M1 Maths

N1-7 Order of Operations

- order of operations conventions

Lead-In Summary Learn Solve Revise Answers

Summary

There is a convention in mathematics that, when performing a calculation with more than one operation, like $5 + 12 \div 3 \times 4$, we do all the $\times$ and $\div$ operations first as we come to them, then we do all the + and – operations. Powers are done before $\times$ and $\div$. And anything in a bracket is worked out first.

There is an implied bracket around anything in the index of a power and also around the numerator and around the denominator of a fraction.

Lead-In

If you put $4 + 3 \times 2$ into a cheap four-function calculator, it will give you 14.

$$4 + 3 \times 2 = 14$$

But a more expensive calculator like a scientific or graphics calculator will give you 10.

$$4 + 3 \times 2 = 10$$

If you have one of each, try it.

Learn

The cheap calculator in the Lead-In does the operations as it comes to them: $4 + 3 = 7$; $7 \times 2 = 14$. The expensive one does the multiplication first, then the addition: $3 \times 2 = 6$; $4 + 6 = 10$.

Mathematicians have an agreement (or convention) about the order in which we perform operations. The convention says that:

If we have a string of operations, we do all the multiplications and divisions first – as we come to them. Then we do all the additions and subtractions – as we come to them.

For example, suppose we have to work out

$$4 + 12 \div 2 \times 3 - 6 \div 3 - 1 + 5$$

First, we go along underlining everything except the + and – signs.

$$\underline{4} + \underline{12 \div 2 \times 3} - \underline{6 \div 3} - \underline{1} + \underline{5}$$

Then we re-write it with each underlined part worked out to a single number. (Some already are a single number, so they don’t change.)

$$\underline{4} + \underline{12 \div 2 \times 3} - \underline{6 \div 3} - \underline{1} + \underline{5}$$
$$4 + 18 - 2 - 1 + 5$$

Then we do the addition and subtraction (as we come to them) to end up with this:

$$\underline{4} + \underline{12 \div 2 \times 3} - \underline{6 \div 3} - \underline{1} + \underline{5}$$
$$= 4 + 18 - 2 - 1 + 5$$
$$= 24$$

Why do we do multiplication and division first?

Consider this problem. Marmaduke goes to his produce store and buys 8 kg of fertiliser at $3/kg, 5 kg of chicken feed at $6/kg and ¼ kg of grass seed at $28/kg. He also returns 10 kg of gypsum (worth $2/kg) for a refund. How much does he have to pay?

We can write out the calculation like this:

$8 \times 3 + 5 \times 6 + 28 \div 4 - 10 \times 2$

If we follow the convention on order of operations, we get this:

$$\underline{8 \times 3} + \underline{5 \times 6} + \underline{28 \div 4} - \underline{10 \times 2}$$
$$= 24 + 30 + 7 - 20$$
$$= 41$$

which is the right answer.

If we just did the operations as we come to them, we would have got 81. Situations where we need to multiply and divide before we add and subtract are actually a lot more common that situations where we just want to do the operations as we come to them. That is why we have this convention.

WARNING!!

There are various slogans like PEMDAS, BEDMAS, BOMDAS, BIDMSA, DIMLAS, DINGBAT etc. designed to help you remember that × and ÷ are done before + and – . DON'T USE THEM – THEY ARE DANGEROUS! They can take over your mind and make you think that addition gets done before subtraction and things like that which are NOT TRUE – repeat – they are NOT TRUE. Addition and subtraction always get done in the order you come to them – whichever comes first. Likewise with multiplication and division.

Just remember that × and ÷ get done before + and –. That's all there is to it.

Practice

Q1 Calculate the following without a calculator:

(a) $6 \times 3 + 4$
(b) $4 + 6 \times 3$
(c) $8 - 3 + 4$
(d) $6 \times 3 \div 2$
(e) $12 - 4 \times 3$
(f) $8 \div 2 \times 5$
(g) $15 - 2 \times 6$
(h) $15 \div 3 + 12$
(i) $8 + 2 \times 3 - 6$
(j) $12 \div 3 - 2 \times 2 + 7$
(k) $4 + 10 \div 2 \times 3 - 1$
(l) $18 - 2 \times 3 \div 6 \times 2 + 5 \times 3$
(m) $5 \times 12 - 12 \div 3 \times 4 + 5$
(n) $4 - 4 + 4 \times 4 \div 4 \times 4 - 4$
(o) $24 \div 2 - 6 \div 2 \times 3 + 4$
(p) $4 \times 2 \div 8 \times 15 - 2 \times 7$
(q) $5 \times 2 - 24 \div 4 \div 2 + 1 \times 3$
(r) $6 + 9 \div 3 - 7 \times 0$
(s) $28 \div 4 + 3 \times 5 - 1$
(t) $8 - 2 + 4 \div 2 \times 5 - 1$

Brackets

Suppose you wanted to add 4 to 5, then multiply the result by 2.

