M1 Maths

M3-4 Length, Area and Volume 3

- perimeters, areas and volumes of compound shapes
- distinguishing length, area and volume formulae by inspection
- the effect on area and volume of multiplying the dimensions of a shape

Summary Learn Solve Revise Answers

Summary

To find the perimeter of a compound shape, calculate the length of each side and add.

To find the area of a compound shape, divide it into simple shapes and add their areas.

To find the volume of a compound shape, divide it into simple shapes and add their volumes.

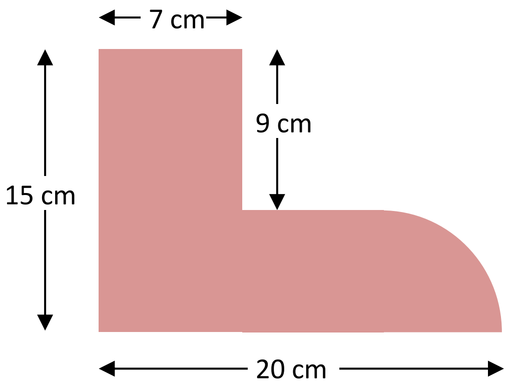
Length formulae have lengths as independent variables, but these are not multiplied together; area formulae have lengths as independent variables multiplied together in pairs; volume formulae have lengths as independent variables multiplied together in threes.

Multiplying the dimensions of a 2D shape by n will multiply the area by n^2; multiplying the dimensions of a 3D shape by n will multiply the volume by n^3.

Learn

Perimeters of compound shapes

A compound shape is a shape made up of two or more simple shapes. The shape below is an example.

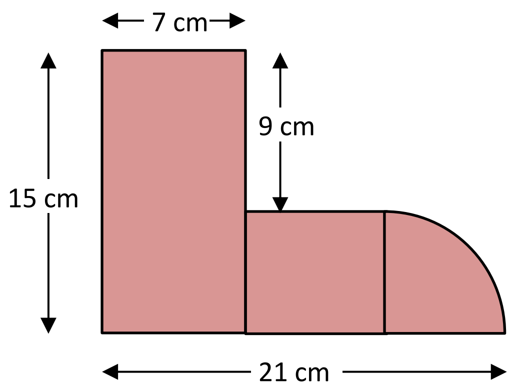


To find its perimeter, we add the lengths of all the sides. The trouble is, there are two sides that don't have the lengths marked. But we can work these out.

In working out the lengths of the unmarked sides, we have to make a couple of assumptions. They are:

- any sides that look parallel are parallel
- any angles that look like right angles are right angles.

If we do this, we can see that the shape is made up of two rectangles and a quarter circle. So we divide it into these shapes like this:



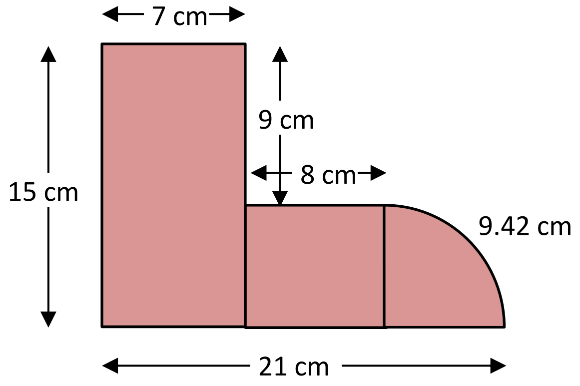
Now we should be able to see that the height of the smaller rectangle is 6 cm (because the opposite sides of a rectangle are equal in length, so the right side of the larger rectangle must be 15 cm and 6 + 9 = 15).

Therefore the radius of the quarter circle is 6 cm.

Therefore the width of the smaller rectangle is 8 cm (because the bottom of the larger rectangle is 7 cm and the bottom (radius) of the quarter circle is 6 cm and 7 + 8 + 6 = 21).

Also, as the radius of the quarter circle is 6, the diameter is 12 and the length of the curved side must be $12\pi \div 4 = 9.42$ cm.

