M1 Maths

N2-4 Ratios

- ratios

Summary Learn Solve Revise Answers

Summary

A ratio is similar to a rate, though it is an amount of one thing corresponding to an amount of something else, not necessarily just one of the other thing. It is written differently. For instance, if the ratio of boys to girls in a club is 2:5 (2 to 5), it means there are 2 boys for every 5 girls.

Various calculations can be performed with ratios. As with rates, most are common sense and can be done in more than one way.

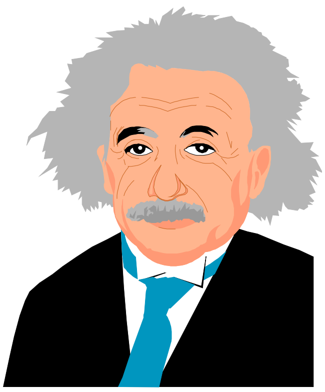
Learn

Preamble – Working Things Out for Yourself

As with Module N2-3 (Rates), once you know what is meant by a ratio (e.g. what is meant by saying that the ratio of boys to girls in a club is 4:3), then you should be able to answer all the questions in this module using just common sense.

You might like to try that. Read the section below headed *Ratios*, then try to answer all the practice questions without reading any further explanations. Go back and read the explanations only if you get totally stuck.

As explained in N2-3, being able to work out how to answer questions of a type you haven't met before is a more important skill than being able to answer the various types of problems involving ratios. If you find you can't answer the questions without reading the instructions first, then you probably need to spend more time developing your problem solving skills so you can work things out for yourself. If you get through all the practice questions without reading the explanations (and get them right), then you are doing a good job of developing your problem solving skills and you will make a good mathematician.



Ratios

A ratio is a comparison between two quantities. We can say that the **ratio** of girls to boys in the Cool Kids Youth Club is 5:2 (pronounced '*5 to 2*'). This means that for every 5 girls in the club there are 2 boys. In other words, we could put the girls in the club in groups of 5 and then there would be just enough boys to put 2 boys with each group of 5 girls.

Note that if we say '*the ratio of girls to boys is 5:2*', the number of girls (5) and the number of boys (2) must be in the same order as the words *girls* and *boys*. So it means 5 girls to 2 boys, not 5 boys to 2 girls. Because we always follow this convention, we don't have to write 5 girls : 2 boys; 5:2 will do. We can swap *boys* and *girls* round, but then we have to swap the 5 and the 2 round too, so we could say '*the ratio of boys to girls is 2:5*'.

Now, of course, saying that the ratio of girls to boys is 5:2 doesn't mean that there are just 5 girls and just 2 boys (though there could be). There could be 10 girls and 4 boys, or 15 girls and 6 boys or 50 girls and 20 boys, and so on. What is important is that there are 5 girls for each lot of 2 boys and 2 boys for each lot of 5 girls. We must be able to put all the members of the club in groups with 5 girls and 2 boys in each group and have none left over.

In a sense, a ratio is another way of writing a rate. A ratio of 5 girls to 2 boys means that there would be 2½ girls for each boy (though we wouldn't cut the girls in half) or 2½ girls per boy. We have then said the same thing as a rate.

Rates are usually written as so many of something for 1 of something else. But ratios are often written with neither of the numbers being 1. We can say the ratio of students to teachers on an excursion is 15:2. That means that there are 15 students for each 2 teachers. This could be written as a rate as 15 students per 2 teachers, but we don't usually write rates like that. We might, however, say that there are 7.5 students per teacher.

So in general, a ratio shows how many of one thing correspond to how many of another thing. Here's another example: in Australia, football fans outnumber cricket fans by 5:3. This means that there are 5 football fans for each 3 cricket fans and that football is a bit more popular than cricket. In Brazil it might is 112:5, showing that there are 112 football fans for each 5 cricket fans and that football is much more popular. In the

West Indies the ratio might be 2:9 meaning that there are 2 football fans for each 9 cricket fans and that cricket is more popular than football.

Changing the numbers in ratios

In the Cool Kids Youth Club, the ratio of girls to boys in 5:2. In other words, there are 5 girls for every 2 boys. This means that, for every 10 girls there will be 4 boys. We could say that the ratio of girls to boys is 10:4. By the same logic, we could also say that the ratio is 15:6, 20:8 and so on.

