

# M1 Maths

## N2-3 Rates

- rates

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### Summary

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A rate is the amount of something for each of something else. For example, if you type at 30 words per minute, this means you type 30 words each minute.

There are three variable quantities here: rate, number of words and number of minutes. You can calculate any of these given the other two just using common sense. There are various other calculations that can be done with rates. These are detailed in the Learn and Practice sections below. Most are just common sense and can be done in a number of different ways.

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### Lead-In

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Test your typing speed. Open a word processor and type for 5 minutes. Time yourself with a watch or phone or the time in the corner of the computer screen. Type something that you know well so that you don't have to stop to think part way through. When you've finished, count the words (or get the program to count them), then work out your typing speed in words per minute.

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### Learn

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#### Preamble - Working Things Out for Yourself

Once you know what is meant by a rate (e.g. what is meant by saying someone types at 40 words per minute), then you should be able to answer all the questions in this module using just common sense.

You might like to try that. Read the next half a page (the section headed *Rates*), then try to answer all the practice questions without reading any further explanations. Go back and read the explanations only if you get totally stuck.

Being able to work out how to answer questions of a type you haven't met before is a more important skill than being able to answer the various types of problems involving rates. If you find you can't answer the questions without reading the

instructions first, then you probably need to spend more time developing your problem solving skills so you can work things out for yourself. If you get through all the practice questions without reading the explanations (and get them right), then you are doing a good job of developing your problem solving skills and you will make a good mathematician.

## Rates

A **rate** is the amount of something for each of something else.

Examples are:

- the number of words typed each minute
- the number of kilometres travelled each hour
- the number of kilometres driven on each litre of petrol
- the number of minutes in each lesson.
- The number of bacteria in each millilitre of blood

The word **per** is usually used when talking about rates, e.g.

words per minute

kilometres per hour

kilometres per litre

minutes per lesson

bacteria per millilitre.

*Per* just means *for each* or *for one*.

When abbreviating units, we usually abbreviate *per* to */*, eg. km/h, km/L, kg/L

## Rate Problems

Rate problems are really just common sense. You don't have to learn any procedures.

- (a) If you type at 20 words per minutes, how many words can you type in 3 minutes?  
In each minute you type 20, so in 3 minutes you will type  $3 \times 20$ , which is 60.



- (b) If you type at 20 words per minutes, how long will it take you to type 100 words?

In each minute you type 20. How many 20's make 100?  $100 \div 20 = 5$ .

- (c) If you type 60 words in 5 minutes how fast can you type, i.e. how many words per minute?

60 words in 5 minutes. Divide by 5 to get 1 minute.  $60 \div 5 = 12$ . Therefore 12 words per minute.

These are the three types of rate problems.

In type (a), you are told the rate and the number of minutes and asked for the number of words.

In type (b), you are told the rate and the number of words and asked for the number of minutes.

In type (c), you are told the number of minutes and the number of words and asked for the rate.

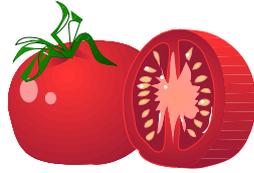
## Practice

- Q1 (a) If Nana can type at 30 words per minute, how many words can she type in 1 minute?
- (b) How many words can Nana type in 5 minutes?
- (c) How long would it take Nana to type 90 words?
- (d) Josh earns \$12 per hour. How much will he earn for 10 hours work?
- (e) How long would it take Josh to earn \$60?
- (f) Jaimie gets paid \$60 for 4 hours work. What is her rate of pay in dollars per hour?
- (g) Maerie cycles 50 km in 5 hours. What speed was she going in km/h?



- (h) Harry drove 200 km in 2 hours. What speed was he doing?
- (i) Jenny walked 20 km in 4 hours. How fast was she walking?
- (j) A car does 8 km/L. How far would it go on 20 L of petrol?
- (k) Harrison pays \$200 per week for his house. How much will he pay to live in it for 5 weeks?
- (l) How long can Harrison live in his house for \$3000?

- (m) Tomatoes cost \$3.50 per kilogram. What is the cost of 2 kg?  
(n) How many tomatoes can you buy for \$14?



