M1 Maths

N1-3 Fraction Conversions

- converting between equivalent common fractions, between mixed numbers and improper fractions and between common fractions, decimal fractions and percentages

Summary Learn Solve Revise Answers

Summary

To convert between equivalent common fractions, multiply or divide the top and bottom by the same thing.

To convert between mixed numbers and improper fractions, draw the fraction divided into parts according to the denominator, then read it the other way. For fractions that are too large to draw, just imagine the drawing.

To convert between common fractions, percentages and decimal fractions, use the methods in the table below.

Conversion	Method	Example
CF → DF	top ÷ bottom	$^{3}/_{5}$ = 3 ÷5 = 0.6
DF → CF	altogether way	2.34 = $^{234}/_{100}$ = $^{117}/_{50}$
% → CF	/100	7.5% = $^{7.5}/_{100}$ = $^{75}/_{1000}$ = $^{3}/_{40}$
CF → %	fraction of 100%	$^{2}/_{5}$ = $^{2}/_{5}$ of 100% = 2 ×20% = 40%
% → DF	/100	125% = $^{125}/_{100}$ = 125 ÷100 = 1.25
DF → %	× 100	0.036 = 0.036 ×100% = 3.6%

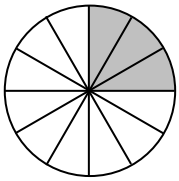
Learn

Converting between equivalent common fractions

Equivalent common fractions are the same amount written in different ways, e.g. $^{1}/_{4}$, $^{3}/_{12}$, $^{6}/_{24}$.

To **convert** between equivalent common fractions, multiply or divide the numerator and denominator by the same number.

If you take ¼ and multiply the numerator and denominator by 3, you get $^{3}/_{12}$.



You should be able to explain, using a picture like this one, why this works. Basically, you are cutting each piece into 3 smaller pieces, so you get 3 times as many pieces each one-third the size.

In the same way, if we take $^{16}/_{20}$ and divide top and bottom by 4, we get $^{4}/_{5}$, which is equivalent to $^{16}/_{20}$.

Simplest form

We sometimes try to write common fractions in their **simplest form**, i.e. with the smallest possible whole numbers for the numerator and denominator. We change fractions to simplest form by dividing top and bottom by the same number until nothing will divide into them both any more.

$^{30}/_{48} = {}^{15}/_{24} = {}^{5}/_{8}$

$^{5}/_{8}$ is the simplest form because nothing goes into 5 and into 8.

Practice

Q1 Write four equivalent fractions for each of the following, two using larger numbers, two using smaller numbers.

(a) $^{8}/_{12}$ (b) $^{4}/_{20}$ (c) $^{16}/_{80}$ (d) $^{20}/_{12}$ (e) $^{32}/_{32}$ (f) $^{96}/_{36}$

Q2 Write the following in simplest form (i.e. as equivalent fractions with the smallest possible numbers).

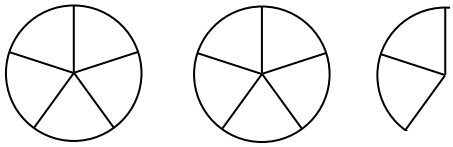
(a) $^{5}/_{10}$ (b) $^{14}/_{21}$ (c) $^{64}/_{24}$ (d) $^{14}/_{35}$ (e) $^{32}/_{15}$ (f) $^{120}/_{200}$

Q3 Explain why multiplying or dividing the top and bottom of a fraction by the same number makes an equivalent fraction.

Converting between mixed numbers and improper fractions

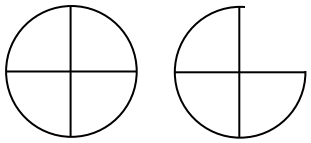
Fractions bigger than 1 can be written as mixed numbers or as improper fractions. A mixed number is a whole number with a fraction, e.g. 1½. An improper fraction is a common fraction with the numerator bigger than the denominator. As an improper fraction, 1½ would be $^{3}/_{2}$.

To convert a mixed number like $2^2/_5$ to an improper fraction, draw it as pizzas like this:



then count the fifths (or whatever). In this case there are 12 fifths ($^{12}/_5$). So $2^2/_5 = {}^{12}/_5$.

To convert an improper fraction like $^7/_4$ to a mixed number, keep drawing pizzas (circles) out of quarters of pizzas, until you have used 7 quarters, then see how many whole pizzas and left over quarters you have.



Answer: $1^3/_4$.