In fact, we could multiply both numbers in the ratio by any number we like (as long as both are multiplied by the same number) and the ratio will be the same.
5:2 = 10:4 = 15:6 = 20:8 = 50:20 and so on. We could write the ratio of girls to boys as any of these (or in an infinite number of other ways like 700:280) and they all mean the same thing.

You might wonder, what happens if there are only 50 girls and 20 boys in the club; could we still write the ratio as 700:280? Well, yes, we could, though we probably wouldn't. It is common practice to express ratios in their simplest form, i.e. with the smallest possible whole numbers. In the club case, it would be 5:2.

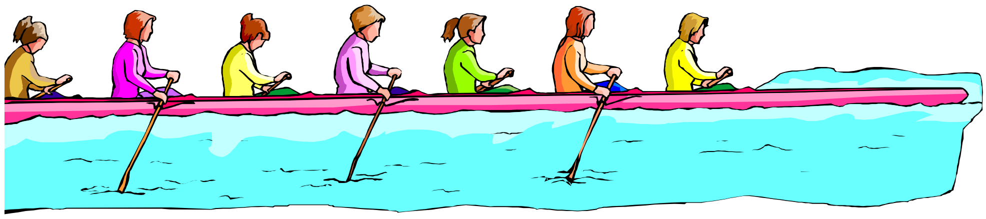
Suppose we know the ratio is 60:24. This isn't simplest form. Just as we can multiply both numbers by the same thing and get the same ratio, we can work back the other way and divide both numbers by the same thing and still get the same ratio. So we could divide both numbers by 2 and say that the ratio is 30:12. Then we divide by 2 again and get 15:6. Then we could divide by 3 and get 5:2. We can go any further because nothing divided into 5 and into 2, so 5:2 is the simplest form.

Expressing ratios in simplest for is very similar to expressing common fractions in simplest form. We just divide the two numbers by whatever we can until we have the smallest possible whole numbers.

20:8 = 10:4 = 5:2 is just like $\frac{20}{8} = \frac{10}{4} = \frac{5}{2}$

Practice

Q1 (a) Complete this sentence: 'If we say that the ratio of boys to girls in a rowing club is 4:5, this means that for every . . . .



(b) Complete this sentence: 'If there are 4 people under 21 for every 3 over 21 at a party, then the ratio of under 21s to over 21s is . . . .

(c) Write the ratio 20:12 in four different forms, two using smaller numbers, two using bigger numbers.

(d) Write the ratio 12:16 in simplest form, i.e. using the smallest possible whole numbers.

(e) Write the ratio 18:8 in simplest form.

(f) Complete this statement: 3:5 = 15: ….

(g) Complete this statement: 24:16 = …. :4

Finding one number, given the other

Suppose the ratio of cordial to water in a drink is 2:9. How much water would you put with 14 L of cordial?

We can solve this as a rate problem in much the same way as the two-step rate problems in Module N2-3 (Rates). We know that 9 L of water is put with 2 L of cordial. This means 4.5 L of water per litre of cordial. So we put 14 × 4.5 L of water with 14 L of cordial. This is 63 L. If you need further help, go back and look at the two-step rate problem section in N2-3.

Practice

Q2 (a) Water and cordial should be mixed in the ratio 7:2. How much water should be put with 4 L of cordial?

(b) Water and cordial should be mixed in the ratio 8:3. How much cordial should be put with 20 L of water?

(c) Gravel and cement are mixed in the ratio 6:1 to make concrete. How many buckets of gravel should one put with 12 buckets of cement?

(d) How many buckets of cement should go with 16 buckets of gravel?

(e) Mashed potato powder and water should be mixed in the ratio 3:5. How much water goes with 750 mL of powder?

(f) How much potato goes with 2 L of water?

(g) The height to width ratio of a picture is 3:4. How wide is it if it is 14 cm high?

(h) The height to width ratio of another picture is 1:1.2. Express this ratio using the smallest possible whole numbers.

(i) The maximum allowable student:teacher ratio for a trip to the meat works is 15:2. How many teachers are needed for 38 students?

(j) The ratio of Carla's mass to Hugo's mass is 3:8. If Carla weighs 42 kg, how much does Hugo weigh?

(k) The ratio of Steph's age to Toni's age is 5:4. If Steph is 20, how old is Toni?

