M1 Maths

N1-9 Decimal Operations 1

- exact and approximate mental/written methods to add and subtract whole numbers and decimal fractions and multiply and divide whole numbers

Summary Learn Solve Revise Answers

Summary

There are numerous ways to add, subtract, multiply and divide decimal numbers, both on paper and in one's head. Different methods are suitable for different numbers. The more strategies you have at your disposal, the better.

One strategy for each type of operation is given in this module. These are easily adapted for getting approximate answers more quickly than exact answers.

Learn

There are lots of ways to add and subtract numbers, and what is best for one pair of numbers won't always be the best for another pair. Likewise, the best method for doing them on paper won't always be the best method for doing them in your head.

One procedure is given below for each operation. Each procedure can be used on all problems involving that operation. These procedures may not be the main ones you have learnt, but they are about the easiest to understand.

+ +

Adding whole numbers

Suppose we want to add 725 to 4 358. The 725 consists of 700, 20 and 5, so we have to add all of these to 4 358

Start with 4 358

Add the 700 by counting up 7 hundreds to 5 058

Add the 20 by counting up 2 tens to 5 078

Then add the 5 by counting up 5 to 5 083

Practice

Q1 Calculate the following without a calculator. You may use any mental or written method.

(a) 168 + 83	(b) 900 + 348	(c) 45 + 6192	(d) 732 + 69
(e) 461 + 927	(f) 230 + 43	(g) 2741 + 87	(h) 3285 + 15
(i) 274 + 83	(j) 5729 + 146	(k) 126 + 451	(l) 45 + 124

Adding decimal fractions

This works the same way except that there are tenths, hundredths etc. to add as well.

Suppose we want to add 25.67 to 358.1. The 25.67 consists of 20, 5, 0.6 and 0.07, so we have to add all of these to 358.1

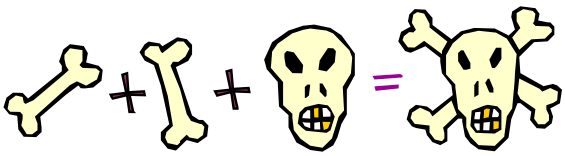
Start with 358.1

Add the 20 by counting up 2 tens to 378.1

Then add the 5 by counting up 5 to 383.1

Then add the 0.6 by counting the tenths place up 6 to 383.7

Then add the 0.07 by counting the hundredths place (which is 0) up 7 to 383.77



Practice

Q2 Calculate the following without a calculator. You may use any mental or written method.

(a) 16.8 + 8.3	(b) 900 + 34.8	(c) 46.1 + 9.27
(d) 230 + 43.8	(e) 0.274 + 0.83	(f) 57.29 + 14.6
(g) 1 385 + 118.3	(h) 900.0 + 374.2	(i) 46.1 + 927.62761
(j) 650.03 + 543.8	(k) 0.074 + 0.0083	(l) 51.7 + 39.62

Subtracting whole numbers

Subtraction is like addition except we count down instead of up.

Let's do 221 – 73

Start with 221

Count down 70 to 152

Count down 3 to 149

Practice

Q3 Calculate the following without a calculator. You may use any mental or written method.

(a) 168 – 83	(b) 900 – 348	(c) 425 – 68	(d) 1732 – 117
(e) 461 – 427	(f) 230 – 43	(g) 41 274 – 1868	(h) 328 – 202
(i) 274 – 83	(j) 5729 – 146	(k) 126 – 41	(l) 475 – 124

Subtracting decimal fractions

Subtraction decimals is like subtracting whole numbers except that there are tenths, hundredths etc. as well.

Let's do 221.58 – 73.2

Start with 221.58

Count down 70 to 152.58

Count down 3 to 149.58

Count down 0.2 to 149.38

Practice

Q4 Calculate the following without a calculator. You may use any mental or written method.

(a) 22.3 – 8.1	(b) 700 – 35.4	(c) 46.1 – 9.27
(d) 310 – 43.8	(e) 0.374 – 0.83	(f) 57.29 – 14.6
(g) 1 385 – 118.3	(h) 900.0 – 374.2	(i) 46.1 – 927.62761
(j) 650.03 – 543.8	(k) 0.074 – 0.0083	(l) 51.7 – 39.62

Approximate methods for adding and subtracting

These procedures above will give exact answers. Any of them can be adapted to give an approximate answer by stopping part way through the procedure.

