1080 (b) $1083.00 (c) $8.30%

Q11 (a) 12.68% (b) 12.68% (c) 12.68% (d) 8.24%
(e) 4.65% (f) 11.85%

Q12 (a) $1411.58 (b) $14 490.55 (c) $16 332. 70 (d) $12 395.86