Map Scales

If you have already done Module M2-1 (Maps and Scales), then you will already be able to answer these types of questions. If not, read on.

Scales on maps are often given as ratios – the ratio of the size of the map to the size of the real bit of land. For instance, if the scale is 1:1000, then it means that something 1 cm long on the map will be 1000 cm long in reality. We call 1000 the scale factor.

To find the real length of a pathway which is 6.4 cm long on the map, we just use the ratio to find the real length. As the first number in the ratio is generally 1, we just multiply by the other number, the scale factor: in this case by 1000. This gives us 6 400 cm. As this distance would be better expressed in metres, we convert it to 64 m. So the real path is 64 m long.

If we know the real length and need to find the length on the map, we divide by the scale factor instead. A 5 km road on a 1:250 000 map will be 5 km ÷ 250 000 = 0.000 02 km. This is .02 m and 2 cm.

To find the scale, given a length on the map and the real length, just put both measurements in the same units, then divide both numbers in the ratio by the smaller one. For instance if a 2000 m race track is 5 cm on the map, the scale is 5 cm : 200 m. This is 5 cm : 20 000 cm, which is 5:20 000, which is 1:4 000. So the scale is 1:4000.

Practice

Q3 (a) On a map the scale is 1:10 000. How wide is a lake that is 2.2 cm wide on the map?

(b) A road is 3 km long. How long would it be on the map in the last question?

(c) Another map has a scale of 1:500 000. How long is an irrigation ditch that is 3.2 cm long on the map?

(d) A beach 1.4 km long is 7 cm long on a map. What is the scale?

Part-part-whole problems

Bronze can be made by mixing copper and tin in the ratio 11:2. So mixing 11 kg of copper with 2 kg of tin will produce 13 kg of bronze. In this situation the 11 kg of copper and the 2 kg of tin are the parts and the 13 kg of bronze is the whole.

If we have 5 kg of tin, how much copper will we need to put with it and how much bronze can we make? We can treat this as a two-step rate problem. The amount of copper per kg of tin is 5.5 kg. So 5 kg of tine will need 27.5 kg of copper. 27 kg of copper and 5 kg of tin makes 32.5 kg of bronze.

If we are asked how much copper and tin are needed to make 20 kg of bronze, we could think as follows. 13 kg of bronze require 11 kg of copper and 2 kg of tin. So 1 kg of bronze will require $^{11}/_{13}$ kg of copper (0.846 kg) and $^{2}/_{13}$ kg of tin (0.154 kg). So 20 kg of bronze require 0.846 × 20 kg of copper (16.92 kg) and 0.154 × 20 kg of tin (3.08 kg).

The same method can be used for sharing problems. Suppose two people invested some money. Sarah put in $5 000 and Remus put in $3 000. They agreed to share the profit in the same ratio as their investments. In other words, the ratio of Sarah's share to Remus' share would be 5000:3000, i.e. 5:3. Sarah would get $5 for every $3 Remus got.

Suppose they made a profit of $400. How should they share it?

Imagine the profit was handed out in small lots. So in each lot, Sarah got $5 and Remus got $3. This would be in the right ratio. In each lot, $8 would be handed out ($5 + $3). How many lots would we get out of the $400? $400 ÷ $8 = 50. So there would be 50 lots. Sarah would get 50 lots of $5, i.e. $250; Remus would get 50 lots of $3, i.e. $150. Just to check, $250 + $150 = $400, so that is right.

Practice

Q4 (a) Brass can be made by mixing 3 kg of copper with 2 kg of zinc. How much copper would you need to make 20 kg of brass?

(b) If you had 7 kg of zinc, how much copper would you need to put with it to make brass and how much brass would you get?

(c) If you had 5 kg of copper, how much zinc would you need to put with it to make brass and how much brass would you get?

(d) Mixing 4 L of pineapple juice with 6 L of ginger ale will make 10 L of punch. How much ginger ale should be put with 2.5 L of pineapple juice and how much punch will it make?

(e) How much pineapple juice should be put with 5 L of ginger ale and how much punch will it make?

(f) How much pineapple juice and ginger ale would be needed to male 42 L of punch?

(g) Daniel and Hariette want to share $100 so that Daniel gets $7 for each $3 Hariette gets (i.e. in the ratio 7:3). How much does each get?

(h) 30 kg of beans are to be divided up in the ratio 2:3. How much is in each portion?