$5 + 4 = 9$ then $9 \times 2 = 18$.

But if you wrote this as $5 + 4 \times 2$, it wouldn't actually mean what you intended, because, using the convention, $5 + 4 \times 2 = 13$.

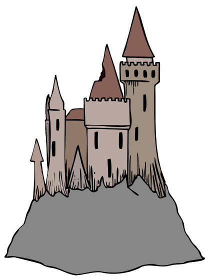
If you want an expression to be calculated using an order of operations different from the convention, this can be done. You just have to put **brackets** (sometimes called *parentheses*, plural of *parenthesis*) around the bit you want done first. So, instead of $5 + 4 \times 2$, we would need to write $(5 + 4) \times 2$.

When working something out, we always do anything inside a bracket first. So, in this case, we would do the $5 + 4$ to get 9, then multiply the 9 by 2 to get 18, which is what we wanted.

$$\begin{aligned} &(\underline{5 + 4}) \times \underline{2} \\ &\underline{9 \quad \times 2} \\ &18 \end{aligned}$$

When doing a calculation with brackets, we go along replacing each set of brackets with a single number. Then we carry on as before.

In the example below, there are brackets and all four operations in a single expression.



$$\begin{aligned} & \underline{24} \div (3 - 1 + 6) \times 2 + 6 \div 3 - (5 - 2) \\ = \quad & \underline{24 \div \quad 8 \quad \times 2} + \underline{6 \div 3} - \underline{3} \\ = \quad & 6 + 2 - 3 \\ = \quad & 5 \end{aligned}$$

If a bracket has × or ÷ and + or − inside it, we work it out doing the convention first, just as if it were an expression by itself. Here is an example:

$$\begin{aligned} & 2 + 3 \times (\underline{10} - \underline{12 \div 3}) \\ = \quad & 2 + 3 \times (10 - \quad 4 \quad) \\ = \quad & \underline{2} + \underline{3 \times \quad 6} \\ = \quad & \underline{2 + 18} \\ = \quad & 20 \end{aligned}$$

Practice

Q2 Calculate the following without a calculator:

(a) 4 × (5 – 2)

(b) 5 – (3 + 2)

(c) 16 ÷ (3 + 1) – 2

(d) 20 – 2 × (6 – 1)

(e) (6 – 1) × 2 – 1

(f) 4 × (2 + 3 + 4) ÷ 3

(g) (5 + 7) ÷ (3 + 3)

(h) 2 × (3 – 2 + 1) + 5

(i) (6 + 2 × 3) + 4 ÷ 2

(j) 3 × (12 – 4 × 2 + 1)

(k) 5 × (12 –9) ÷ 3 × (4 + 5)

(l) 4 – (4 + 4) × 4 ÷ (4 × 4) + 4

(m) 3 – 4 × (7 – 12 ÷ 6 × 3 + 1) ÷ 8 + 4

(n) $2 \times (12 - 15 \div 3 + 2) - 4 \div 2$
(o) $60 \div (2 + 4 \times 5 - 5 \times 2) \times 6 - 4 \times 5$
(p) $12 - (40 \div 8 + 4 \times 2 - 5) \div 4 - 1$
(q) $(13 - 5 \times 2 + 9) \div 3 \times (6 - 4)$
(r) $6 \times 2 + (12 - 4 \times 2) \times 5 - (3 + 9 \div 3 - 2) + 4$
(s) $4 + (3 + 2 \times (8 - 1) - 2) - 5$
(t) $12 \div (5 + 21 \div (13 - 2 \times 3) - 6) \times (5 - 2 \times 2 + 4) + 8$

Powers

Suppose we have 5×2^3. This expression has multiplication and a power. If we do the multiplication first, we get $15^3 = 3375$; if we do the power first, we get $5 \times 8 = 40$.

By the convention, powers are done before multiplication and division (though after brackets). So 40 is the correct answer. This should be fairly intuitive because 2^3 looks more like a single number than 5×2. If we wanted 3375, we would have to write $(5 \times 2)^3$.

So the order of operations convention can be summed up as:

brackets, then powers, then × ÷, then + −

Implied Brackets

In $2^2 + 3$, clearly we should do the power first to get $4 + 3$, which is 7.

