M1 Maths

N1-10 Rounding and Approximation

- rounding of numbers
- approximate calculations

Summary Lead In Learn Solve Revise Answers

Summary

We can round a number to a certain decimal place by changing all the digits after that place to zero.

When rounding to a place after the decimal point, we sometimes refer to this as rounding to a certain number of decimal places.

We can round a number to a certain number of significant figures by counting along that many digits starting with the first non-zero digit and changing everything after those digits to zero.

When rounding, if the first digit we change to zero is 5 or more, we add one to the last digit we keep. This is called rounding up.

By rounding numbers to one significant digit (the leading digit), we can change them to numbers we can do arithmetic on in our heads. This allows us to get approximate answers to difficult arithmetic calculations mentally.

Lead-In

Jamie and Steph were gardening. Jamie wanted to tie up 7 tomato plants. Steph had a piece of string 120 cm long. Jamie asked how long each piece should be if she wanted to cut it into 7 equal lengths.

Steph used the calculator on her phone to divide 120 by 7 and told Jamie that each piece would be 17.142857143 cm long.

Jamie laughed.

Learn

You can't cut a piece of string 17.142857143 cm long. For one thing, you wouldn't be able to do it to the nearest billionth of a centimetre – that's less than the width of an atom; and even if you could, the pieces would be a bit stretchy and frayed at the ends. You can't get this sort of accuracy when cutting string.

The best you could probably hope for is to get each piece accurate to the nearest tenth of a centimetre, say 17.1 cm.

Giving her answer as 17.142857143, Steph had been much too precise. The **precision** of an answer (i.e. how many decimal places are given) shouldn't be more than its **accuracy** (i.e. how close the given answer is to the real value). 17.1 cm would have been a more appropriate answer.

What Steph should have done was **round** her number to the nearest tenth of a centimetre.

Rounding to the Nearest Whatever

To **round to the nearest** tenth, Steph could put a vertical line after the tenths digit and then change all the digits after the line to zeros. Then of course, being on the end of the number and after the decimal point, she can ignore them. On paper you might do that like this:

17.1|42857143

17.1|00000000

This number is then just 17.1

If she had wanted to round it to the nearest hundredth (because she thought she could cut the string to the nearest hundredth of a centimetre), she would have put the vertical line after the hundredths digit like this

17.14|2857143

17.14|0000000

giving an answer or 17.14

Rounding to the nearest millionth would look like this

17.142857|143

17.142857|000

Bigger numbers like the population of Australia, 25 627 048, we might want to round to the nearest hundred thousand (because it is constantly changing and would be out of date very quickly). We do that by putting the vertical line after the hundred thousands digit and changing all digits to the right of it to zero, like this

25 627 048

25 600 000

We would then give the answer as 25 600 000. This approximation will be correct for quite a while.

24 735.629 184 rounded to the nearest hundred would be 24 700

rounded to the nearest one (or nearest whole number), it would be 24 735

rounded to the nearest thousandth, it would be 24 735.629

Rounding to So Many Decimal Places

Sometimes, when rounding to a digit after the decimal point, we say we are **rounding to so many decimal places**. We can call 'rounding to the nearest thousandth' 'rounding to three decimal places. This is because we keep three decimal places in the rounded number.

So if we were to the round 24 735.629 184 to one decimal place, it would be 24 735.6

Rounding Up

Say we were rounding 12.947 to the nearest whole number. By the method above, we would round like this:

12.|947

12.|000

But of course, 12.947 is closer to 13 than to 12, so it would have been better to round to 13. So that we do this right, we always check the first digit that we are changing to zero. If it is 5 or more, then we add 1 to the last digit that we are keeping. Like this:

12.|947

13.|000

This is called **rounding up**. Here are some other examples:

29 62|7 339	34.7|8123	562|388.27
29 63|0 000	34.8|0000	562|000.00
	= 34.8	= 562 000

If the number you need to put up is already a 9, then change it to zero instead and add one to the number before it, like this:

31.9|63

32.0|00

Practice

Q1 Round the numbers below as specified

(a)	45 234.887	nearest 10
(b)	6.94271	nearest hundredth
(c)	14.7714936	nearest ten-thousandth
(d)	826.724	nearest whole number
(e)	44.1928	2 decimal places
(f)	83 423 912	nearest million
(g)	0.002643391	4 decimal places
(h)	45.7329	1 decimal place
(i)	0.66666666	nearest tenth
(j)	27.2495322	3 decimal places
(k)	499 679	nearest thousand
(l)	2.00455	2 decimal places
(m)	45.774	nearest ten
(n)	327.4	nearest thousand

Significant Figures

A student was doing some maths problems. The first one gave an answer of 8 427 436.921 on the calculator. He asked the teacher how much should round his answers. The teacher looked at the answer and said 'round all your answers to the nearest thousand.' He gave the answer as 8 427 000. All good.

