M1 Maths

N1-8 Common Fraction Operations 1

- mental methods to perform simple operations on common fractions
- approximation to perform more difficult operations

Summary Learn Solve Revise Answers

Summary

Adding and subtracting fractions with the same denominator is easy. If fractions have different denominators, sometimes, one of them can be changed to an equivalent fraction so they have the same denominator.

To multiply fractions, think of × as 'of'. To divide fractions, think of one number divided by another as 'how many of the second number make the first number'.

To approximate operations on common fractions, draw the fractions (e.g. as pizzas) and use visualisation and the strategies above to get approximate answers.

Learn

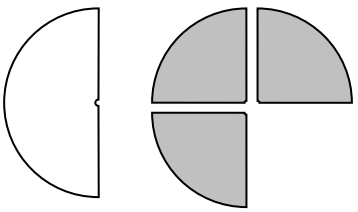
There are formal methods for adding, subtracting, multiplying and dividing common fractions which you will meet later. However, students have a tendency to memorise these methods without them making total sense, then to forget them and not be able to work them out again.

In this module, we won't meet the formal methods, but instead will use informal methods based on a feel for the fractions and common sense. If you master these, then the formal methods will make sense and (a) you'll be less likely to forget them and (b) if you do forget them, you should be able to work them out again.

Performing simple operations on common fractions mentally

Adding and Subtracting

It is easy to add 2 quarters to 3 quarters – obviously it makes 5 quarters: imagine if someone cut some pizzas into quarters and gave you 2 of them, then another 3.



You would end up with 5 quarters. This, of course, is $1\frac{1}{4}$. Adding fractions with the same denominator is really a no-brainer.

Adding fractions with different denominators can be more tricky. You will not be expected to add ¾ to $\frac{5}{6}$ yet, but ½ + ¾ is reasonably easy because you can change the ½ to $\frac{2}{4}$ and then you are adding ¾ and $\frac{2}{4}$ like we did before.

You should be able to add fractions of different denominators if it is easy to re-write one of the fractions so it has the same denominator as the other. You should be able to do $\frac{1}{3} + \frac{5}{12}$ by changing the $\frac{1}{3}$ to $\frac{4}{12}$, then adding to make $\frac{9}{12}$, which, of course, is ¾.

It is the same for subtracting. You would do $\frac{7}{10} - \frac{1}{5}$ by thinking of $\frac{1}{5}$ as $\frac{2}{10}$, then taking the 2 tenths from the 7 tenths leaving 5 tenths. $\frac{5}{10}$ is, of course, ½.

Practice

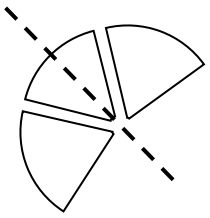
Q1 Calculate the following in your head. Give the answers as common fractions or whole numbers. You may draw pictures to help you. You might like to do them on a calculator too to see if you get the same answer – and to reinforce your skills with fractions on a calculator.

(a) ¼ + ¼	(b) ½ + ½	(c) ½ + ¼	(d) ½ + ¾
(e) $1 - \frac{2}{5}$	(f) $2 - 1\frac{1}{2}$	(g) $1\frac{1}{2} + \frac{3}{4}$	(h) $1 - \frac{1}{2}$
(i) $2 - \frac{3}{4}$	(j) $2\frac{3}{5} + \frac{2}{5}$	(k) $2\frac{1}{5} - \frac{2}{5}$	(l) $4 - 2\frac{3}{4}$

Multiplying and Dividing

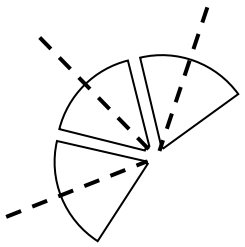
To multiply fractions, you have to know that, with fractions, × means 'of '. So $\frac{1}{2} \times \frac{4}{5}$ means ½ of $\frac{4}{5}$. Imagine a pie cut into fifths and you take four of them. Then you have to share those four with your sister. Half of your 4 fifths is 2 fifths – you get 2 fifths each. So ½ of $\frac{4}{5}$ is $\frac{2}{5}$.

$\frac{1}{2}$ of $\frac{3}{5}$ is a bit harder because you can't divide 3 by 2 evenly: it would make $1\frac{1}{2}$ fifths, which is not a standard way to write a fraction.