For example, suppose we are doing 4000 – 1385.

Take away the 1000 to leave 3000, then take away the 300 to leave 2700. Stopping there gives an answer which might be near enough for some purposes.

Practice

Q5 Do the following calculations approximately in your head to decide which is the closest of the four answers given

(a) 9 200 + 473.8	A 9 000	B 10 000	C 11 000	D 13 000
(b) 0.086 + 0.15278	A 0.24	B 0.42	C 0.9	D 1.01
(c) 600 + 593.8	A 1 000	B 1 100	C 1 200	D 1 300
(d) 2.93 + 0.0431	A 2.9	B 3.3	C 3.9	D 4.7
(e) 9200 – 4738	A 3000	B 4500	C 6000	D 7500
(f) 86 + 15278	A 100	B 15 000	C 16 000	D 80 000
(g) 7195 + 1239	A 8100	B 8200	C 8300	D 8400
(h) 7195 – 1239	A 5900	B 6000	C 6100	D 6200
(i) 0.186 – 0.15278	A 0.03	B 0.3	C 0.9	D 1.01
(j) 600 – 593.8	A 5	B 10	C 50	D 100
(k) 0.086 – 0.015278	A 0.01	B 0.1	C 1	D 10

As with adding and subtracting, there is more than one way to multiply and to divide. One method is given below for multiplication and one for division.

× ×

Multiplying

First we need to know how to multiply by 10, 100, 1000 etc. We do this by adding 1, 2 or 3 zeros according to how many zeros on the number we are multiplying by.

So $4 \times 10 = 40$, $6 \times 100 = 600$, $15 \times 10\,000 = 150\,000$, $367 \times 10 = 3670$ etc.

Then we need to be able to do multiplications like 400×30. We just do 4×3 to get 12, then add the two zeros from the 400 and the one zero from the 30 to make 12 000. This is the answer.

Why does this work? Well, 400×30 $= 4 \times 100 \times 3 \times 10$

$= 4 \times 3 \times 100 \times 10$

$= 12 \times 1000$

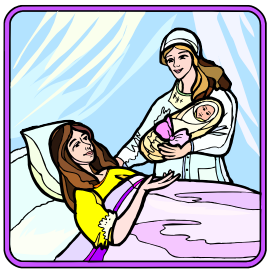
$= 12\,000$

which is 12 with three zeros added.

In the same way, $50 \times 4\,000 = 200\,000$

$6 \times 700 = 4\,200$

$4\,000 \times 70\,000 = 280\,000\,000$ and so on



Practice

Q6 Calculate the following without a calculator.

(a) 8×100 (b) 22×10 (c) 10×1000

(d) 40×10 (e) $600 \times 10\,000$ (f) 24×100

Q7 Calculate the following without a calculator. You may use any mental or written method.

(a) 600×400 (b) 900×30 (c) 50×660

(d) $5\,000 \times 8$ (e) $20 \times 60\,000$ (f) 2×400

(g) $8\,000 \times 700$ (h) $20\,000 \times 50$ (i) $500 \times 8\,000$

(j) $400 \times 9\,000$ (k) $50\,000 \times 2\,000$ (l) 100×500

Now suppose we wanted to multiply 135 by 27.

This is 135 lots of 27. That is 100 lots of 27 plus 30 lots of 27 plus 5 lots of 27.

100 lots of 2.7 is 100 lots of 2 plus 100 lots of .7;

30 lots of 27 is 30 lots of 20 plus 30 lots of 7;

5 lots of 27 is 5 lots of 20 plus 5 lots of 7

We can lay all these out in a grid or table like this:

	100	30	5
20			
7			

Then we fill in the squares by multiplying the number above it by the number to the left of it.

	100	30	5
20	2000	600	100
7	700	210	35

Then we add up all the bits to get 3 645.

Practice

Q8 Calculate the following without a calculator. You may use any mental or written method.