- (o) Iron weighs  $7.5 \text{ g/cm}^3$ . How much would  $10 \text{ cm}^3$  weigh?  
(p) How much would 1 L of iron weigh?  
(q) Air weighs  $1.2 \text{ g/L}$ . How much would 5 L weigh?  
(r) How much space would 120 g of air take up?  
(s) If 10 L of aluminium weighs 27 kg, what is the density of aluminium in kg/L

## Bigger Numbers

So far the problems have involved simple numbers. You need to solve problems involving more difficult numbers. For example, a question might be:

*A plane flies at  $425 \text{ km/h}$ . How long will it take to fly  $118 \text{ km}$ ?*



For each of the three types of question, (a), (b) and (c), you find the third quantity by either multiplying or dividing the two given quantities. Deciding whether to multiply or divide is fairly obvious when the numbers are simple. It can be a bit less obvious when the numbers are bigger.

If it's not obvious, then you need to picture the situation in your mind or on paper. With the question about the plane you might use this approach:

Picture the plane going  $425 \text{ km}$  in an hour. You might do this by imagining your desk is a map and  $425 \text{ km}$  is the length of the desk. It takes the plane an hour to fly across the desk. Then picture  $118 \text{ km}$  on the desk. This is about a quarter of the way across – not exactly, but roughly is all we need.

So if it takes an hour to fly across the desk, it will take about a quarter of an hour to fly a quarter of the way across.

So you have an estimate of the answer – about a quarter or  $0.25$ .

Now decide which operation will give you an answer of about 0.25: is it  $425 \times 118$ ,  $425 \div 118$  or  $118 \div 425$ . You might see that it has to be  $118 \div 425$ . If you can't see that, just do all three calculations on your calculator and see which one gives you about 0.25.

$$425 \times 118 = 50\,150 \text{ – obviously not right}$$

$$425 \div 118 = 3.601\dots \text{ – that doesn't seem to be right either}$$

$$118 \div 425 = 0.2776\dots \text{ - that is close to 0.25.}$$

So the answer must be 0.2776 hours.

If you want this in minutes, multiply by 60 to get 16.6 minutes.

You will get good at rate problems as you practice. The more you do, the more you will be able to do them straight off.

## Practice

- Q2 (a) If Cynthia can type at 25 words per minute, how many words can she type in 8 minutes?
- (b) How many words can Cynthia type in 60 minutes?
- (c) How long would it take Cynthia to type a 9000-word essay?
- (d) Hairy earns \$12.60 per hour. How much will he earn for 17 hours work?
- (e) How long would it take Hairy to earn \$600?
- (f) Stevie gets paid \$84.60 for 4.5 hours work. What is her rate of pay in dollars per hour?
- (g) Magot cycled 115 km in 5 hours. What speed was she going in km/h?
- (h) Tomatoes cost \$3.79 per kilogram. How many kilos can you buy for \$2350?
- (i) Jenny crawled 20 km in 16 hours. How fast was she crawling?
- (j) A car does 8.9 km/L. How far would it go on 45 L of petrol?
- (k) Harrison pays \$320 per week for his house. How long can he live in it for \$10 000?
- (l) Drurgle drove 2460 km in 2.92 hours. What speed was he doing?
- (m) Lead has a density of 11.6 kg/L. How much would 16 L weigh?
- (n) What would be the volume of 1 t of lead?

## Two-step problems

Consider these questions:

1. If 5 people need 15 kg of food for a hike, how much do they need per person?

2. How much would be needed for 7 people?

The first question asks you to find the rate, i.e. the amount of food for 1 person. It is 3 kg/person. Then you can use this answer to work out that 7 people would need 21 kg.

Sometimes you will be given the second question without the first – like this:

If 5 people need 15 kg of food for a hike, how much would be needed for 7 people?

In this case you work it out using the same two steps as before. First work out the rate in kg/person (3), then use this to work out the amount for 7 people (21 kg).



## Practice

- Q3
- (a) If 5 people need 15 kg of food for a hike, how much will 11 people need?
  - (b) If 4 people need 20 kg of food, how much will 9 people need?
  - (c) If 8 people need 12 kg of food, how much will 18 people need?
  - (d) If 10 people need 18 kg of food, how much will 4 people need?
  - (e) If Fred earns \$28 for 4 hours work, how much will he earn for 7 hours?
  - (f) If Julia earns \$160 for 5 hours work, how much will she earn for 7 hours?
  - (g) If Mungo earns \$92 for 8 hours, how much would he earn for 3 hours?
  - (h) A car travels 270 km on 15 L of petrol. How far would it go on 110 L?
  - (i) A truck goes 90 km on 16 L, how far would it go on 5L?

