M1 Maths

N2-6 Negative Operations

- mental/written methods to perform operations on positive and negative numbers

Summary Lead-In Learn Solve Revise Answers

Summary

One way to add and subtract positive and negative numbers is to use a number line. Locate the first number on the line, then move up or down the number of places according to the second number – up for adding, down for subtracting, the opposite for a negative number.

To multiply or divide positive and negative numbers, perform the operation as if they were both positive, then, if one of the numbers was negative, change the sign of the answer to negative; if two were negative, change the sign back to positive; if three were negative, change it back to negative and so on.

Lead-In

Take a scientific calculator and do some additions using negative numbers as well as positive numbers, for example $4 + -3$, $-2 + -5$. Look at the answers. When you have done a few, start to try to predict what the answers will be before you do them.

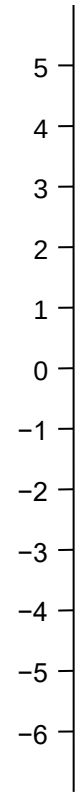
Then try subtraction.

Later, try multiplication and division. Multiplication and division are actually the easiest to predict.

Learn

Adding positive and negative numbers using a number line

To add and subtract when some of the numbers are negative, you can use a number line like the one to the right. Of course you can draw it horizontally if you like.



Suppose you need to add 2 to 3 (i.e. work out 3 + 2). You find 3 on the number line. Knowing that adding to 2 makes the number higher, you just move 2 up the number line. This brings you to 5. So 5 is the answer.

For positive numbers, this should be fairly obvious, but it works the same starting with negative numbers too. So to work out –1 + 4, start at –1, then move up 4 places. This takes you to 3. So –1 + 4 = 3. In the same way –5 + 2 = –3.

Fractions work the same way.

To do 3.4 + 1.7, start at 3.4, then move up 1.7 to 5.1

To do –4.8 + 3.3, start at –4.8, then move up 3.3 to –1.5

To do –3.5 + 5.7, start at –3.5, then move up 5.7 to 2.2

So, to add a number we move up the number line. To add 3, we move up 3; to add 2, we move up 2, to add 1, we move up 1, to add 0, we move up 0 (stay where we are).

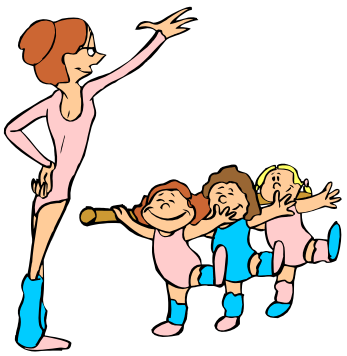
To add –1 we move up –1. –1 is the opposite of 1, so moving up –1 is the same as moving down 1. So to add –1, we move down 1; to add –2, we move down 2 and so on.

So 4 + –1 = 3 5.2 + –1.3 = 3.9 2 + –6 = –4 –2 + –5 = –7

Practice

Q1 Draw your own number line from –10 to 10 and use it to work out the following:

(a) 1 + 3	(b). –2 + 5	(c) –4 + 1	(d) –3 + 3
(e) –1 + 3.2	(f) –4.2 + 2	(g) –2 + 0	(h) 2 + –1
(i) 4 + –5	(j) –3 + –2	(k) –8 + 2	(l) –6 + –4
(m) –4 + –5	(n) 2 + –2	(o) –1 + 5	(p) –3.6 + –1
(q) 0 + –0.24	(r) 1 + –1	(s) 10 + –4	(t) 4 + 2



Subtracting using a number line

Subtracting is the opposite of adding. So to subtract, we move in the opposite direction to what we would if we added.

Obviously, to do 7 – 3, we start at 7 and move down 3 to get 4

To do 2 – 5, start at 2, then move down 5 to get –3

To do –3 – 4, start at –3, then move down 4 to –7.

So, to subtract a number we move down the number line. To subtract 3, we move down 3; to subtract 2, we move down 2, to subtract 1, we move down 1, to subtract 0, we move down 0 (stay where we are), to subtract –1 we move down –1. –1 is the opposite of 1, so moving down –1 is the same as moving up 1. So to subtract –1, we move up 1; to subtract –2, we move up 2 and so on.

So $4 - -1 = 5$ $\quad -1 - -2 = 1$ $\quad -5 - -2 = -3$

Just remember that subtracting is the opposite of adding and negative is the opposite of positive. So:

- adding a positive number moves you up;
- adding a negative number is the opposite of adding a positive number so it moves you in the opposite direction, i.e. down;
- subtracting a positive number is the opposite of adding a positive number, so it moves you in the opposite direction, i.e. down;
- subtracting a negative number is the opposite or subtracting a positive number, so it moves you in the opposite direction, i.e. up.

Thinking about opposites is the key.

At first you will need to draw a number line to do these calculations, but later, you will be able to picture the number line in your head and use that.