(a) 68 × 3 (b) 93 × 4 (c) 45 × 6 (d) 732 × 6

(e) 461 × 8 (f) 230 × 15 (g) 274 × 21 (h) 328 × 50

(i) 2714 × 83 (j) 572 × 106 (k) 126 × 41 (l) 45 × 124

(m) 270 × 51 (n) 4297 × 92 (o) 4208 × 187 (p) 732 × 190

÷ ÷

Dividing

Suppose we want to divide 6234 by 5.

Imagine this is $6234 to be shared equally between 5 people. Also imagine that we have the money as 6 $1000 notes, 2 $100 notes, 3 $10 notes and 4 $1 notes.

Start with the $1000 notes. We can give each person 1 of these. This uses 5 of them and leaves one over which can't be shared equally.

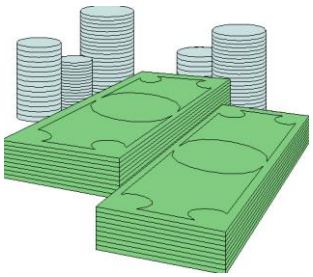
So we change the spare one for 10 $100 notes. Along with the 2 $100 notes we already have, we now have a total of 12 $100 notes. So we share these. Each person gets 2, and there are 2 left over.

Change the 2 left-over $100 notes for 20 $10 notes. Along with the 3 we already have, this gives us a total of 23 $10 notes. So we share these. Everyone gets 4 and there are 3 left over.

We change the 3 left-over $10 notes for 30 $1 notes. Along with the 4 we started with, we now have 34 $1 notes. So we share these. Each person gets 6 and there are 4 left over.

So everyone gets 1 $1000 note, 2 $100 notes, 4 $10 notes and 6 $1 notes, and there is $4 left over

So 6234 ÷ 5 = 1246 remainder 4



Laying out division

We can lay this division out as follows:

6 2 3 4

÷ 5

Share the 6 $1000 notes. They get 1 each. Write the 1 underneath. There is 1 over, which we change to 10 $100 notes, giving us a total of 12 $100 notes. So we change the 2 to a 12 like this:

$6^{1}2$ 3 4

÷ 5

1

Then share the 12 $100 notes. They get 2 each, so write the 2 underneath. There are 2 left over, which we change to 20 $10 notes making 23 $10 notes. So we change the 3 to 23.

$6^{1}2^{2}3$ 4

÷ 5

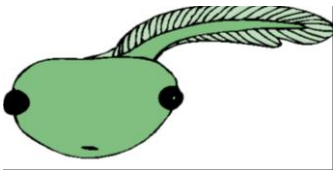
1 2

And so on until we have:

$6^{1}2^{2}3^{3}4$

$\div 5$

1 2 4 6 r 4



Practice

Q9 Calculate the following using any mental or written method.

(a) $64 \div 4$	(b) $96 \div 2$	(c) $16 \div 3$	(d) $224 \div 5$
(e) $413 \div 1$	(f) $4956 \div 2$	(g) $500 \div 8$	(h) $294 \div 7$
(i) $3852 \div 5$	(j) $4826 \div 4$	(k) $310 \div 6$	(l) $990 \div 5$
(m) $34\,912 \div 7$	(n) $16\,000 \div 9$	(o) $9\,461 \div 8$	(p) $4008 \div 6$

3 ways of writing remainders

When dividing, we often get a remainder. For example, $11 \div 4 = 2$ remainder 3 or 2 r 3.

This remainder can be left as a remainder or it can be written as a common or decimal fraction. You should be able to do all of these and be able to decide which is most appropriate.

To write it as a common fraction just make the remainder into the numerator and make the number you are dividing by the denominator. So $11 \div 4 = 2$ r $3 = 2\frac{3}{4}$.

If you want the remainder as a decimal fraction, just put a decimal point and a few zeros after the number you are dividing, and keep the division going until nothing is left to carry, or until you can see that the decimal is recurring, or until you have gone far enough for the accuracy you need.