(i) Divide $200 in the ratio 5:3.

(j) A 66 cm line is divided in the ratio 4:7. How long is the shorter part?

(k) A line is divided in the ratio 3:2. What fraction of the line is the longer part?

(l) In making cordial, water and syrup are mixed in the ratio 5:1. How much syrup is needed to make 15 L of cordial?

(m) How much cordial could you make if you used 2.2 L of syrup?

(n) How much water would you have to put in the cordial in the last question?

(o) Fiona and Vance want to share $4000 in the ratio 3:4. How much will Vance get if they do it to the nearest dollar?

(p) The ratio of Sam's mass to Julie's mass is 3:5. Together they weigh 104 kg. How much does Julie weigh?

(q) A 6000 m^2 paddock is divided into two so that the areas are in the ratio 7:5. How much bigger is the bigger one than the smaller one?

Multi-part Ratios

Ratios can have more than two quantities. For example, when making concrete we might mix 4 buckets of gravel with 2 buckets of sand and 1 bucket of cement. The ratio of gravel to sand to cement can be written 4:2:1.

Suppose we wanted to know how much gravel and cement to put with 11 buckets of sand. Well 1 bucket of sand goes with 2 buckets of gravel and ½ a bucket of cement. So 11 buckets of sand go with 22 buckets of gravel and 5½ buckets of cement.

You should be able to answer all the same types of questions with multi-part ratios as you can with two-part ratios without further instructions. Common sense should be enough.

Practice

Q5 (a) Bronson makes cakes using butter, sugar and flour in the ratio 2:1:5. If he uses 600 g of butter, how much sugar and flour will he use?

(b) How many grams of cake mixture will be made in the previous question?

(c) How much of each ingredient would Bronson need to make 1.8 kg of cakes?

(d) If, after mixing the butter and sugar, Bronson finds the mix weighs 960 g, how much flour should he add?

(e) Mawler makes cakes using a ratio of butter, sugar and flour of 4:2:12. Write this ratio in simplest form.

(f) How much butter and flour would Mawler's cakes contain if they contained 1.5 kg of sugar?

(g) Albert, Basil and Charlie have just robbed a bank and stolen $90. If they share this in the ratio 1:2:3, how much would each get?

(h) Three sisters, Sarah, Hannah and Miah, share an inheritance in the ratio 4:5:7. If Sarah gets $2870, how much does Miah get?

(i) How much did Sarah, Hannah and Miah inherit between them?

(j) A 78 cm line is divided in the ratio 3:5:5. How long is the shortest part?

(k) The ratio of Sam's, Kim's and Skinny's masses is 4:5:2. Together they weigh 242 kg. How much does each weigh?

(l) A 7000 m^2 block of land is divided into three so that the areas are in the ratio 7:2:5. How much bigger is the biggest part than the smallest part?

(m) If you divided $3600 between four people in the ratio 2:2:3:5, how much would each get?

Odds

In Module P1-1 (Probability), we talked about the likelihood of something in terms of the probability. This is the most common way in mathematics. But there is another way that uses ratios. We can talk about how likely something is in terms of odds. Gamblers often use odds instead of probability.

Suppose that the probability that it will rain on any given day in April is ¼. In the long run, out of every 4 April days there will be 1 wet one and 3 dry ones.

We say that the odds of it raining are 3 to 1 against or 3:1 against. The odds of it being dry are 3 to 1 on (3:1 on).

If the probability of it raining were 29%, then the odds of it raining would be 71 to 29 against.

The odds are the ratio of it happening to not happening or vice versa. We generally put the bigger number first, then say *against* if the probability is less than 50%, or *on* if it is more than 50%.

Sometimes, the *against* is omitted. If it is said that the odds are 40 to 1, then this means 40 to 1 against. The *on* is never omitted.

If the odds of Grommit winning the race are 7 to 2 against, then the probability that he will win is $^2/_9$ and the probability that he won't is $^7/_9$. Just add the 7 and the 2 to get the 9. If the odds of Grommit winning the race are 7 to 2 on, then the probability that he will win is $^7/_9$ and the probability that he won't is $^2/_9$.

Make sure that you understand this and that you can convert any probability to odds and any odds to probability.

