M1 Maths

N2-5 Decimal Operations 2

- exact and approximate mental/written methods to multiply and divide whole numbers and decimal fractions

Summary Lead-In Learn Solve Revise Answers

Summary

To multiply by 10, 100, 1000 etc., we just move the decimal point 1, 2, 3 places to the right. To divide by we move the decimal point to the left.

Decimal fractions can be written as whole numbers divided by 10, 100 etc. Once we have done this, we can multiply two decimal fractions by multiplying the whole numbers and moving the decimal point accordingly. To divide by a decimal fraction, we remember that dividing by a small number (<1) makes the answer bigger and vice versa, so we move the decimal point the correct way.

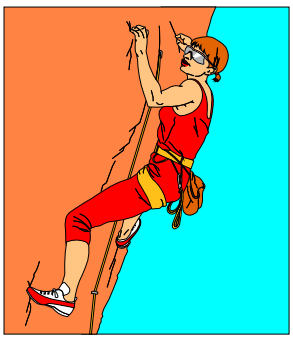
Lead-In

Think of a number. Add another number to it. Do you get a bigger number?
Does subtracting make a number smaller?
What about multiplying? What about dividing?

Take a counter. Put it at the bottom of the diamond on the next page where it says 10.

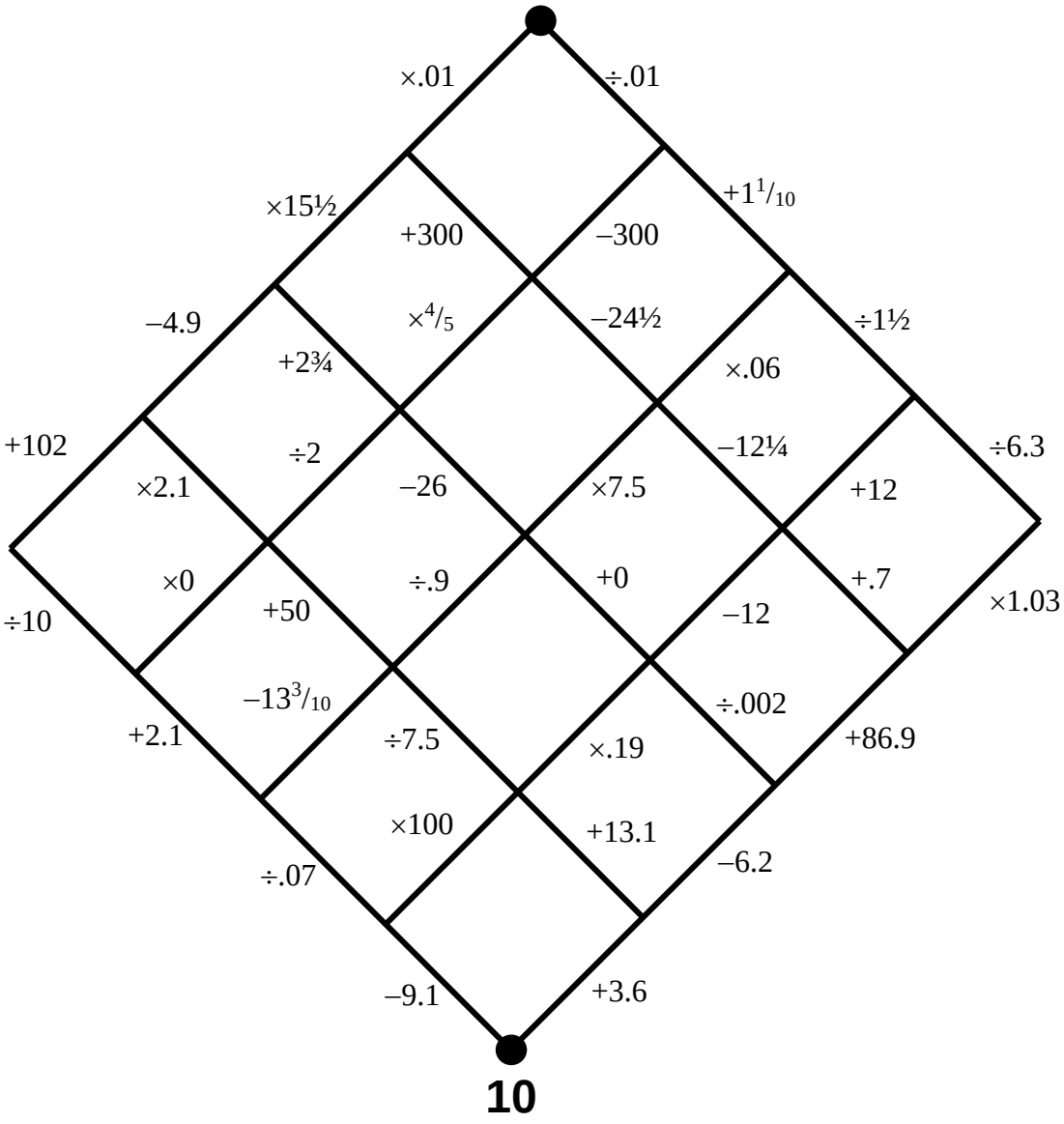
Put 10 into your calculator. Then move upwards to the next intersection, either to the left or to the right. It you go left, subtract 9.1 from the 10 on your calculator and press = (or ENTER or EXE). If you go right, add 3.6 and press =.