Practice

Q2 Draw your own number line from –10 to 10 and use it to work out the following:

(a) $3 - 1$	(b). $-2 - 3$	(c) $2 - 6$	(d) $-3 - 1$
(e) $-1 - 3.2$	(f) $-4.2 - 2.1$	(g) $-2 - 0$	(h) $3 - 2$
(i) $3 - 1$	(j) $3 - 0$	(k) $3 - -1$	(l) $3 - -2$
(m) $3 - -3$	(n) $3 - -6$	(o) $-1 - -5$	(p) $-3.6 - -1$
(q) $0 - -0.24$	(r) $1 - -1$	(s) $-10 - -4$	(t) $-4 - 2$
(u) $2 + -4$	(v) $3 - 7$	(w) $-10 + 4$	(x) $-4 + -2$

An alternative way of looking at it – as money and debts

There is another way to think about adding and subtracting. You can think about positive numbers as money and negative numbers as debts.

7 + 3 means you have \$7 and you have \$3, so obviously you have \$10 in total.
So 7 + 3 = 10

7 + – 3 means you have \$7, but also a debt of \$3. You can pay off the debt using \$3 of your \$7. This will leave you with \$4..
So 7 – 3 = 4

– 7 + 3 means you have a \$7 debt and \$3. The \$3 will pay of part of your debt leaving you with a debt of \$4.
So –7 + 3 = –4

–7 + –3 means you have a \$7 debt and a \$3 debt. This makes you \$10 in debt.
So –7 + –3 = –10

7 – 3 means you have \$7, but someone takes \$3 from you. This leaves you with \$4.
So 7 – 3 = 4

–7 – 3 means you are \$7 in debt, then someone demands \$3 from you. So now you are \$10 in debt.
So –7 – 3 = –10

–7 – –3 means you have \$7 worth of debts, but someone takes \$3 worth of those debts from you. This means they pay off \$3 worth of your debts or that they give you \$3. This leaves you with only \$4 worth of debts.
So –7 – –3 = –4

–7 – –11 means you are \$7 in debt, then someone takes away \$11 worth of debts by giving you \$11. So you end up with \$4 left over.
So –7 – –11 = 4

7 – –3 means you have 7 dollars, and someone gives you \$3. You then have \$10.
So 7 – –3 = 10

When you get experienced at adding and subtracting negatives, you will probably use this type of thinking more than number lines.

Practice

Q3 Do these without a calculator, thinking about them whichever way you prefer.

(a) $4 + -3$	(b) $-2 - 5$	(c) $2 - -4$	(d) $-1.5 + -3.2$
(e) $4 - -5$	(f) $-3 + -2$	(g) $-12 + 2$	(h) $-8 - -4$
(i) $-4 + -5$	(j) $2 + -2$	(k) $-1 - 5$	(l) $-3.6 - -1$
(m) $3 - -1.24$	(n) $-1 + -1$	(o) $10 + -4$	(p) $21.3 - -7$
(q) $-16 + 5.5$	(r) $3 - -8$	(s) $-4 + 8$	(t) $6 - -10$
(u) $2 + -5$	(v) $-8 - -5$	(w) $1 - -3.5$	(x) $-1 + -1$

Multiplying and dividing with negative numbers

This is actually easier than adding and subtracting.

$6 \times 2 = 12$	$6 \times 2 = 12$
6×-2 is the opposite of 6×2, so it is -12	$6 \times -2 = -12$
Also -6×2 is the opposite of 6×2, so it is -12	$-6 \times 2 = -12$
-6×-2 is the opposite of -6×2 or of 6×-2, so it is 12	$-6 \times -2 = 12$

The same goes for dividing.

$6 \div 2 = 3$	$6 \div 2 = 3$
$6 \div -2$ is the opposite of $6 \div 2$, so it is -3	$6 \div -2 = -3$
Also $-6 \div 2$ is the opposite of $6 \div 2$, so it is -3	$-6 \div 2 = -3$
$-6 \div -2$ is the opposite of $-6 \div 2$ or of $6 \div -2$, so it is 3	$-6 \div -2 = 3$

You can think of this in terms of money and debts if you want to. For example:
-6×-2 means someone takes away two \$6 debts. In other words they give you \$12.

So $-6 \times -2 = 12$.

Practice

Q4 Work out the following without a calculator

(a) 4×3	(b) 4×-3	(c) -4×3	(d) -4×-3
(e) $6 \div 2$	(f) $-6 \div -2$	(g) $-6 \div 2$	(h) $6 \div -2$
(i) -4×-5	(j) $2 \div -2$	(k) -1×5	(l) $-3)6 \times -1$
(m) 0×-5	(n) $-1 \div -1$	(o) $10 \div -4$	(p) $-21 \div 7$
(q) $-2 \times 5)5$	(r) 3×-8	(s) $-4 \div 8$	(t) $6 \div -10$
(u) $2 \div -5$	(v) $-8 \div -5$	(w) $1 \times -3)5$	(x) -1×-1
(y) $96 \div -6$	(z) -3×5		

Using your knowledge from Modules N1-6 (Powers) and N1-7 (Order of Operations) along with your knowledge from the current module, you should be able to do the following.