Practice

Q6 Convert the following probabilities to odds

(a) $^2/_5$ (b) $^9/_{10}$ (c) 15% (d) 96%

Q7 Convert the following odds to probabilities

(a) 3 to 1 against (b) 9:1 on

(c) 15 to 2 on (d) 100:8

Bad Questions

Some textbook questions ask you to express a ratio like 3:5 as a fraction. This can be a bit confusing, because ratios and fractions are quite different things. It's a bit like being asked to express a bowl of fruit as an equation.

It can be said that, if the ratio of boys to girls is 3:5, then there are $^3/_5$ as many boys as girls, but it can also be said that there are $^5/_3$ as many girls as boys, that $^3/_8$ of the people are boys and that $^5/_8$ of the people are girls. So, there is not one logical best answer. Though the expected answer is usually $^3/_5$.

The best approach to such questions is to ignore them or to explain to the person who asked them that they are bad questions.

Solve

Q51 80% of defence force employees are male. What is the ratio of males to females in the defence forces?

Q52 In a group of 200 people, the ratio of live ones to dead ones is 9:1. How many more living people will need to be added to the group to make it 99:1?

Q53 If 6 more cats booked into Casey's Cat and Dog Home, the ratio of cats to dogs would be 3:2. If, instead, 4 more dogs booked in, the ratio would be 9:10. What is the ratio at present?

Revise

Revision Set 1

Q61 Write the ratio 8:10 in two other forms, one with bigger numbers, one with smaller numbers.

Q62 Write the ratio 28:18 in simplest form, i.e. using the smallest possible whole numbers.

Q63 If the ratio of animals to keepers in a zoo is 10:3, how many keepers are there if there are 120 animals?

Q64 In the last question, how many animals are there per keeper?

Q65 Catroona and Bloopy share their $48 profit in the ratio 3:5. How much does each get?

Q66 Sand, gravel and cement are mixed in the ratio 4:5:2. If a job requires 30 kg of cement, how much sand and gravel will it take and how much concrete will be made?

Q67 Convert the following probabilities to odds: (i) $^2/_7$ (ii) 95%

Q68 Convert the following odds to probabilities: (i) 3 to 2 against (ii) 4 to 1 on

Answers

Q1 (a) 4 boys there are 5 girls (b) 4:3
(c) 40:24, 60:36, 10:6, 5:3 (d) 3:4 (e) 9:4 (f) 3:5 = 15:25
(g) 24:16 = 6:4 (h) 220 m (i) 30 cm (j) 16 km

Q2 (a) 14 L (b) 7.5 L (c) 72 (d) $2^2/_3$ (e) 1250 mL
(f) 1.2 L (g) 10.5 cm (h) 5:6 (i) 6 teachers (j) 112 kg
(k) 16

Q3 (a) 220 m (b) 30 cm (c) 16 km (d) 1:20 0000

Q4 (a) 12 kg (b) 10.5 kg, 17.5 kg (c) 3.33 kg, 8.33 kg
(d) 3.75 L, 6.25 L (e) 3.33 L, 8.33 L (f) 16.8 L, 25.2 L
(g) Daniel $70, Hariette $30 (h) 12 kg, 18 kg
(i) $125, $75 (j) 24 cm (k) $^3/_5$

	(l) 2.5 L	(m) 13.2 L	(n) 11 L	
	(o) $2286	(p) 65 kg	(q) 1000 m^2	
Q5	(a) 300 g, 1500 g	(b) 2400	(c) 450 g butter, 225 g sugar, 1125 g flour	
	(d) 1600 g	(e) 2:1:6	(f) 3 kg butter, 9 kg flour	
	(g) Albert $15, Basil $30, Charlie $45		(h) $5022.50	
	(i) $11480	(j) 18 cm	(k) Sam 88 kg, Kim $110 kg, Skinny 44 kg	
	(l) 2500 m^2	(m) $600, $600, $900, $1500		
Q6	(a) 2:3 against	(b) 9:1 on	(c) 17:3 against	(d) 24:1 on
Q7	(a) ¼	(b) 9/10	(c) 0.882	(d) 7.4%

Q51 4:1 Q52 1800 Q53 9:8

Q61 e.g. 16:20, 4:5 Q62 14:9 Q63 36 Q64 3.3 Q65 $18 and $30

Q66 60 kg sand, 75 kg gravel, 165 kg concrete Q67 (a) 2:5 against (b) 19:1 on

Q68 (a) 0.4 (b) 0.8