M1 Maths

N1-6 Powers

- whole number powers and square roots
- expressing composite numbers as products of prime factors in power form

Summary Lead-In Learn Solve Revise Answers

Summary

5^3 means three 5s multiplied together, i.e. 5×5×5. It is pronounced '5 to the power of 3'

5^3 is called a power; the 5 is called the base; the 3 is called the index or exponent.

'to the power of 2' is often called 'squared'; 'to the power of 3' is often called 'cubed'.

The square root of a number like 49 is the number which when squared will make 49; $7^2 = 49$, so the square root of 49 is 7. It is written: $\sqrt{49} = 7$.

A composite number can be written as a product of prime factors in power form. For example, $720 = 2^4 \times 3^2 \times 5$.

Lead-In

5+5+5+5+5+5+5+5 can be written more simply as 5×8, pronounced 5 times 8.

5×5×5×5×5×5×5×5 can be written more simply as 5^8, pronounced 5 to the power of 8.

Learn

Powers

5^2, pronounced *5 to the power of 2*, means 2 fives multiplied together, i.e. 5×5, i.e. 25

5^3, pronounced *5 to the power of 3*, means 3 fives multiplied together, i.e. 5×5×5, i.e. 125

5^4, pronounced *5 to the power of 4*, means 4 fives multiplied together, i.e. 5×5×5×5, i.e. 625

and so on.

2^8, pronounced *2 to the power of 8*, means 8 twos multiplied together, i.e. 2×2×2×2×2×2×2×2, i.e. 256

10^{14} means 14 10s multiplied together,

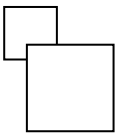
i.e. 10×10×10×10×10×10×10×10×10×10×10×10×10×10, which is 100 000 000 000 000.

Note that 5^4 does **not** mean 5×4. Try to remember that! 5×4 = 5+5+5+5, 5^4 = 5×5×5×5.

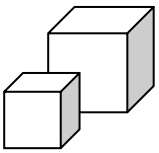
$$5 \times 4 = 5 + 5 + 5 + 5 = 20$$

$$5^4 = 5 \times 5 \times 5 \times 5 = 625$$

'to the power of 2' is often pronounced '**squared**', so 5^2 is often pronounced '5 squared'



'to the power of 3' is often pronounced '**cubed**', so 8^3 is often pronounced '8 cubed'



The other powers do not have special names: 2^4 is always pronounced '2 to the power of 4'

2^4 is called a **power**. The 2 is the **base** and the 4 is the **index** or **exponent**.

In 10^7, the base is 10 and the index is 7; 10^7 is the power.

Note that 5^1 is 5 and 5^0 is 1 (not zero). You can remember these if you realise that each time the index increases by 1, the power is multiplied by the base, so each time the index reduces by 1, the power is divided by the base.

$5^4 = 625$

$5^3 = 625 \div 5 = 125$

$5^2 = 125 \div 5 = 25$

$5^1 = 25 \div 5 = 5$

$5^0 = 5 \div 5 = 1$

Practice

Q1 (a) In what two ways can 4^2 be pronounced?

(b) What does it mean?

(c) What is its value?

Q2 (a) How is 5^3 pronounced?

(b) What does it mean?

(c) What is its value?

Q3 (a) How is 2^4 pronounced?

(b) What does it mean?

(c) What is its value?

Q4 Calculate: (a) 2^5 (b) 10^6 (c) 1^4 (d) 0^9

(e) 3^5 (f) 4^1 (g) 3^0 (h) 92^0

Powers on a Calculator

Suppose you had to work out 2^{14} on your calculator. You could press 2, then × 2 thirteen times. But scientific and graphics calculators have a '*to the power of*' button. It will probably be marked ^ or x^y or y^x. So you can just press 2 ^ 14 = .

Practice

Q5 Use your calculator to find:

(a) 2^{11} (b) 3^6 (c) 1.1^{24} (d) 0.95^{40}

(e) 0.763^8 (f) 2^7 (g) 1^{75} (h) 0^{80}

Square Roots

The square of 7 is 49. We say that 7 is the **square root** of 49.

The square root of a number is the number which will make it when squared.

The square root of 25 is 5; the square root of 36 is 6.

Estimating square roots

The square root of 28 is somewhere between 5 and 6. It is approximately 5.3.

28 is not a perfect square, i.e. it is not the square of a whole number. So the square root of 28 is not a whole number. We have to estimate it.

You need to be able to estimate the square roots of numbers and get the answer within 0.3. You do it this way. Firstly decide which two whole numbers the square root will be between. 28 is between 25 and 36 (the squares of 5 and 6), so the square root of 28 is between 5 and 6. Then decide which number it is closest to. 28 is closer to 25 than 36, so the square root will be closer to 5 than 6. It is about 5.3.

The symbol for the square root of 28 is $\sqrt{28}$. Your calculator has a button that calculates square roots. It has a $\sqrt{\ }$ sign. It should tell you that $\sqrt{28}$ = 5.291502622…

Practice

Q6 Calculate: (a) $\sqrt{36}$ (b) $\sqrt{100}$ (c) $\sqrt{16}$ (d) $\sqrt{1}$
(e) $\sqrt{0}$ (f) $\sqrt{144}$

Q7 Estimate the square roots of the following. Then check with your calculator. Your answer must be within 0.3.
(a) 26 (b) 47 (c) 57 (d) 12 (e) 38.9
(f) 5 (g) 1.9

Products of Prime Factors in Power Form

In Module N1-1, we saw how to write a composite number as a product of prime factors, e.g. $360 = 2\times2\times2\times3\times3\times5$.