For bigger numbers, it is helpful to be able to imagine the drawings without actually drawing them.

Practice

Q4 Convert the following mixed numbers to improper fractions.

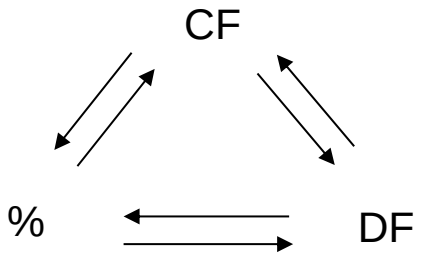
(a) $2^1/_2$ (b) $1^3/_8$ (c) $3^3/_4$ (d) $7^1/_3$ (e) $20^4/_5$ (f) $63^1/_2$

Q5 Convert the following improper fractions to mixed numbers.

(a) $^6/_5$ (b) $^{13}/_4$ (c) $^{20}/_3$ (d) $^{19}/_7$ (e) $^{53}/_5$ (f) $^{187}/_6$

Converting between common fractions, decimal fractions and percentages

There are six conversion as shown in this diagram:



Common fractions to decimal fractions

$^4/_5$ means 4÷5. Use written division or a calculator to divide 4 by 5 to get 0.8

Decimal fractions to common fractions

If we have 0.374, we just think about this in the altogether way (see Module N1-2). The fraction ends in the thousandths place, so the denominator is 1000. Ignoring the decimal point, the fraction is 374, so the numerator is 374. The fraction is $\frac{374}{1000}$. This is a common fraction. We can then simplify it to $\frac{187}{500}$ if we wish.

Percentages to common fractions

Percent just means hundredths, so 35% means $^{35}/_{100}$. This can be simplified to $^7/_{20}$.

17.2% means $^{17.2}/_{100}$, which is $^{172}/_{1000}$ (multiplying top and bottom by 10 to get whole numbers). This can be simplified to $^{43}/_{250}$.

$6^1/_4$% means 6.25%, which is $^{6.25}/_{100}$, which is $^{625}/_{10\,000}$. This can be simplified to $^1/_{16}$.

Common fractions to percentages

$^3/_5$ means $^3/_5$ of a whole or $^3/_5$ of 100%.

We get $^1/_5$ of 100% by dividing 100% by 5 giving us 20%.

We then we get $^3/_5$ by multiplying by 3 to get 60%.

So $^3/_5$ as a percentage is 60%.

Percentages to decimal fractions

42.5% means $^{42.5}/_{100}$ or 42.5 ÷ 100.

By moving the decimal point 2 places to the left, or by using a calculator, this is 0.425.

Decimal fractions to percentages

To change a percent to a decimal fraction, we divide by 100.

So, to go back to a percent, we just multiply by 100..

0.15 is 0.15 × 100%, which is 15%

3.2 = 3.2 × 100% = 320%

0.047 = 0.047 × 100% = 4.7%

Summary

The table below summarises the conversions between common fractions, decimal fractions and percentages. A good way to make sure you know all the conversions is to write out this table from memory.

Conversion	Method	Example
CF → DF	top ÷ bottom	$^3/_5 = 3 \div 5 = 0.6$
DF → CF	altogether way	$2.34 = {}^{234}/_{100} = {}^{117}/_{50}$
% → CF	/100	$7.5\% = {}^{7.5}/_{100} = {}^{75}/_{1000} = {}^{3}/_{40}$
CF → %	fraction of 100%	$^2/_5 = {}^2/_5$ of 100% = 2 ×20% = 40%
% → DF	/100	$125\% = {}^{125}/_{100} = 125 \div 100 = 1.25$
DF → %	× 100	0.036 = 0.036 ×100% = 3.6%

Going the long way

If you forget a conversion, it is sometime possible to go the other way. For instance, if you forget how to change a decimal fraction to a common fraction, you can change it to a percent, then change the percent to a common fraction.

Practice

Q6 Copy and complete the following table.

Common fraction	Decimal fraction	Percent
		27%
	0.56	
$^3/_5$		
		4.2%
	0.295	
$^4/_7$		
		120%
	3.4	
$^8/_5$		
		11.98%
	0.0045	
$2\,^3/_4$		

Conversions to remember

The conversions below are worth remembering.

Common fraction	Decimal fraction	Percent
$^1/_{10}$	0.1	10%
$^1/_5$	0.2	20%
$^1/_4$	0.25	25%
$^1/_3$	$0.3\dot{}$	$33.3\dot{}\%$
$^1/_2$	0.5	50%
1	1	100%

Solve

Q51 If you ate $^3/_8$ of a pizza, what fraction would you have left?

