M1 Maths

N2-1 Number Sets

- the various sets of numbers

Summary Learn Solve Revise Answers

Summary

The set of natural (or counting) numbers, N, is the numbers we use to count things: 1, 2, 3, 4, . . . and so on for ever.

The set of whole numbers, W, consists of the natural numbers plus 0.

The integers, Z, are the whole numbers, plus their negative equivalents.

The rational numbers, Q, are the set of numbers which can be expressed as an integer divided by another integer. As decimals, they either terminate or recur.

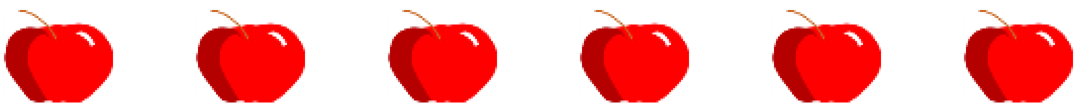
The irrational numbers cannot be so expressed. They include surds and, as decimals, neither terminate nor recur.

The rational and irrational numbers together are called the real numbers, R.

Learn

Counting Numbers or Natural Numbers

The **counting numbers** are the numbers we count in:
1, 2, 3, 4, 5, 6, . . . and so on for ever



The counting numbers are also called the **natural numbers**.

Whole Numbers

The **whole numbers** are the natural numbers plus zero, i.e.
0, 1, 2, 3, 4, 5, ... and so on for ever.

Integers

The **integers** are the whole numbers plus their negative equivalents:
. . . –3, –2, –1, 0, 1, 2, 3, 4, 5, . . .

Rational Numbers

The **rational numbers** are the integers plus numbers which can be expressed as common fractions. They are numbers which can be written as an integer over (divided by) an integer, i.e. as a **ratio** of two integers. Examples are $^3/_4$, $^{-148}/_{1983}$, $^{17}/_1$, $^0/_{12}$ etc.

All rational numbers can be written as whole numbers, as terminating decimals or as recurring decimals. All whole numbers, terminating decimals and recurring decimals are rational numbers.

A terminating decimal is one that has a finite number of decimal places, e.g. –3.78, 16.4865427236783. A recurring decimal is one which has an infinite number of decimal places, but the same sequence of numbers keeps repeating for ever, e.g. 92.361361361361…, 0.33333…, 0.0036429494949494949…

The denominator of a rational number can always be expressed as a product of primes. If this product contains only 2s and 5s, then the decimal equivalent will terminate. Otherwise it will recur.

Irrational Numbers

Irrational numbers are numbers which are not rational. $\sqrt{2}$ is one. It cannot be written as a ratio of two integers. As a decimal fraction it doesn't terminate or recur, i.e. it has an infinite number of decimal places and the sequence doesn't end up repeating the same numbers over and over.

Many square roots are irrational. Irrational roots are sometimes called surds. $\sqrt{2}$ is a surd. $\sqrt{3}$ is a surd. $\sqrt{4}$ is not a surd because it is 2 which is rational.

If we divide the circumference (perimeter) if a circle by its diameter (distance across it), we get the number 3.1415926… This number is called π (pi) and is also irrational.

The exact length or mass of a real object will always be an irrational number. Can you see why? The approximation we make when measuring will generally be a rational number, though.

Real Numbers

The rational and the irrational numbers together are called the **real numbers**. So the real numbers are all the types are numbers that you would be familiar with.

Mathematicians have invented imaginary numbers, however, which are not real numbers. Imaginary numbers involve the square root of –1, which is not a real number: no real number squared gives –1.

The real and imaginary numbers together are called the complex numbers. Imaginary numbers / complex numbers can be useful in advanced maths and you may meet them in late high school or university, but you don't need to know about them now.

Abbreviations for number sets

Mathematicians often use single-letter abbreviations for some of the number sets. All are upper case letters. Ones you should know are:

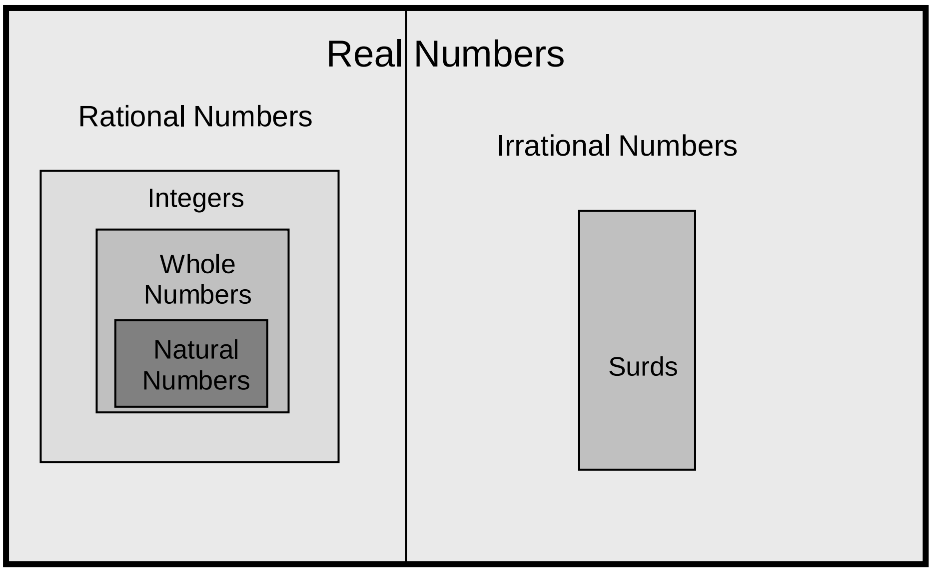
N – Natural numbers   W – Whole numbers   Z – Integers

Q – Rational numbers   R – Real numbers

We don't normally use a letter for the irrational numbers. That is somewhat irrational.