Where factors are repeated, it can by tidier (and quicker) to write the product using powers. In this way, $360 = 2^3\times3^2\times5$.

Practice

Q8 Write the following numbers as products of prime factors in power form.
(a) 56 (b) 270 (c) 308 (d) 1170 (e) 133 875

Solve

Q51 Complete this table by finding and continuing the patterns of subtracting and dividing:

$10^5 = 100\,000$
$10^4 = 10\,000$
$10^3 = 1000$
$10^2 = \ldots$
$10^1 = \ldots$
$10^0 = \ldots$
$10^{-1} = \ldots$
$10^{-2} = \ldots$
$10^{-3} = \ldots$

Q52 What would be the divisibility rules for 12 and 15?

Q53 Write as a product of primes in power form: $2^3 \times 5^5 \times 7^4 \times 19 \times 30$ (Note that 30 is not prime.)

Q54 What is the highest common factor of $2^3 \times 5^2 \times 7^2 \times 13$ and $2 \times 5^4 \times 11$?
[Note that highest common factor means the largest number which is a factor of both numbers.]

Revise

Revision Set 1

Q61 (a) How is 2^5 pronounced? (b) What does it mean? (c) What is its value?

Q62 (a) Write $7\times7\times7\times7\times7$ as a power (b) Write '4 cubed' as a power

Q63 Calculate: (a) 10^6 (b) 1^{12} (c) 6^1 (d) 5^0 (e) 0^3

Q64 Calculate: (a) $\sqrt{49}$ (b) $\sqrt{1}$

Q65 Estimate the square roots of the following. Then check with a calculator. Your answer must be within 0.3. (a) 7 (b) 52.6

Q66 Write 8400 as a product of prime factors in power form.

Revision Set 2

Q71 (a) Write 3 to the power of 4 as a power (b) What does it mean?
(c) What is its value?

Q72 (a) Write '9 squared' as a power (b) Write $5\times5\times5$ as a power

Q73 Calculate: (a) 10^4 (b) 2^6 (c) 52^0 (d) 5^1 (e) 0^{237}

Q74 Calculate: (a) $\sqrt{400}$ (b) $\sqrt{16}$

Q75 Estimate the square roots of the following. Then check with a calculator. Your answer must be within 0.3. (a) 11 (b) 97

Q76 Write 7425 as a product of powers of primes.

Revision Set 3

Q81 (a) How is 5^3 pronounced? (b) What does it mean? (c) What is its value?

Q82 (a) Write '11 squared' as a power (b) Write $6\times6\times6\times6$ as a power

Q83 Calculate: (f) 10^1 (b) 2^8 (c) 6^3 (d) 1^0 (e) 1000^2

Q84 Calculate: (a) $\sqrt{9}$ (b) $\sqrt{64}$

Q85 Estimate the square roots of the following. Then check with a calculator. Your answer must be within 0.3. (a) 15 (b) 149

Q86 Write 12 250 as a product of primes in power form.

Answers

Q1 (a) 4 to the power of 2 or 4 squared
(b) two 4s multiplied together
(c) 16

Q2 (a) 5 to the power of 3 or 5 cubed
(b) three 5s multiplied together
(c) 125

Q3 (a) 2 to the power of 4
(b) four 2s multiplied together
(c) 16

Q4 (a) 32 (b) 1 000 000 (c) 1 (d) 0 (e) 243 (f) 4
(g) 1 (h) 1

Q5 (a) 2048 (b) 729 (c) 9.85 (d) 0.1285
(e) 0.1149 (f) 128 (g) 1 (h) 0

Q6 (a) 6 (b) 10 (c) 4 (d) 1 (e) 0 (f) 12

Q7 (a) 5.10 (b) 6.86 (c) 7.55 (d) 3.46 (e) 6.24 (f) 2.24
(g) 1.38

Q8 (a) $2^3 \times 7$ (b) $2 \times 3^3 \times 5$ (c) $2^2 \times 7 \times 11$ (d) $2 \times 3^2 \times 5 \times 13$ (e) $3^2 \times 5^3 \times 7 \times 17$

Q51 (a) $10^0 = 1$, $10^{-1} = 0.1$ $10^{-2} = 0.01$, $10^{-3} = 0.001$

Q52 12: Divides by 3 and by 4 15: Divides by 3 and by 5

Q53 $2^4 \times 3 \times 5^6 \times 7^4 \times 19$

Q54 50 (2×5^2)

Q61 (a) 2 to the power of 5 (b) five 2s multiplied together (c) 32

Q62 (a) 7^5 (b) 4^3

Q63 (a) 1 000 000 (b) 1 (c) 6 (d) 1 (e) 0

Q64 (a) 7 (b) 1

Q65 (a) 2.65 (b) 7.25

Q66 $2^4 \times 3 \times 5^2 \times 7$

Q71 (a) 3^4 (b) four 3s multiplied together (c) 81

Q72 (a) 9^2 (b) 5^3

Q73 (a) 10 000 (b) 64 (c) 1 (d) 5 (e) 0

Q74 (a) 20 (b) 4

Q75 (a) 3.32 (b) 9.85

Q76 $3^3 \times 5^2 \times 11$

Q81 (a) 5 to the power of 3 or 5 cubed (b) three 5s multiplied together (c) 125

Q82 (a) 11^2 (b) 6^4

Q83 (a) 10 (b) 256 (c) 216 (d) 1 (e) 1 000 000

Q84 (a) 3 (b) 8

Q85 (a) 3.87 (b) 12.2

Q86 $2 \times 5^3 \times 7^2$