Q52 If you ate ¼ of a cake, and your mother ate $^3/_8$, what fraction would be left?

Q53 If Jo had $^2/_5$ of a litre of Coke and Billy had 0.35 L, how much would they have between them? Answer as a common fraction and as a decimal fraction.

Q54 What is $^2/_7 \times 5$ as a mixed number?

Q55 What is (a) half of $^6/_{10}$? (b) half of $^7/_{10}$?

Q56 What is (a) 25% of $^4/_5$? (b) ¾ of $^{12}/_{25}$?

Q57 If everybody eats $^5/_8$ of a pizza, how many people can be fed with 20 pizzas?

Q58 Find a common fraction, a decimal fraction and a percent which add to 1. How many combinations can you find?

Revise

Revision Set 1

Q61 Write two equivalent fractions for $^{12}/_{16}$, one using larger numbers, one using smaller numbers.

Q62 Write $^{64}/_{36}$ in simplest form

Q63 Convert $3^4/_5$ to an improper fraction.

Q64 Convert $^{23}/_6$ to a mixed number.

Q65 Copy and complete the following table.

Common fraction	Decimal fraction	Percent
		7%
	0.4	
$^8/_5$		

Answers

Q1 (a) e.g. $^{2}/_{3}$, $^{4}/_{6}$, $^{16}/_{24}$, $^{24}/_{36}$
(b) e.g. $^{1}/_{5}$, $^{2}/_{10}$, $^{8}/_{40}$, $^{12}/_{60}$
(c) e.g. $^{1}/_{5}$, $^{2}/_{10}$, $^{32}/_{160}$, $^{48}/_{240}$
(d) e.g. $^{5}/_{3}$, $^{10}/_{6}$, $^{40}/_{24}$, $^{200}/_{120}$
(e) e.g. $^{1}/_{1}$, $^{2}/_{2}$, $^{64}/_{64}$, $^{41}/_{41}$
(f) e.g. $^{16}/_{6}$, $^{8}/_{3}$, $^{192}/_{72}$, $^{960}/_{360}$

Q2 (a) ½ (b) $^{2}/_{3}$ (c) $^{8}/_{3}$ (d) $^{2}/_{5}$ (e) $^{32}/_{15}$ (f) $^{3}/_{5}$

Q3 If you multiply top and bottom by say 3, then you make the pieces a third the size, but you take three times as many, so you end up with the same amount.

Q4 (a) $^{5}/_{2}$ (b) $^{11}/_{8}$ (c) $^{15}/_{4}$ (d) $^{22}/_{3}$ (e) $^{104}/_{5}$ (f) $^{127}/_{2}$

Q5 (a) $1^{1}/_{5}$ (b) $3^{1}/_{4}$ (c) $6^{2}/_{3}$ (d) $2^{5}/_{7}$ (e) $10^{3}/_{5}$ (f) $31^{1}/_{6}$

Q6

Common fraction	Decimal fraction	Percent
$^{27}/_{100}$	0.27	27%
$^{14}/_{25}$	0.56	56%
$^{3}/_{5}$	0.6	60%
$^{21}/_{500}$	0.042	4.2%
$^{59}/_{200}$	0.295	29.5%
$^{4}/_{7}$	0.5714	57.14%
$^{6}/_{5}$	1.2	120%
$^{17}/_{5}$	3.4	340%
$^{8}/_{5}$	1.6	160%
$^{599}/_{5000}$	0.1198	11.98%
$^{9}/_{2000}$	0.0045	0.45%
$2^{3}/_{4}$	2.75	275%

Q51 $^{5}/_{8}$ Q52 $^{3}/_{8}$ Q53 ¾, 0.75 Q54 $1^{3}/_{10}$

Q55 (a) $^{3}/_{10}$ (b) $^{7}/_{20}$ Q56 (a) $^{1}/_{5}$ (b) $^{9}/_{25}$ Q57 32

Q58 e.g. ½, 0.2, 30%; there are endless combinations

Q61 e.g. ¾, $^{15}/_{20}$ Q62 $^{16}/_{9}$ Q63 $^{19}/_{5}$ Q64 $3^{5}/_{6}$

Q65

Common fraction	Decimal fraction	Percent
$^{7}/_{100}$	0.07	7%
$^{2}/_{5}$	0.4	40%
$^{8}/_{5}$	1.6	160%