Let's say you go right. Your calculator should now show 13.6. Now move upwards again – either left or right. If you move left, you add 13.1 to the 13.6 and press =. This will give you 26.7.



Keep doing this until you have had 8 moves and you are at the top of the diamond. See what number is on your calculator.

The aim is to get to the top with the highest number you can on your calculator. It is very unlikely that you will get the highest possible number first time. Try a few times and see what you can get. It is possible to get over a million.



Learn

Multiplying and dividing decimals by 10, 100, 1000 etc.

Take a number like 41.27. If we want to multiply it or divide it by 10 or 100 or 1000 or . . . (i.e. a 1 followed by some zeros), we follow three simple steps

1. We fill a few decimal places before and after the number with zeros: 00041.27000. This is still the same number, but normally we just don't write in that there are 0 hundreds, 0 thousands, 0 ten-thousandths etc.

2. Then we move the decimal point to the right or left. The number of places we move it is the same as the number of zeros in the power of 10. So, if we are multiplying or dividing by 1000, we move it 3 places like this:

0 0 0 4 1.2 7 0 0 0

↓

0 0 0 4 1 2 7 0.0 0

or like this:

0 0 0 4 1.2 7 0 0 0

↓

0 0.0 4 1 2 7 0 0 0

3. We check whether we have moved it the right way (left or right). To do this we remember that if we multiply, we make the number bigger, if we divide, we make the number smaller. In this case, we multiplied, so the number should get bigger. The first move above makes the number bigger; the second move makes it smaller. So the first one is correct. The answer is 00041270.00

4. Finally we remove the zeros that aren't needed, i.e. those at the beginning before the decimal point and at the end after the decimal point, to get

41 270

Once you get good at this, you might do all these steps together rather than separately. Steps 2 and 3 need to be done together anyway. Also you will probably only write in the extra zeros if and when you need them because the decimal point moves past the end of the number.

Why does this work?

It is possible to do the steps above without understanding why they work. But it is always good, when learning maths, to understand why procedures work. The best thing is to try to work out yourself why it works before you read on. If you cannot work it out, then read on.

What we are doing when we multiply 41.27 by 1000 is to multiply each digit by 1000. So we multiply 40 by 1000, 1 by 1000, 0.2 by 1000 and 0.07 by 1000.

When we multiply 40 by 1000 it becomes 40 000. This is a 4 in the 10 000s place. This is 3 places to the left of the 10s place where it was. So what happens is that the 4 moves 3 places to the left. The same happens to all the other digits. So all the digits move 3 places to the left relative to the decimal point. This looks exactly the same as the decimal point moving 3 places to the right relative to the numbers. As it is easier to move the decimal point than to move all the digits, that's what we do.

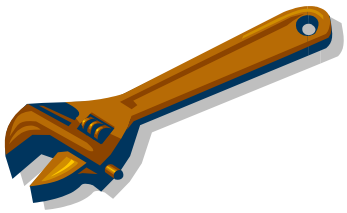
If you have 5.21 × 1000 ÷ 100, we just start with the 5.21, move the decimal point three places to the right to get 5210 then move it two places to the left to get 52.1.

Alternatively, we could look at the × 1000 ÷ 100 and see that this will be 3 right and 2 left, which comes to 1 right, giving us 52.1 as before.

Practice

Q1 Work out the following.