But what about 2^{2+3}? Here we have a power and an addition. The convention tells us we should do the power first. But that would turn the expression into $2^2 + 3$. In actual fact, the index of a power is considered to have a bracket around it: even if it isn't shown, it is implied.

So $2^{2+3} = 2^{(2+3)} = 2^5 = 32$.

Also consider $\frac{13-5}{4}$. Here we have subtraction and division. Again the convention tells us that division should be done first, but looking at the expression, it might seem more natural to do the subtraction first. And we do. There is an implied bracket around the numerator of any fraction and around the denominator – even if it isn't shown.

So $\frac{13-5}{4} = \frac{(13-5)}{4} = \frac{8}{4} = 2$.

When using less sophisticated calculators, these implied brackets must actually be inserted. For instance, to do $\frac{13-5}{4}$, we have to enter (1 3 – 5) ÷ 4 = and to do 2^{2+3}, we have to enter 2 ^ (2 + 3) = .

Some more sophisticated calculators actually allow you to enter the numbers into a fraction or power layout on the screen and then you don't need to manually enter the brackets.

Practice

Q3 Find the following without a calculator:

(a) 2×3^2
(b) $(2 \times 3)^2$
(c) $3^2 + 1$
(d) $3^{(2+1)}$
(e) 3^{2+1}
(f) $\frac{5+1}{2}$
(g) $\frac{15}{2+3}$
(h) $\frac{15 \div 3}{2+3}$
(i) $\frac{2^5 \div 4}{7+3^2}$
(j) $24 - 3 \times \left(\frac{2^4-1}{11-2\times3} - 1\right)^2 \div 4$

Redo questions a-j with a calculator to make sure you get the same answers

Solve

Q51 Work out $1 + 1 - 1 \times 1 \div 1 + 1 - 1 \times 1 \div 1 + 1 - 1 \times 1 \div 1 + 1 - 1 \times 1 \div 1 + \ldots$ going on until there are a thousand 1s.

Q52 Work out $2 + 2 - 2 \times 2 \div 2 + 2 - 2 \times 2 \div 2 + 2 - 2 \times 2 \div 2 + 2 - 2 \times 2 \div 2 + \ldots$ going on until there are a thousand 2s.

Q53 Work out $3 - \frac{3}{3} \div 3 \times 3 + 3^3 \div (3 \times 3)$

Q54 $(7 + 7) \times 7 \div 7 = 14$; $7^{7-7} + 7 = 8$; $77 - 7 \times 7 = 28$; $\sqrt{7 \times 7} + 7 + 7 = 21$.

Try to make every number from 0 to 50 using fours 7s and any operation symbols.

Revise

Revision Set 1

Q61 Calculate the following without a calculator:

(a) $5 \times 3 + 4$
(b) $17 - 6 \times 2$
(c) $12 \div 4 \times 2$
(d) $10 - 2 + 5 - 8$
(e) $5 \times (3 + 5)$
(f) $(6 \times 2) + 7$
(g) $16 - (3 + 1) \times 2$
(h) $4 - 4 + 4 \times 4 \div 4 \times 4 - 4$
(i) $3 - 4 \times (7 - 12 \div 6 \times 3 + 1) \div 8 + 4$
(j) 2^{2+3}
(k) $\frac{20}{10-2\times3}$
(l) $\frac{2^3\times6}{7^2-5^2}$

Answers

Q1	(a) 22	(b) 22	(c) 9	(d) 9		
	(e) 0	(f) 20	(g) 3	(h) 17		
	(i) 8	(j) 7	(k) 18	(l) 31		
	(m) 49	(n) 12	(o) 7	(p) 1		
	(q) 10	(r) 12	(s) 21	(t) 15		
Q2	(a) 12	(b) 0	(c) 2	(d) 10		
	(e) 9	(f) 12	(g) 2	(h) 9		
	(i) 14	(j) 15	(k) 45	(l) 6		
	(m) 6	(n) 16	(o) 10	(p) 9		
	(q) 8	(r) 32	(s) 14	(t) 38		
Q3	(a) 18	(b) 36	(c) 10	(d) 27	(e) 27	(f) 3
	(g) 3	(h) 1	(i) ½	(j) 21		
Q51	1	S2 0	S3 5			
Q61	(a) 19	(b) 5	(c) 9	(d) 5	(e) 40	(f) 19
	(g) 8	(h) 12	(i) 6	(j) 32	(k) 5	(l) 2