Relation between the Various Sets of Numbers

The relationship between the various sets of numbers can be expressed in a diagram as follows. This kind of diagram is sometimes called a Venn diagram.



Note: The size of the areas in the diagram is schematic only and doesn't indicate the relative sizes of the different sets of numbers. For example there are an infinite number of rationals between every two integers and there are an infinite number of irrationals for every rational.

Something you don't need to know (though it's interesting)

Why is a surd called a surd?

About 500 BC Greek mathematicians believed that all mathematics and geometry could be explained in terms of integers. All quantities were considered to be a ratio of two integers. They knew that the length of the diagonal of a unit square was $\sqrt{2}$ and they didn't know what two integers made $\sqrt{2}$, but they believed that some day someone would work it out.

But before they did, someone else managed to prove that $\sqrt{2}$ was not the ratio of two integers. This was, of course, very deflating for the mathematical community. Rumour has it that whoever proved it was on a ship at the time with some other mathematicians and that the other mathematicians threw him overboard in disgust. That story probably has as much truth as the apple falling on Newton's head, causing him to discover gravity, but it's a popular tale.

Anyway, the Greeks accepted that irrational numbers do exist. They named them *a logos*, meaning *without logic* or *illogical*. But the word *logos* has two meanings – *logic* and *word*.

After Greek civilisation was succeeded by Arab civilisation, the Arabs started to translate the old Greek texts into Arabic. The person who translated the bit about irrational numbers couldn't decide whether *logos* was meant as *logic* or as *word*. He had a 50-50 chance, but he went for the wrong one and irrational numbers became numbers *without a word* in Arabic.

Later on, the Arabic texts got into the hands of European scholars who wrote in Latin. They couldn't make a lot of sense of *numbers without a word*, so they translated them as *surdus* meaning *deaf*. So irrational numbers became deaf numbers.

The English word *surd* comes from the Latin *surdus*. So *surd* means *deaf number*.

Practice

Q1 Write the meanings of the various sets of numbers.

Q2 Draw the Venn diagram showing the relations between the sets of numbers.

Q3 Copy the following table and complete it with ticks and crosses.

	–6	0.3877	$\sqrt{5}$	2π	$1^7/_9$	$\sqrt{16}$	0	∞
Natural number								
Whole number								
Integer								
Rational number								
Irrational number								
Real number								

Q4 Write the single-letter abbreviations for each of the following sets.

(a) natural numbers (b) whole numbers (c) integers

(d) rational numbers (e) real numbers

Solve

Q51 There are two whole numbers which are neither prime nor composite. What are they?

Q52 For each of the following, say whether it is a rational number and whether it is a real number:

(a) $1 \div 4$ (b) $4 \div 1$ (c) $0 \div 4$ (d) $4 \div 0$

(e) $1 \div \pi$ (f) $0 \div \pi$ (g) $\pi \div \pi$ (h) $\left(\sqrt{2}\right)^2$

Q53 Is your exact mass in kilograms a rational number?

Revise

Revision Set 1

Q61 Copy the following table and complete it with ticks and crosses.

	–½	31	1	$\sqrt{6}$	π	$\sqrt{9}$	0	$0.\dot{3}$
Natural number								
Whole number								
Integer								
Rational number								
Irrational number								
Real number								

Q62 Write the single-letter abbreviations for each of the following sets.

(a) natural numbers (b) whole numbers (c) integers

(d) rational numbers (e) real numbers

Answers

Q1 See pages 1-2 of this module.
Q2 See page 3 of this module.
Q3

	–6	0.3877	$\sqrt{5}$	2π	$1^7/_9$	$\sqrt{16}$	0	∞
Natural number	×	×	×	×	×	✓	×	×
Whole number	×	×	×	×	×	✓	✓	×
Integer	✓	×	×	×	×	✓	✓	×
Rational number	✓	✓	×	×	✓	✓	✓	×
Irrational number	×	×	✓	✓	×	×	×	×
Real number	✓	✓	✓	✓	✓	✓	✓	×

Q4 (a) N (b) W (c) Z (d) Q (e) R

Q51 0, 1
Q52 (a) rational and real (b) rational and real (c) rational and real
(d) not real, therefore not rational (e) irrational, real
(f) rational and real (g) rational and real (h) rational and real
Q53 no, because if measure infinitely accurately, the decimal places would go on for ever without repetition

Q61

	–½	31	$\sqrt{-4}$	$\sqrt{6}$	π	$\sqrt{9}$	0	$0.\dot{3}$
Natural number	×	✓	×	×	×	✓	×	×
Whole number	×	✓	×	×	×	✓	✓	×
Integer	×	✓	×	×	×	✓	✓	×
Rational number	✓	✓	×	×	×	✓	✓	✓
Irrational number	×	×	×	✓	✓	×	×	×
Real number	✓	✓	×	✓	✓	✓	✓	✓
Imaginary number	×	×	✓	×	×	×	×	×
Complex number	✓	✓	✓	✓	✓	✓	✓	✓

Q62 (a) N (b) W (c) Z (d) Q (e) R