## Round the Other Way

Note that in Q3(a) we were told that 15 kg of food was needed for 5 people and we were asked the amount of food for 11 people, so we first worked out the rate – 3 kg per person, then the amount for 11 people – 33 kg.

But suppose the question was: If 5 people need 15 kg of food for a hike, how many people could go if we had 24 kg of food?

We can still work it out the same way: find the rate (3 kg per person), then find the number of people that need 24 kg. It is  $24 \div 3 = 8$ . This second step is the same kind of

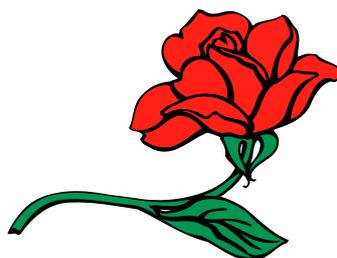
calculation you worked on in Q2.

Alternatively, you could find the rate round the other way, i.e. the number of people per kg of food. This would be  $5 \div 15 = 0.333\dots$  people/kg. Then 24 kg would be enough for  $24 \times 0.333\dots = 8$  people.

Both methods work equally well and you can please yourself which you use. All rate problems are really just common sense after all, and there is usually more than one way to do them. There are other possible methods that haven't been mentioned here too. Feel free to use any method as long as it consistently gives the right answer and as long as you can explain if needed to.

## Practice

- Q4
- (a) 5 people need 20 kg of food for a hike. How many could go if there was 32 kg?
  - (b) 6 people need 15 kg of food for a hike. How many could go if there was 65 kg?
  - (c) A motorbike goes 225 km on 5 L of petrol. How much petrol would it use going 135 km?
  - (d) A car goes 140 km on 7 L, how many litres would it take to go 200 km?
  - (e) 5 cm<sup>3</sup> of gold weighs 95 g. How much would 12 cm<sup>3</sup> weigh?
  - (f) What would be the volume (in cm<sup>3</sup>) of a 570 g gold bar?
  - (g) 20 L of air weighs 25g. How much would 530 L weigh?
  - (h) What would be the volume of 170 g of air?
  - (i) Jade gets paid \$85 for 5 hours work. How long would she have to work to earn \$1000?
  - (j) How much would Jade earn if she worked 21 hours?
  - (k) A plane travels 3200 km in 5 hours. How many hours would it take to do a journey of 7140 km?
  - (l) A gardener needs 150 L of mulch to plant 18 roses. How much would he need to plant 100 roses?



- (m) Another gardener needs 4 kg of fertiliser to fertilise 50 cucumber plants. How many could he fertilise with just 1.6 kg?

- (n) A weed keeps growing at 2.4 cm per day. How long would it take to grow 31 cm?
- (o) 50 mice weigh the same as 14 rats. How many mice would it take to balance 35 rats?
- (p) How many rats would it take to balance 75 mice?
- (q) Sally's hair grows 15 cm in 50 weeks. How many weeks will it take to grow 54 cm?
- (r) 5 ounces of gold is worth \$6250. How much is 3.1 ounces worth?
- (s) How many ounces of gold could you buy for \$20 000?
- (t) 7 hours pay for Harry is \$245. How much is 5 hours pay?
- (u) Half hour's pay is \$18. How much is  $\frac{2}{3}$  hour pay?
- (v) 5% of my money is \$20. How much is 8% of it?
- (w) 15% of my money is \$36. How much is 40% of it?
- (x) 30% of my money is \$750. How much is 100% of it?
- (y) 40% of my money is \$120. How much do I have?

Questions like these are sometimes called *proportion problems*, but *two-step rate problems* is a good term for them.

## Changing Units

If you walk at 3 km/h, that means you will walk 3 km every hour (as long as you keep walking at the same speed). 3 km is 3000 m (because 1 km is 1000 m). So your speed is also 3000 m/h. This should be fairly obvious.