$1\ 1\ .\ ^{3}0\ ^{2}0\ 0$

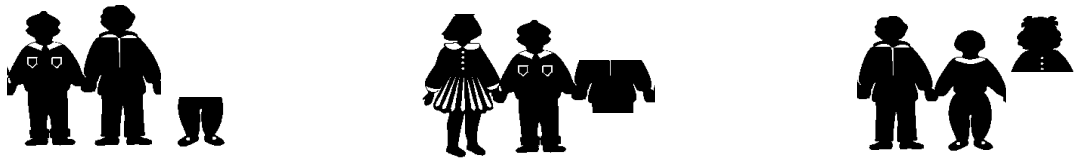
$\div 4$

2 . 7 5

Use a common fraction if the answer would normally be given as a common fraction, eg. 7 pizzas shared between 4 people is $1\frac{3}{4}$ each

Use a decimal fraction if the answer would normally be given as a decimal fraction, eg. 4.2 kg of rice shared between 5 bowls is 0.84 kg per bowl;.

Use a remainder if the remainder wouldn't normally be divided up, e.g. 7 orphans are shared between 3 families. They wouldn't have $2\frac{1}{3}$ each.



So we would say '2 each with one left over' or 2 r 1

Practice

Q10 Work out the following without a calculator. Give the remainders as remainders, as common fractions and as decimal fractions.

(a) $17 \div 2$ (b) $116 \div 5$ (c) $500 \div 3$ (d) $95 \div 7$

(e) $1808 \div 4$ (f) $2 \div 5$ (g) $23 \div 8$ (h) $700 \div 6$

Q11 For each of the following situations, decide whether the remainder should be given as a remainder, as a common fraction or as a decimal fraction:

(a) \$53 is shared between 4 people

(b) 7 pizzas are shared between 4 people

(c) 13 guinea pigs are to be shared between 3 children

Approximate methods for multiplying and dividing

Suppose we are doing 327×45.

We might start the table, but only fill in the squares which will have the biggest numbers:

	300	20	7
40	12000	800	
5	1500		

Adding these up gives 14 300, which is reasonably close to the accurate answer of 14 715.

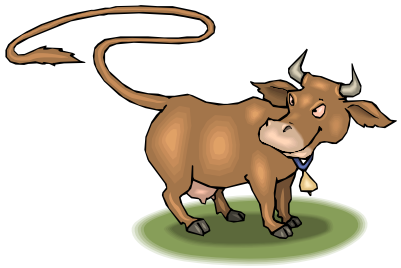
If we needed only a very rough answer, we could just do $300 \times 40 = 12\,000$.

Likewise with division. If we needed to do 5127 ÷ 7, we could just go as far as

$$\begin{array}{l} 5\,^{5}1\;\;2\;\;7 \\ \underline{\qquad\div 7} \\ 0\;\;7 \end{array}$$

Then put 0s in the tens and ones places to give 700.

Rounding Up



Sometimes, when doing approximate calculation, it can be worth rounding up. For example, if you were doing 38 × 29, you could do 30 × 20 to get 600. But 38 is closer to 40 than to 30 and 29 is closer to 30 than to 20, so it would be better to do 40 × 30 to get 1 200. The exact answer is 1 102, which is a lot closer to 1200 than to 600. Be aware of this while doing the following practice questions.

Practice

Q12 Do the following calculations approximately in your head to decide which is the closest of the four answers given

(a) 278 × 31	A 300	B 1000	C 10 000	D 90 000
(b) 612 ÷ 7	A 8	B 32	C 70	D 170
(c) 63 × 492	A 25 000	B 30 000	C 35 000	D 40 000
(d) 2137 ÷ 4	A 200	B 500	C 2 000	D 5 000

Solve

Q51 Find $^1/_9$ as a decimal (divide 1 by 9). Then multiply the decimal by 9. This should give you $^9/_9$, which is 1. Comment on the result.

Q52 If we needed to get an approximate answer to 37×37×37 in our heads, we would use rounding. Would we get a closer answer if we rounded all three numbers up to 40 or if we rounded two up to 40 and one down to 30?

Q53 Use what you learnt in the last question to get an approximate answer to 1.3^{12} in your head.

Q54 10^3 is 1000, i.e.1 followed by 3 zeros; 10^8 is 1 followed by 8 zeros; 10^{34} is 1 followed by 34 zeros. In your head, work out the following, giving the answers as powers.

(a) $10^8 \times 10^3$ (b) $10^{34} \times 10^8$ (c) $10^{34} \div 10^8$

Revise

Revision Set 1

Q61 Calculate the following without a calculator. You may use any mental or written method.