(a) 2.39 × 10	(b) 68 × 100	(c) 37 ÷ 1000	(d) 14.9 ÷ 100
(e) 0.003 ÷ 10	(f) 0.026 × 100	(g) 1 ÷ 10 000	(h) 390 × 100
(i) 1.79 × 1000 ÷ 100		(j) 442 × 100 ÷ 10 000	
(k) 0.32 ÷ 1000 × 100		(l) 175.2 × 10 ÷ 1000	
(m) 0.0055 × 100 ÷ 10		(n) 0.0187 ÷ 10 000 × 100	
(o) 1.4 ÷ 10 × 100 ÷ 10 000		(p) 130 × 100 ÷ 10 000 × 100	



Writing decimals as whole numbers divided by 10, 100, 1000 etc.

Using the above idea in reverse, we can write any decimal fraction as a whole number divided by 10 or 100 or 1000 or . . . .

For example 5.27 can be written as 527 ÷ 100. To get the whole number, just re-write the decimal number without the decimal point. To know what to divide by, you can use guess and check or just look at how many places the decimal point has moved (assuming that it is now at the end of the number.

This is similar to writing 48 000 as 48 × 1000.

Practice

Q2 Write the following as a whole number multiplied or divided by 10, 100, 1000 etc.

(a) 3.56	(b) 0.0004	(c) 0.0723	(d) 45 000
(e) 0.003	(f) 41.27	(g) 560	(h) 390 000
(i) 10.5	(j) 0.0314	(k) 400.5	(l) 300
(m) 0.45	(n) 83.9	(o) 52 000	(p) 3.90

Multiplying decimals

We learnt to multiply whole numbers in Module N1-9 (Decimal Operations 1). To multiply decimals, we can write the numbers as whole numbers multiplied or divided by 10, 100 etc. For instance:

$$\begin{aligned} 0.03 \times 5\,000 &= 3 \div 100 \times 5 \times 1000 \\ &= 3 \times 5 \times 1000 \div 100 \\ &= 15 \times 1000 \div 100 \\ &= 150 \end{aligned}$$

$$\begin{aligned} 2.7 \times 0.244 &= 27 \div 10 \times 244 \div 100 \\ &= 27 \times 244 \div 10 \div 100 \\ &= 6588 \div 1000 \\ &= 6.588 \end{aligned}$$

Note that we can multiply 27 by 244 the way explained in N1-9.

Practice

Q3 Work out the following without a calculator.

(a) 40×20	(b) 6×300	(c) 30×0.2	(d) 400×0.005
(e) 0.03×0.8	(f) 0.007×40	(g) 8000×0.02	(h) $100 \times .0003$
(i) 3.2×200	(j) 0.004×20	(k) 0.21×500	(l) 0.6×0.04
(m) 400×0.7	(n) 1.15×0.02	(o) 560×0.2	(p) $0.005 \times 40\,000$
(q) 10.5×20	(r) 0.0314×500	(s) 60×0.002	(t) 300×4.6
(u) 0.03×0.21	(v) 2.13×0.22	(w) 520×1.6	(x) 3.90×0.0042

Approximate answers

To get approximate answers, as with multiplying by whole numbers, we just stop before we go right through the calculation. You must get the decimal point in the right place though or else the answer will be way out.

Dividing

There are many ways to divide whole numbers and decimal fraction. One method is explained below. It will work for all the numbers you will need to divide. If you have your own methods, feel free to use them instead.

Note that we normally express remainders in the decimal fraction form when dividing whole numbers and decimal fractions.

Dividing by small whole numbers

In dividing 11 by 4 above, we really divided 11.000 by 4. So we have already divided decimals by whole numbers.

The only thing different with decimal fractions is that the digits after the decimal point are not all 0s.

Let's divide 34.16 by 5:

$$\begin{array}{r} 34.16 \\ \underline{\div\ 5\quad\quad} \end{array}$$

The finished product will look like this.

$$\begin{array}{r} 3^{3}4.\ {}^{4}1^{1}0 \\ \underline{\div\ 5\quad\quad} \\ 0\ \ 6.\ \ 8\ \ 2 \end{array}$$

So the answer is 6.82. Do this yourself step by step if you need to.

Practice

Q4 Work out the following without a calculator.