Suppose we wanted to know what 3000 m/h is in metres per minute. Think about it this way: if you walk 3000 m in an hour, how far will you walk in a minute? Obviously less distance – one sixtieth of the distance in fact. So divide 3000 by 60. This gives us 50 m/min.



## Practice

- Q5
- (a) If I walk 8 km/h, what is that in m/h?
  - (b) If I crawl 2 km/h, what is that in m/h?
  - (c) If a caterpillar crawls 120 m/h, how many metres per minute is that?
  - (d) A plane flies 600 km/h. What is that in km/min?
  - (e) Write 5 km/h in m/h.

- (f) Write 6000 m/h in m/min
- (g) A car travels 200 m/min. What is that in km/min?
- (h) Willard can walk at 400 cm/s. What is that in m/s?
- (i) Adrienne can run at 4 m/min. What is that in m/h?
- (j) Jeremy can run at 4 m/s. What is that in m/min?
- (k) A printer can print 180 words/min. How many words per second is that?
- (l) If Stella earns \$150 per hour, what is her rate of pay in dollars per minute?
- (m) What is Stella's rate of pay in cents per minute?
- (n) Iron has a density of 7.5 g/mL. What is that in g/L?
- (o) What is the density of iron in kg/L?

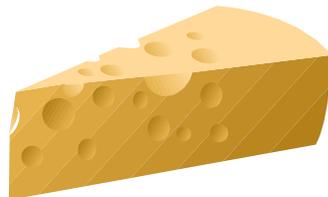
Some of the following ones require you to change both units. Change one and find the new rate, then change the other.

- (p) Harry walks 6 km/h. What is that in m/min?
- (q) What is Harry's walking speed in m/s?
- (r) Lead has a density of 11.6 kg/L. What is that in g/mL?
- (s) Kat is paid 50c/min. What is that in \$/h?
- (t) Macadamias cost \$30/kg. What is that in c/g?
- (u) Almonds are 2.2 c/g. What is that in dollars per kilogram?
- (v) A Nissan does 12 km per litre of petrol. What is that in cm/mL?
- (w) A Ford does 345 cm/mL. What is that in km/L?

## Non-standard rates

The standard way of expressing a rate is as the number of units of one quantity for **one** unit of the other. So km/h means the number of km in **one** hour.

But occasionally you will see rates like L/100 km for fuel consumption of a car or \$/100 g for the price of food on supermarket shelves.



These should be fairly easy to interpret. If a car uses 8 L/100km, then it will use 0.08L/km (just divide 8L by 100 to find the fuel use in 1 km).

## Practice

- Q6 Write the following as standard rates
- (a) 8 L per 100 km
  - (b) 20 L/100 km
  - (c) \$4.50 per 100 g
  - (d) \$30 per 5 kg
  - (e) 300 customers every 2.5 hours
- Q7 Write the following in the non-standard units requested
- (a) \$3.50 for 500 g in \$/100g
  - (b) 37L for 250km in L/100 km
  - (c) 26 words/minute in words per 20 minutes
  - (d) \$40 per 50 g in \$ per half kilogram
  - (e) 23.4 g/L in g/100 mL
  - (f) 75 g/L in mg/100 mL
  - (g) \$24 for 3 kg in c/100 g
- Q8 Decide which is the best buy (smallest amount per g or per 100 g or whatever) out of each of these pairs
- (a) A: \$4.50 for 500 g or B: \$2.70 for 250 g
  - (b) A: \$5.70 for 250 g or B: \$8.10 for 375 g
  - (c) A: \$4.60 for 155 g or B: \$3.20 for 125 g
  - (d) A: \$5 for 700 g or \$8 for 1.15 kg
  - (e) A: \$18 for 12 kg or \$30 for 20 kg
  - (f) 60 for \$12.50 or 100 for \$19.90

## Swapping units

Suppose a car goes 10 km on 1 L of fuel, i.e. it goes 10 km/L. Suppose too that we want to know how many litres of fuel it uses per kilometres, i.e. its fuel consumption in L/km. In other words we want to swap the units round.



To do this, we think like this. We need to know how many litres it takes to go 1 km. On 1 L it goes 10 km, so it will take a tenth of that to go 1 km.  $1 \div 10 = 0.1$ , so it takes 0.1 L/km.