(a) 247 + 88 (b) 6000 – 152

Q62 Calculate the following without a calculator. You may use any mental or written method.

(a) 28.4 + 9.7 (b) 60 – 12.8 (c) 41.3 – 0.27 (d) 132.17 + 54.3

Q63 Do the following calculations approximately in your head to decide which is the closest of the four answers given.

(a) 437.92 + 26.845	A 450	B 500	C 600	D 700
(b) 69 – 0.083	A 1	B 61	C 68	D 69

Q64 Calculate the following without a calculator. You may use any mental or written method.

(a) 307 × 24 (b) 2 687 ÷ 5

Q65 Do the following calculations approximately in your head to decide which is the closest of the four answers given

(a) 384 × 19	A 800	B 4000	C 8000	D 40 000
(b) 15 278 ÷ 7	A 200	B 800	C 2 000	D 8 000

Answers

Q1	(a) 251	(b) 1 248	(c) 6 237	(d) 801
	(e) 1 388	(f) 273	(g) 2 828	(h) 3 300
	(i) 357	(j) 5 878	(k) 557	(l) 169
Q2	(a) 25.1	(b) 934.8	(c) 55.37	
	(d) 273.8	(e) 1.104	(f) 71.89	
	(g) 1 503.3	(h) 1 274.2	(i) 973.72761	
	(j) 1 193.83	(k) 0.0823	(l) 91.32	
Q3	(a) 85	(b) 552	(c) 357	(d) 1615
	(e) 34	(f) 187	(g) 39406	(h) 126
	(i) 192	(j) 5583	(k) 85	(l) 351
Q4	(a) 14.2	(b) 664.6	(c) 6.833	
	(d) 266.2	(e) -0.456	(f) 42.69	
	(g) 1 266.7	(h) 525.8	(i) -881.52761	
	(j) 106.23	(k) 0.0657	(l) 12.08	
Q5	(a) B	(b) A	(c) C	(d) A
	(e) B	(f) B	(g) D	(h) B
	(i) A	(j) A	(k) B	
Q6	(a) 800	(b) 220	(c) 10 000	
	(d) 400	(e) 6 000 000	(f) 2 400	

Q7 (a) 240 000 (b) 27 000 (c) 33000
(d) 40 000 (e) 1 200 000 (f) 800
(g) 5 600 000 (h) 1 000 000 (i) 4 000 000
(j) 3 600 000 (k) 100 000 000 (l) 50 000

Q8 (a) 204 (b) 372 (c) 270 (d) 4 392
(e) 3 688 (f) 3 450 (g) 5 754 (h) 16 400
(i) 225 262 (j) 60 632 (k) 5 166 (l) 5 580
(m) 13 770 (n) 395 324 (o) 786 896 (p) 139 080

Q9 (a) 16 (b) 48 (c) 5.3333 (d) 44.8
(e) 413 (f) 2 478 (g) 62.5 (h) 42
(i) 770.2 (j) 1 206.5 (k) 51.6666 (l) 198
(m) 4 987.43 (n) 1 777.78 (o) 1 182.625 (p) 668

Q10 (a) 8r1 8½ 8.5
(b) 23r1 $23\frac{1}{5}$ 23.2
(c) 166r2 $166\frac{2}{3}$ 166.666…
(d) 13r4 $13\frac{4}{7}$ 13.571…
(e) 452
(f) 0r2 $\frac{2}{5}$ 0.4
(g) 2r7 $2\frac{7}{8}$ 2.875
(h) 116r4 $116\frac{2}{3}$ 166.66…

Q11 (a) decimal fraction (b) common fraction (c) remainder

Q12 (a) C (b) C (c) B (d) B

Q51 $\frac{1}{9}$ = 0.111 recurring; $0.111^{\cdot} \times 9 = 0.999^{\cdot}$ This proves that $0.999^{\cdot} = 1$.
Q52 Two up and one down
Q53 Round 8 factors down and 4 up to get 16
Q54 (a) 10^{11} (b) 10^{42} (c) 10^{26}

Q61 (a) 335 (b) 5 848
Q62 (a) 38.1 (b) 47.2 (c) 41.03 (d) 186.47
Q63 (a) A (b) D
Q64 (a) 7368 (b) 537 r4
Q65 (a) C (b) C