(a) $4.28 \div 2$ (b) $16.34 \div 5$ (c) $0.0051 \div 3$ (d) $116 \div 7$

(e) $1.9058 \div 4$ (f) $2.96 \div 3$ (g) $0.01 \div 8$ (h) $7.004 \div 6$

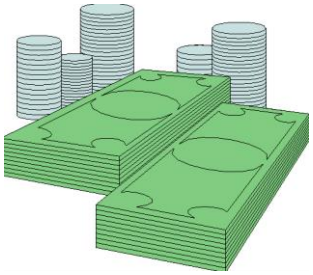
Jules: If I cut a cake into three equal pieces, each will be $1 \div 3$, i.e. 0.333 of the cake, right? If I then put the pieces back together again, I will have 3×0.333, i.e. 0.999 of the cake. So what happened to the other 0.001?

Harry: You'll find it on the knife.

Dividing by large whole numbers

By large whole numbers we mean numbers like 50, 300, 40 000 etc. – numbers where all but the first digit are zeros. You do not need to be able to divide by numbers like 42, 796 etc. Use a calculator for these.

Suppose you share some money between 300 people.



Obviously, each share will be less than if you shared between 3 people – 100 times less in fact. To divide the money between 300 people, we could divide it into 3 piles, then divide each pile into 100 smaller piles. That way we would get the 300 piles we need.

To divide by 300, we just divide by 3, then divide the answer by 100. In the same way, to divide by 40, divide by 4, then by 10. To divide by 6 000, divide by 6, then by 1000.

As an example, let's divide 4.7 by 500.

4.7 ÷5 = 9.4 9.4 ÷ 100 = 0.094 So 4.7 ÷ 500 = 0.094

Practice

Q5 Work out the following without a calculator.

(a) 6.26 ÷ 200	(b) 5.34 ÷ 50	(c) 0.0072 ÷ 30	(d) 4 ÷ 700
(e) 2.876 ÷ 4	(f) 2.96 ÷ 3 000	(g) 0.01 ÷ 800	(h) 7.2 ÷ 60

Dividing by decimal fractions

You need to be able to divide by numbers like 0.03, 0.6, 0.0004 – numbers where all but the last digit are zeros. You do not need to be able to divide by numbers like 0.56, 0.00731 etc. Again, use a calculator for these.

Suppose you have a number. The bigger the number you divide it by, the smaller the result. Dividing by 30 gives a smaller result than dividing by 3. [If you divide the money into 30 piles, the piles will be smaller than if you divide it into 3 piles.]

Conversely, the smaller the number you divide it by, the bigger the result. So, if you divide say 6 by 0.3, you should get a bigger result than if you divide by 3. This is one of the things you should have discovered by doing the Lead-In. If you think of division in terms of how many lots of 0.3 make 6, it should be obvious that you need more lots of 0.3 than of 3 to make 6.

Suppose we want to do $6 \div 3$. As we know, the answer is 2. So, if we did $6 \div 0.3$, the answer would be bigger – 10 times bigger. $6 \div 0.3 = 20$.

In the same way, to work out $14 \div 0.005$, do $14 \div 5$, which is 2.8. Then, because 0.005 is 1000 times smaller than 5, i.e. $5 \div 1000$, the answer will be 1000 times bigger. It will be 2 800.

Another way of thinking about it

Here is an alternative way of thinking about dividing by large numbers and decimals.

Suppose we need to do $41.372 \div 0.04$.

We know that fractions can be written as division ($\frac{3}{4} = 3 \div 4$) and so division can be written as a fraction ($3 \div 4 = \frac{3}{4}$). We use this idea to rewrite $41.372 \div 0.04$ as $\frac{41.372}{0.04}$.

Then we can change this to an equivalent common fraction by multiplying the top and bottom by a power of 10 so as to make the bottom a whole number. Because the bottom has 2 decimal places we multiply by 100 to get $\frac{4137.2}{4}$.

We then change this back to division form, i.e. $4137.2 \div 4$. Then we do the division as normal.

In fact we don't really have to physically change the division to a fraction – we could just multiply both numbers in the division by 100 and it comes to the same thing.

In the same way, if we had to do $74.8 \div 200$, we would divide both numbers by 100 to get $0.748 \div 2$ and then do this to get 0.374.

Practice

Q6 Work out the following without a calculator.

