

M1 Maths

M6-2 Exact Trig Values

- exact values of the trig functions of multiples of 30 and 45 degrees

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Summary

You need to remember the sines, cosines and tangents of 0 , 30° , 45° , 60° and 90° (0 , $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$ and $\frac{\pi}{2}$). They can be obtained using the unit circle and triangle diagrams.

The sines, cosines and tangents of multiples of 30 and 45 outside the first quadrant can be obtained from the first quadrant values with a circle diagram.

Learn

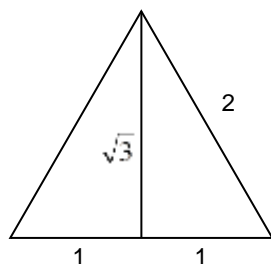
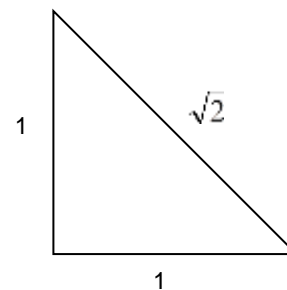
First Quadrant

You need to know the exact values of the sines, cosines and tangents of 0° , 30° , 45° , 60° and 90° (0 , $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$ and $\frac{\pi}{2}$). They are listed in the table below.

		sin	cos	tan
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	–

How do we get these? The values for 0° and 90° can be obtained from the unit circle diagram.

The values for 45° can be obtained by considering a 45° - 45° - 90° triangle and Pythagoras' Theorem as shown here. Make sure you can see how it is done.



The values for 30° and 60° can be obtained by considering a 30° - 60° - 90° triangle, which is half of an equilateral triangle, and Pythagoras' Theorem as shown here. Make sure you can see how this is done too.

Knowing how the values are derived gives you a fall-back in case you forget them. And it's good style.

Practice

Q1 Learn the values, then check yourself by copying and completing this table.

	0°	30°	45°	60°	90°
sin					
cos					
tan					

Q2 Copy and complete this table.

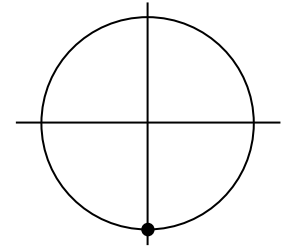
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin					
cos					
tan					

Angles outside the first quadrant

For these we draw a unit circle.

For multiples of 90° , we just read the values off the unit circle diagram.

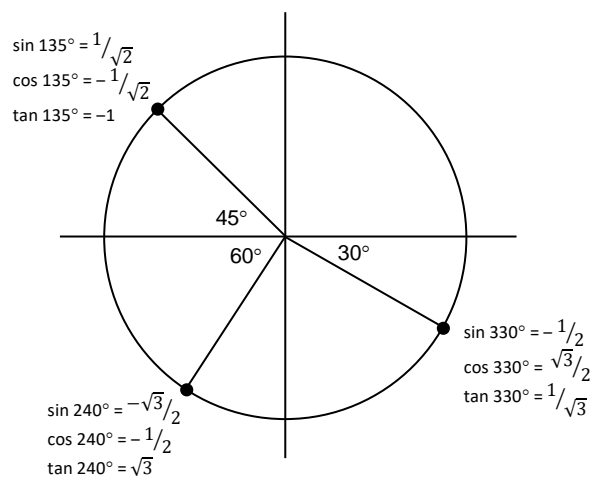
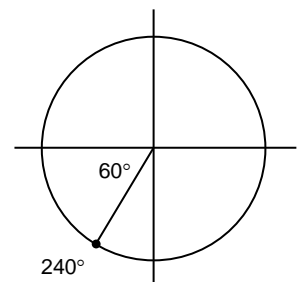
For example, $\sin 270^\circ = -1$; $\cos 270^\circ = 0$; $\tan 270^\circ$ is not defined.



For other multiples of 30° , 45° and 60° , plot the angle on a circle diagram and find how many degrees it is from the x -axis (positive or negative).

If it is 30° , then the trig functions will be the same as for 30° ;
if it is 45° , then the trig functions will be the same as for 45° ;
if it is 60° , then the trig functions will be the same as for 60° ;

... except that you have to decide whether it's positive or negative. You do this by seeing if the trig function is positive or negative in that part of the circle. Some examples are shown below.



Some people like to use the CAST rule to decide between positive and negative. This provides a way of getting the right answer without really understanding what's going on. As such, it is not a recommended way to learn. It will destroy your ability to think for yourself and eventually turn you into a zombie. If you really want to be a zombie though, a quick Internet search will find the rule.

Practice

Q3 Use unit circle diagrams to find the sin, cos and tan of each of the following angles. Check your answers on your calculator.

- (a) 120° (b) 225° (c) 330° (d) -45° (e) 765° (f) 600°
(g) $-\frac{5\pi}{4}$ (h) $\frac{7\pi}{2}$ (i) $\frac{43\pi}{6}$ (j) $-\frac{9\pi}{4}$ (k) 3π (l) $\frac{83\pi}{4}$

Solve

Q51 Find all the angles between 1000° and 1500° which have a cosine of $-\frac{1}{\sqrt{2}}$.

Q52 Given that $\sin(A + B) = \sin A \cos B + \cos A \sin B$ for any angles A and B , find the exact value (not a decimal approximation) of $\sin 75^\circ$ without using a calculator.

Revise

Revision Set 1

Q61 Complete this table.

	0°	30°	45°	60°	90°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin					
cos					
tan					

Q62 Use unit circle diagrams to find the sin, cos and tan of each of the following angles. Check your answers on your calculator.

- (a) 135° (b) 330° (c) -300° (d) 5π (e) $\frac{25\pi}{6}$ (f) $\frac{15\pi}{4}$

Answers

Q1, 2 Check against the table on the first page.

Q51 $1215^\circ, 1305^\circ$

Q52 $\frac{1+\sqrt{3}}{2\sqrt{2}}$

Q61 Check against the table on the first page.