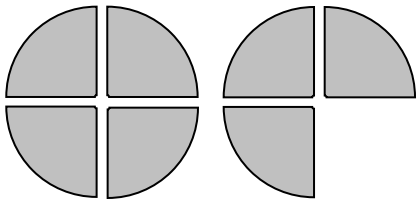


You then need to change$1\frac{1}{2}$ fifths to an equivalent fraction by multiplying top and bottom by 2 to give $\frac{3}{10}$.

Alternatively, imagine you have $^3/_5$ of a pie and you have to share it equally with your sister. What you could do is cut each fifth in halves, so instead of 3 fifths, you then have 6 tenths. Then you can share the 6 tenths, giving you 3 tenths each. So $\frac{1}{2} \times {}^3/_5$ is $^3/_{10}$.



For division, use the 'how many of these make that?' model. $1\frac{3}{4} \div \frac{1}{4}$ means how many $\frac{1}{4}$s make $1\frac{3}{4}$? Obviously, the answer is 7, so $1\frac{3}{4} \div \frac{1}{4} = 7$. You are not expected to do anything much harder than this with division at this stage.



Practice

Q2 Calculate the following in your head. Give the answers as common fractions or whole numbers. You may draw pictures to help you. You might like to do them on a calculator too.

(a) $\frac{1}{2}$ of $\frac{1}{2}$ (b) How many $\frac{1}{4}$s in $\frac{3}{4}$ (c) How many $\frac{1}{2}$s in 3 (d) $\frac{3}{4}$ of 2

(e) $2 \times {}^2/_5$ (f) $^2/_5 \times 4$ (g) $\frac{3}{4} \div \frac{1}{4}$ (h) $2 \div \frac{1}{2}$

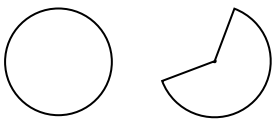
(i) $^2/_5 \times 5$ (j) $1\frac{1}{2} \div \frac{3}{4}$ (k) $4\frac{1}{2} \div 1\frac{1}{2}$ (l) $\frac{3}{4} \times 8$

Approximating operations on common fractions mentally

When approximating operations on common fractions, use a combination of visualising the fractions and common sense. You do not need any particular new knowledge.

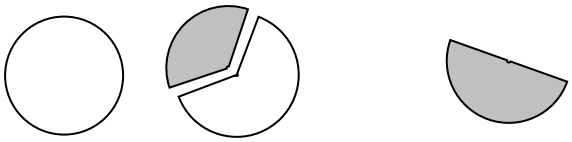
When adding fractions, picture the size of each fraction. It can help to think of them in terms of cups of water or pizzas or something else that is easy to visualise. Then picture putting them together and see roughly how much you end up with.

For example, if you had to add $1^2/_3$ to $^5/_6$, you might use pizzas.

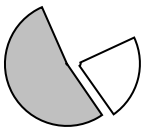
Picture $1^2/_3$ like this  and $^5/_6$ like this 

Then imagine putting these together. Obviously, you would get more than 2, but less than 3, say $2^1/_2$.

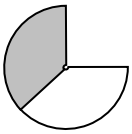
Alternatively, imagine cutting enough off the $5/6$ to make the $1\frac{2}{3}$ up to 2. Then picture what would be left of the $5/6$ – maybe about ½. So you would have the 2 plus about another ½, i.e. about $2\frac{1}{2}$.



Likewise for subtracting fractions. Suppose you were estimating $\frac{3}{4} - \frac{1}{5}$. Picture ¾ of a pizza. Picture $1/5$ of a pizza taken away from it. This will leave you with about a half – maybe a bit more than a half.



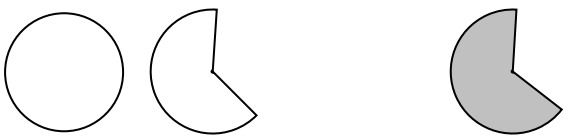
When multiplying fractions, you just have to remember that 'multiplied by' means 'of ', so ½ × ¾ means the same as ½ of ¾. Then picture ¾, then picture half of that. The answer is a bit less than a half – about $3/8$.



When dividing fractions, use the 'how many of these make that?' model for division. For example, to work out $1\frac{1}{4} \div ½$, think "how many halves make $1\frac{1}{4}$. Well, 2 halves make 1 and another half of a half makes the quarter, so the answer is $2\frac{1}{2}$.

An alternative way for all operations is to picture the common fractions and estimate or calculate what they would be as decimal fractions. Then do the operation with the decimal fractions (either exactly or approximately). Then, if you need the answer as a common fraction, either write it as a common fraction or picture it and estimate what it would be as a common fraction.