Note that 0.1 is  $1 \div 10$ . To switch the units in a rate, just divide 1 by the rate. Another way of saying this is 'take the reciprocal of the rate'. This works both ways. So,

$$20 \text{ km/L} = 1 \div 20 \text{ L/km} = 0.05 \text{ L/km}$$

$$0.05 \text{ L/km} = 1 \div 0.05 \text{ km/L} = 20 \text{ km/L}$$

$$5 \text{ words/min} = 1 \div 5 \text{ min/word} = 0.2 \text{ min/word}$$

$$4 \text{ g/mL} = 0.25 \text{ mL/g}$$

$$0.2 \text{ g/mL} = 5 \text{ mL/g}$$

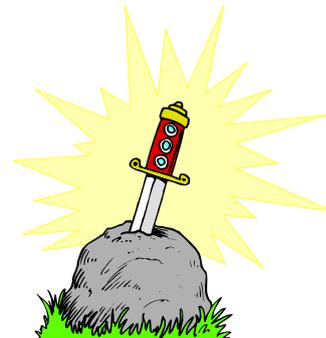
$$0.04 \text{ km/min} = 25 \text{ min/km}$$

$$12 \text{ \$/h} = 0.0833 \text{ h/\$} = 5 \text{ min/\$}$$

## Practice

Q9 Write these rates around the other way

- (a) 5 km/L
- (b) 10 km/h
- (c) 8 words/min
- (d) 0.2 words per second.
- (e) 8 min/km
- (f) 4 L/kg?
- (g) 200 kg/m<sup>3</sup>
- (h) 0.001 kg/L
- (i) 0.2 \\$/min
- (j) 3.6 tonnes/day
- (k) 0.29 cm/year
- (l) 163.7 miles/hour



## Double Rates

If you pick olives at Oily Pete's Olive Farm, you are paid \$250 for a day's work. Pete has a lot of Olives to pick and needs many pickers over several days. His wages bill is \$250 per picker per day. This is a **double rate** in the sense that there are two *pers*.

If Pete employs 10 pickers for 8 days, he will need to pay  $10 \times 8 \times \$250$ , or \$20 000. We just multiply the wage per picker per day by the number of pickers and then by the number of days. This should make sense and be reasonably obvious.

Another way to think about it is that he pays \$250 per picker, i.e.  $10 \times \$250$  or \$2500 and so \$2500 per day, i.e.  $8 \times \$2500$  or \$20 000.

## Practice

- Q10 (a) How much would Pete pay if he employed 6 pickers for 12 days?  
(b) How much would Pete pay if he employed 2 pickers for  $5\frac{1}{2}$  days?  
(c) If Smelly Ellie pays \$25 per person per hour to pick garlic, how much would she have to pay if she employed 5 people for 8 hours?  
(d) If Sam paid \$32 000 to 20 people who worked for 5 days, how much did he pay per person per day?  
(e) Ernest pays \$24 per person per hour. How many people could he employ for an 8-hour day if he could pay \$960.  
(f) Hiring a Bullet Bus costs \$4 per seat per hour. What would it cost to hire a 15-seater Bullet Bus for 4 hours?  
(g) If you could spend \$704, how many hours could you hire a 32-seater Bullet Bus for?



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## Solve

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- Q51 If 6 kg of gold has a volume of 306 mL, what will be the mass in tonnes of 163 L of gold?
- Q52 If 4 people can pick 720 kg of strawberries in 8 hours, how many kilos could 9 people pick in 6 hours?
- Q53 If 6 people can pack 126 boxes in 3 hours, how long will it take 8 people to pack 900 boxes?
- Q54 Daphne walks at 5 km/h, but her husband, Albert, can only manage 4 km/h. When they walk together, Daphne walks ahead, then turns around and walks back past him, then turns around again and overtakes him, and so on. That way they stay reasonably close together. What percentage of the time does Daphne spend walking the opposite direction to Albert? What percentage of the route does she cover 3 times?
- Q55 If a man can see 30 km from the top of a tower, how far can half a man see from the top of two towers?