Practice

Q5 Work out the following without a calculator

(a) $4 - -2 + -5$	(b) $-1 \times (2 + -3)$	(c) $4 + -2 \times 5$
(d) $6 - -3 \times -3$	(e) $(-3)^2$	(f) $-(2 + 3)$
(g) $-(2 \times -3)$	(h) $(-2)^3$	(i) $5 - (2 + -4)^2$
(j) $(-1)^3$	(k) $(-1)^4$	(l) $(-1)^5$
(m) $(-1)^{77}$	(n) $(-2)^2 - (-2)^3$	(o) $(-3)^2$
(p) -3^2 *	(q) 5^2	(r) $(-5)^2$
(s) $\sqrt{25}$ **	(t) $\sqrt{64}$	(u) $\sqrt{-100}$
(v) $-3^3 \div (-3)^3$	(w) $\sqrt{(-2 \times -2)}$	(x) $41 - -4 \times -12$
(y) $6 - -3 \times 5 - -8 \div 2 \times (-3)^2$		
(z) $-82 \times (4 - -6 \times -1) \div (-2 - -12)$		

* Note that order of operations conventions say that powers get done before +, –, × and ÷ and before negative signs. So the answer to Q5p is –9.

** You will see from 5(q) and 5(r) that $5^2 = 25$ and $(-5)^2 = 25$. So 25 has two square roots: 5 and –5. However, we use the $\sqrt{\ }$ sign to indicate the positive square root. This is true for all positive numbers. Negative numbers do not have real square roots. No real number multiplied by itself makes a negative number.

Solve

Q51 If $a = 2$, $b = -3$ and $c = -5$, find

(a) abc (b) $\frac{ab}{c}$ (c) b^2c (d) $(ab)^3c$

(e) $ab + c$ (f) $ac - 2b$ (g) $b^2 - c^2$ (h) $c - 3b$

Q52 Gary should have calculated $(-4)^2$, but instead calculated -4^2. How far out would his answer have been?

Q53 What would you get if you multiplied all the integers from -5 to 5 inclusive?

Q54 What is $10 - 9 - 8 - 7 - \ldots\ldots - -8 - -9 - -10$?

Revise

Revision Set 1

Q61 Work out the following without a calculator.

(a) $3 + -5$ (b) $2 - 6$ (c) $-7.5 + 5.5$ (d) $-2 - -5$

(e) 4×-5 (f) $-6 \div -1$ (g) $-7 \div 2$ (h) $(-4)^2$

Answers

Q1	(a) 4	(b) 3	(c) –3	(d) 0
	(e) 2.2	(f) –2.2	(g) –2	(h) 1
	(i) –1	(j) –5	(k) –6	(l) –10
	(m) –9	(n) 0	(o) 4	(p) –4.6
	(q) –0.24	(r) 0	(s) 6	(t) 6
Q2	(a) 2	(b) –5	(c) –4	(d) –4
	(e) –4.2	(f) –6.3	(g) –2	(h) 1
	(i) 2	(j) 3	(k) 4	(l) 5
	(m) 6	(n) 9	(o) 4	(p) –2.6
	(q) 0.24	(r) 2	(s) –6	(t) –6
	(u) –2	(v) 4	(w) –6	(x) –6
Q3	(a) 1	(b) –7	(c) 6	(d) –4.7
	(e) 9	(f) –5	(g) –10	(h) –4
	(i) –9	(j) 0	(k) –6	(l) –2.6
	(m) 4.24	(n) –2	(o) 6	(p) 28.3
	(q) –11.5	(r) 11	(s) 4	(t) 16
	(u) –3	(v) –3	(w) 4.5	(x) –2
Q4	(a) 12	(b) –12	(c) –12	(d) 12
	(e) 3	(f) 3	(g) –3	(h) –3
	(i) 20	(j) –1	(k) –5	(l) 6
	(m) 0	(n) 1	(o) –2.5	(p) 3
	(q) –10	(r) –25	(s) –0.5	(t) –0.6
	(u) –0.4	(v) 1.6	(w) –5	(x) 1
	(y) –16	(z) –15		

Q5	(a) 1	(b) 1	(c) –6	(d) –3
	(e) 9	(f) –5	(g) 6	(h) –8
	(i) 1	(j) –1	(k) 1	(l) –1
	(m) –1	(n) 12	(o) 9	(p) –9
	(q) 25	(r) 25	(s) 5	(t) 8
	(u) none	(v) 1	(w) 2	(x) –7
	(y) 57	(z) 16.4		

Q51	(a) 30	(b) 1.2	(c) −45	(d) 1080
	(e) −11	(f) −4	(g) −16	(h) 4
Q52	32			
Q53	0			
Q54	20			

Q61	(a) –2	(b) 4	(c) –2	(d) 3
	(e) –20	(f) 6	(g) –3.5	(h) 16