For example, if you were doing $1\frac{2}{3} \times \frac{5}{7}$, you might picture them like this:



then estimate that they are 1.7 and 0.7, then estimate that 1.7×0.7 is about 1.2. This is $1\frac{2}{10}$.

The best way to get good at all this is by doing lots of practice. There is an exercise below. You can make up your own questions too and check your answers using a calculator.

Practice

Q3 For each of the following, write A, B, C or D depending on which option is closest to the answer. Do them in your head. Don't use a calculator.

(a) $\frac{2}{7} + \frac{5}{7}$	A 0.25	B 0.75	C 1	D 2
(b) $\frac{9}{10} - \frac{7}{8}$	A 0	B 0.79	C 1	D 2
(c) $1\frac{3}{10} - \frac{3}{4}$	A 0.1	B 0.5	C 1	D 2
(d) $1\frac{1}{12} + \frac{8}{9}$	A 0.2	B 0.6	C 1	D 2
(e) ½ of $\frac{17}{5}$	A 0.6	B 1.6	C 2.6	D 3.6
(f) $\frac{3}{4} \times 3\frac{7}{8}$	A 1	B 2	C 3	D 4
(g) $4\frac{3}{4} \div 5$	A 1	B 2	C 3	D 4
(h) $2\frac{1}{15} \div \frac{1}{2}$	A 1	B 2	C 3	D 4
(i) $3\frac{2}{5} \div 1\frac{5}{9}$	A 1	B 2	C 3	D 4
(j) $\frac{13}{7}$ of $1\frac{1}{11}$	A 1	B 2	C 3	D 4
(k) $\frac{7}{8} \div \frac{2}{15}$	A 1	B 2	C 3	D 4
(l) $\frac{3}{10} - \frac{3}{8}$	A 0	B 0.5	C 1	D 20
(m) $1\frac{1}{2} \times 1\frac{1}{2}$	A 1	B 2	C 3	D 4
(n) $3\frac{6}{7} \div 2$	A 1	B 2	C 3	D 4
(o) $2 \div \frac{16}{33}$	A 1	B 2	C 3	D 4

Solve

Q51 Roughly how much pizza would you eat if you ate half a pizza, then a quarter of a pizza, then and eighth of a pizza, then a sixteenth and so on for a long time?

Q52 What is $\frac{3}{8}$ of 0.2

Q53 What is 15% of ¾?

Revise

Revision Set 1

Q61 Calculate the following in your head. Give the answers as common fractions or whole numbers. You may draw pictures to help you.

(a) ½ + ¾ (b) ¼ of ½ (c) ¾ × 12

(d) How many $\frac{1}{6}$s in 3 (e) $4 - \frac{12}{5}$ (f) $6 \div 1\frac{1}{2}$

Q62 For each of the following, write A, B, C or D depending on which option is closest to the answer. Do them in your head. Do not use a calculator.

(a) $^2/_9 + {}^5/_9$	A 0.25	B 0.75	C 1	D 2
(b) $^4/_7 \times 2{}^7/_8$	A 1	B 1.5	C 2	D 2.5
(c) $4{}^3/_4 \div {}^2/_5$	A 2	B 4	C 7	D 10
(d) $^7/_{10} - {}^3/_8$	A 0	B 0.5	C 1	D 4
(e) $4 \div {}^5/_{16}$	A 2	B 8	C 15	D 25

Answers

Q1	(a) ½	(b) 1	(c) ¾	(d) 1¼
	(e) $^3/_5$	(f) ½	(g) 2¼	(h) ½
	(i) 1¼	(j) 3	(k) $1{}^4/_5$	(l) 1¼
Q2	(a) ¼	(b) 3	(c) 6	(d) 1½
	(e) $^4/_5$	(f) $^8/_5$	(g) 3	(h) 4
	(i) 2	(j) 2	(k) 3	(l) 6
Q3	(a) C	(b) A	(c) B	(d) D
	(e) B	(f) C	(g) A	(h) D
	(i) B	(j) B	(k) D	(l) A
	(m) B	(n) B	(o) D	

Q51 1 Q52 0.075 Q53 $^9/_{80}$

Q61	(a) 1¼	(b) $^1/_8$	(c) 9	(d) 18	(e) $1{}^3/_5$	(f) 4
Q62	(a) B	(b) B	(c) D	(d) B	(e) C	