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## Revise

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### Revision Set 1

- Q61 If Dravinia can type at 42 words per minute, how many words can she type in  $2\frac{1}{2}$  minutes?
- Q62 Josh earns \$120 for 15 hours work. How much does he earn per hour?
- Q63 A car travels at 75 km/h. How long will it take to drive 400 km?
- Q64 If 8 people need 20 kg of food, how much will 15 people need?
- Q65 A motorbike goes 420 km on 10.5 L of petrol. How much petrol would it use going 240 km?
- Q66 Jeronimo is paid \$45/h. What is that in c/min?
- Q67 (a) If Pansy's marmalade costs \$3.80 for 450g, what is that in \$/100 g?  
(b) If Violet's marmalade costs \$5 for 600g, is that more or less than Pansy's marmalade per 100 g?
- Q68 Write 1.29 g/L round the other way.
- Q69 If running a heater costs 40c per kilowatt per hour, what would it cost to run a 4 kilowatt heater for 12 hours?

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## Answers

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- Q1 (a) 30 (b) 150 (c) 3 minutes (d) \$120 (e) 5 hours  
(f) \$15/h (g) 10 km/h (h) 100 km/h (i) 5 km/h (j) 160 km  
(k) \$1000 (l) 15 weeks (m) \$7 (n) 4 (o) 75 g  
(p) 7.5 kg (q) 6 g (r) 100 000cm<sup>3</sup> (s) 2.7 kg/L
- Q2 (a) 200 (b) 1500 (c) 360 minutes (d) \$214.20  
(e) 47h 38m (f) \$18.8/h (g) 23 km/h (h) 620 kg  
(i) 1.25 km/h (j) 400.5 km (k) 31¼ weeks (l) 842.46 km/h  
(m) 185.6 kg (n) 86.2 L
- Q3 (a) 33 kg (b) 45 kg (c) 27 kg (d) 7.2 kg (e) \$49  
(f) \$224 (g) \$34.50 (h) 1980 km (i) 28.125 km
- Q4 (a) 8 (b) 26 (c) 3 km (d) 10 (e) 228 g  
(f) 30 cm<sup>3</sup> (g) 662.5 g (h) 136 L (i) 58.82 h (j) \$357  
(k) 11.16 h (l) 833.3 L (m) 20 (n) 12.9 days (o) 125  
(p) 21 (q) 180 (r) 3875 (s) 16 (t) \$175  
(u) \$24 (v) \$32 (w) \$96 (x) \$2500 (y) \$300
- Q5 (a) 8000 (b) 2000 (c) 2 (d) 10 (e) 5000  
(f) 100 (g) 0.2 (h) 4 (i) 240 (j) 240  
(k) 3 (l) 2.50 (m) 250 (n) 7500 (o) 7.5  
(p) 100 (q) 1.667 (r) 11.6 (s) 30 (t) 3  
(u) 22 (v) 1200 (w) 3.45
- Q6 (a) 0.08 L/km (b) 0.2 L/km (c) 4.5 c/g (d) \$6/kg (e) 120 customers per hour
- Q7 (a) 70 c/100 g (b) 14.8 L/100 km (c) 520 words per 20 minutes  
(d) \$400 per half kilogram (e) 2.34 g/1000 mL (f) 7500 mg/100 mL  
(g) 80 c/100 g
- Q8 (a) A (b) B (c) B (d) A (e) same (f) B

- Q9 (a) 0.2 L/km (b) 0.1 h/km (c) 0.125 min/word (d) 5 s/word (e) 0.125 km/min  
 (f) 0.25 kg/L (g) 0.005 m<sup>3</sup>/kg (h) 1000 L/kg (i) 5 min/\$ (j) 0.278 days/t  
 (k) 3.45 years/cm (l) 0.0061 h/mile
- Q10 (a) \$18 000 (b) \$2750 (c) \$1000 (d) \$320 (e) 5  
 (f) \$240 (g) 5<sup>1/2</sup>
- Q51 3.196 t Q52 1215 kg Q53 7 h 4 min Q54 10%, 30%
- Q55 Silly question – he'd be dead.
- Q61 105 Q62 \$8 Q63 5.3 hours Q64 37.5 kg  
 Q65 6L Q66 75 c/min Q67 (a) 0.8444 \$/100 g (b) less  
 Q68 0.775 L/g Q69 \$19.20