The next question had an answer of 29.338. So the student duly rounded it to the nearest thousand, which gave him the answer 0. Not so good.

To get around this problem, we often **round to a certain number of significant figures** instead of to the nearest whatever or to so many decimal places. Rounding to 3 significant figures means keeping the first non-zero digits and the two after it and changing the rest to zero (rounding up if necessary).

Using this approach, the student's rounded answers would have been 8 430 000 and 29.3. Much more useful.

Here are a few more examples:

28 344	rounded to 2 significant figures is	28 000
0.00049237	rounded to 2 significant figures is	0.00049
34.82669	rounded to 4 significant figures is	34.83
0.3517	rounded to 1 significant figure is	0.4
65.8	rounded to 1 significant figure is	70

Sometimes rounding to one significant figure is called '**rounding to the leading digit**'. You can probably see why.

Practice

Q2 Round the numbers below as specified

(a)	45 234.887	3 significant figures
(b)	6.94271	5 significant figures
(c)	14.7714936	1 significant figure
(d)	826.724	2 significant figures
(e)	44.1928	3 significant figures
(f)	83 423 912	4 significant figures
(g)	0.002643391	1 significant figure
(h)	45.7329	3significant figures
(i)	0.66666666	leading digit
(j)	27.2495322	6 significant figures
(k)	499 679	2 significant figures
(l)	2.00455	leading digit
(m)	45.774	2 significant figures
(n)	327.4	leading digit

Approximation

Rounding to one significant figure (or to the leading digit) can allow up to get approximate answers in our heads to arithmetic calculations which would be impossible without a calculator or pencil and paper.

For instance, suppose we needed to do 638.42 × 38.716. Most people can't do that in their heads, but they can get a reasonable **approximation** by rounding both numbers to the leading digit, then multiplying.

638.42 rounds to 600 and 38.716 rounds to 40.

600 × 40 = 24 000

The more accurate answer is 24 717. So we were close. In a lot of situations, an approximate answer is good enough.

Practice

Q3 Do the following approximately in your head by rounding the numbers to the leading digit.

(a) 48.6×61

(b) 274.118×29.743

(c) 6.22×1.4

(d) 0.0045×33.9

(e) 0.082×0.00047

(f) $62\,936 \times 9\,273$

(g) $634.5 \times 487.9 \times 36.2$

(h) $7892 \div 214$

(i) $5225.38 \div 48.2$

(j) $5.885 \div 2.716$

(k) $0.0356 \div 10.55$

(l) $0.351 \div 0.000389$

When approximating like this, sometimes we can get a closer answer by rounding a bit differently. For example, if we were approximating, 34.26×53.19, rounding both numbers down to 30×50 would give us a bit of an under-estimate (1500 instead of 1822). If we rounded one up and one down, we might get a closer result: 30×60 gives us 1800 – quite a bit closer.

When adding and subtracting numbers approximately, they need to be rounded to the same decimal place. For instance, to perform $254.76 + 6.92$, we would probably round both numbers to the nearest whole number and add to get $255 + 7 = 262$.

Solve

Q51 A rectangular patio 18.78 m by 5.22 m needs to be tiled. Estimate mentally the number of square metres of tiles that would be needed.

Q52 If the tiles in the last question were 32.6 cm square, roughly how many would be needed to do the job.

Q53 A light year is the distance light travels in a year. Light travels at 299 792.458 km/s. Use rounding to approximate the length of a light year in kilometres.

Q54 Josh gets an annual salary of $94 255.85. He works 7 hours 51 minutes a day, 5 days a week. 48 weeks a year. Estimate his pay per hour.

Q55 The surface of the Earth is about 510 100 000 km^2. About two thirds of that is ocean. The average depth of the ocean is about 3.69 km. Make an approximation of the volume of the ocean in km^3.

Q56 28.5 is equally close to 28 and 29. Why do we always round it up?