(a) $7.32 \div 0.2$ (b) $22 \div 0.005$ (c) $0.6 \div 0.3$ (d) $4 \div 0.05$

(e) $2.376 \div 0.004$ (f) $4.96 \div 0.7$ (g) $0.01 \div 0.8$ (h) $5.2 \div 600$

Approximate answers

To get approximate answers, as with dividing by whole numbers, we just stop before we go right through the calculation. Again, you must get the decimal point in the right place though or else the answer will be way out.

Practice

Q7 Do the following calculations approximately in your head to decide which is the closest of the four answers given

(a) 278.34×31.9	A 90	B 900	C 9000	D 90 000
(b) $612.853 \div 7$	A 8.5	B 32	C 70	D 170
(c) 0.094×0.23	A 0.002	B 0.02	C 0.2	D 2.0

Solve

Q51 6000 soldiers in the desert need 0.003 m^3 of water a day each. How many cubic metres of water will they need to last them 30 days?

Q52 How many long will 200 m^3 of water last 4000 soldiers?

Q53 Without a calculator, multiply 4×10^5 by 3×10^{-2}.

Q54 Without a calculator, multiply 2.1×10^{44} by 5×10^{33}.

Q55 Without a calculator, divide 8.4×10^{15} by 2×10^6.

Revise

Revision Set 1

Q61 Work out the following.

(a) 2.39×10	(b) 680×100	(c) $37 \div 1000$	(d) $0.0149 \div 100$
(e) 40×20	(f) 6×300	(g) 30×0.2	(h) 400×0.005
(i) 0.03×0.8	(j) 0.007×40	(k) 8000×0.02	(l) $100 \times .0003$

Q62 Calculate $34 \div 5$ without a calculator. Give your answer with

(a) a remainder (b) a common fraction (c) a decimal fraction

Q63 Calculate the following without a calculator.

(a) $4.6 \div 4$	(b) $20.8 \div 9$	(c) $40 \div 5000$	(d) $0.045 \div 30$
(e) $5 \div 0.002$	(f) $32.8 \div 0.4$	(g) $0.007 \div 0.05$	

Answers

Q1	(a) 23.9	(b) 6 800	(c) 0.037	(d) 0.149
	(e) 0.0003	(f) 0.26	(g) 0.0001	(h) 39 000
	(i) 17.9	(j) 4.42	(k) 0.032	(l) 1.752
	(m) 0.055	(n) 0.000187	(o) 0.0014	(p) 130
Q2	(a) 356	(b) 4	(c) 723	(d) 45
	(e) 3	(f) 4127	(g) 56	(h) 39
	(i) 105	(j) 314	(k) 4005	(l) 3
	(m) 45	(n) 839	(o) 52	(p) 39
Q3	(a) 800	(b) 1 800	(c) 6	(d) 2
	(e) 0.024	(f) 0.28	(g) 160	(h) 0.03
	(i) 640	(j) 0.08	(k) 105	(l) 0.024
	(m) 280	(n) 0.023	(o) 112	(p) 200
	(q) 210	(r) 15.7	(s) 0.12	(t) 1 380
	(u) 0.0063	(v) 0.4686	(w) 832	(x) 0.01638
Q4	(a) 2.14	(b) 3.268	(c) 0.0017	(d) 16.57…
	(e) 4.7645	(f) 0.986…	(g) 0.00125	(h) 1.1673
Q5	(a) 0.0313	(b) 0.1068	(c) 0.00024	(d) 0.00571…
	(e) 0.719	(f) 0.000986…	(g) 0.0000125	(h) 0.12
Q6	(a) 36.6	(b) 4400	(c) 2	(d) 80
	(e) 594	(f) 7.0857	(g) 0.0125	(h) 0.00866
Q7	(a) C	(b) C	(c) B	

Q51 540 Q52 $16^{2}/_{3}$ days Q53 12×10^{3} or 1.2×10^{4}

Q54 10.5×10^{77} or 1.05×10^{78} Q55 4.2×10^{9}

Q61	(a) 23.9	(b) 68 000	(c) 0.0037	(d) 0.00149
	(e) 800	(f) 1 800	(g) 6	(h) 2
	(i) 0.024	(j) 0.28	(k) 160	(l) 0.03
Q62	(a) 6r4	(b) $6^{4}/_{5}$	(c) 6.8	
Q63	(a) 1.15	(b) 2.3111…	(c) 0.008	(d) 0.0015
	(e) 1000	(f) 82	(g) 0.14	