Revise

Revision Set 1

Q61 Round the numbers below as specified

(a)	26 234.887	nearest 100
(b)	6.85271	nearest tenth
(c)	14.7714936	3 decimal places
(d)	826.724	2 significant figures
(e)	46.1928	the leading digit
(f)	83 423 912	nearest million
(g)	0.002143391	5 decimal places
(h)	45.7329	1 decimal place
(i)	0.7777777	nearest thousandth
(j)	0.00045812	to 1 significant figure

Q62 Round to the leading digit to get approximate answers to the following. Note if you get closer than the given answer by modifying the method, that is good.

(a) 234×71

(b) 3.88×0.029

(c) $42\,790 \times 18.315$

(d) 0.00043×0.0613

(e) $6294 \div 29.69$

(f) $0.0432 \div 18$

(g) $37.66 \div 0.0023$

Revision Set 2

Q71 Round the numbers below as specified

(a)	0.04743391	3 decimal places
(b)	45.7329	3 significant figures
(c)	0.3877777	nearest thousandth
(d)	27.2495322	leading digit
(e)	699 679	nearest hundred
(f)	0.00045511	4 significant figures
(g)	45.774	nearest hundred

(h)	7.4072	1 decimal place

Q72 Round to the leading digit to get approximate answers to the following. Note if you get closer than the given answer by modifying the method, that is good.

(a) 134×77

(b) 13.29×0.029

(c) $42.791 \times 28\,315.6$

(d) 0.0093×0.000484

(e) $16\,294 \div 38.63$

(f) $0.00132 \div 21$

(g) $19.66 \div 0.00023$

Revision Set 3

Q81 Round the numbers below as specified

(a)	26 234.887	4 significant figures
(b)	6.85271	nearest thousandth
(c)	14.7714936	1 decimal place
(d)	27.2495322	5 decimal places
(e)	499 679	leading digit
(f)	2.00455	3 significant figures
(g)	45.774	nearest tenth
(h)	327.4	1 significant figure

Q82 Round to the leading digit to get approximate answers to the following. Note, if you get closer than the given answer by modifying the method, that is good.

(a) 44×98

(b) 7.81×0.029

(c) 4620×181.3

(d) 0.043×0.0021

(e) $9.217 \div 26.4$

(f) $28\,432 \div 593$

(g) $14.66 \div 0.0054$

Answers

Q1	(a) 45 230	(b) 6.94	(c) 14.7715	(d) 827	(e) 44.19	(f) 83 000 000
	(g) 0.0026	(h) 45.7	(i) 0.7	(j) 27.250	(k) 500 000	(l) 2.00
	(m) 50	(n) 0				
Q2	(a) 45 200	(b) 6.9427	(c) 10	(d) 830	(e) 44.2	(f) 83 420 000
	(g) 0.003	(h) 45.7	(i) 0.7	(j) 27.2495	(k) 500 000	(l) 2
	(m) 46	(n) 300				
Q3	(a) 3000	(b) 9000	(c) 6	(d) 0.12	(e) 0.00004	(f) 540 000 000

	(g) 12 000 000	(h) 40	(i) 100	(j) 2	(k) 0.004	(l) 0.001

Q51 (a) 100 Q52 900 Q53 10^{13} Q54 $50 Q55 1 200 000 000

Q56 The 5 will often be followed by other digits, so always rounding up makes the rule simple.

Q71	(a) 26 200	(b) 6.9	(c) 14.771	(d) 830	(e) 50	(f) 83 000 000
	(g) 0.00214	(h) 45.7	(i) 0.778	(j) 0.0005		
Q72	(a) 14 000	(b) 0.12	(c) 800 000	(d) 0.000024	(e) 200	(f) 0.002
	(q) 20 000					
Q81	(a) 0.047	(b) 45.7	(c) 0.388	(d) 30	(e) 699 700	(f) 0.0004551
	(g) 0	(h) 7.4				
Q82	(a) 8000	(b) 0.3	(c) 1 200 000	(d) 0.0000045	(e) 500	(f) 0.00005
	(g) 200 000					
Q91	(a) 26 230	(b) 6.853	(c) 14.8	(d) 27.24953	(e) 500 000	(f) 2.00
	(g) 45.8	(h) 300				
Q92	(a) 4000	(b) 0.24	(c) 1 000 000	(d) 0.00008	(e) 0.3	(f) 50
	(g) 2000					