We can write these lengths on the diagram so it looks like this:



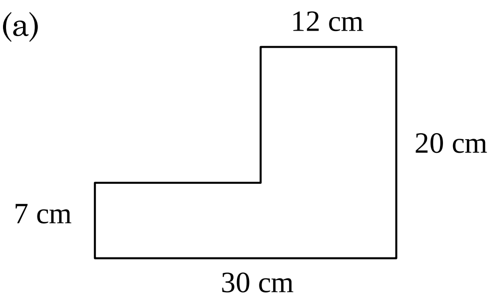
Then we can add up all side lengths all the way round to get
21 + 15 + 7 + 9 + 8 + 9.42 = 69.42 cm. So the perimeter is 69.42 cm.

There is a bit of detective work in working out all the unmarked sides and you may have to think hard.

Practice

Q1 Find the perimeters of these shapes.

(a)



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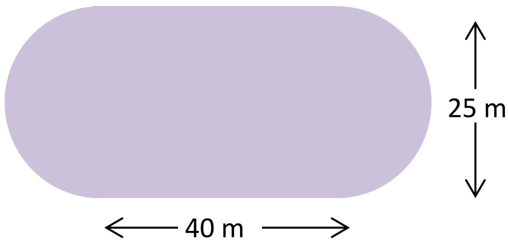

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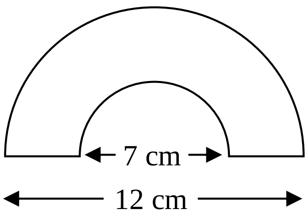

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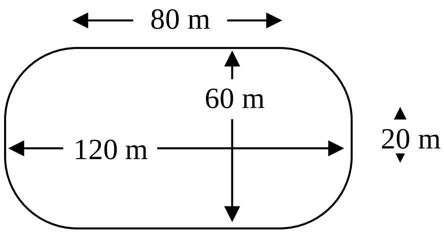

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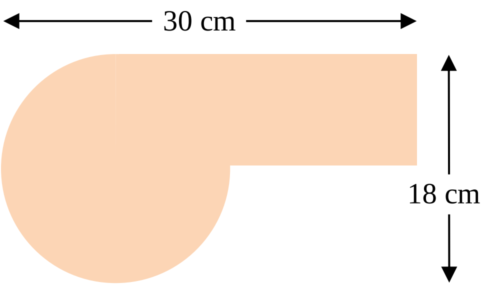
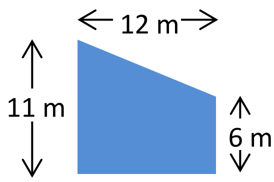


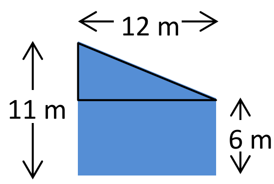
Pythagoras

Sometimes, Pythagoras' theorem has to be used to find side lengths before calculating the perimeter.

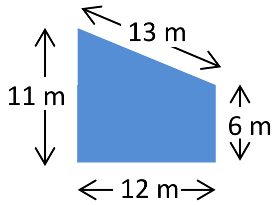


For example, to find the perimeter of this trapezium, we would have to use Pythagoras to find the length of the sloping side.

We would draw in the right-angle triangle like this:



Then we could see that its base is 12 cm and its height 5 m.
So its hypotenuse would be $\sqrt{12^2 + 5^2} = 13$ m.



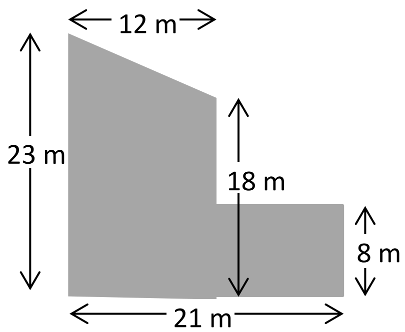
So the side lengths would be as shown to the left.

And the perimeter of the shape would be 11 + 13 + 6 + 12 = 42 m.

Practice

Q2 Find the perimeters of the following shapes.

(a)



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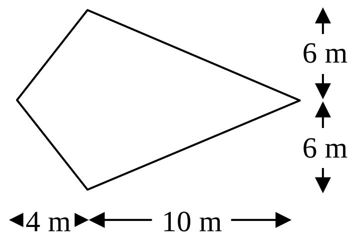

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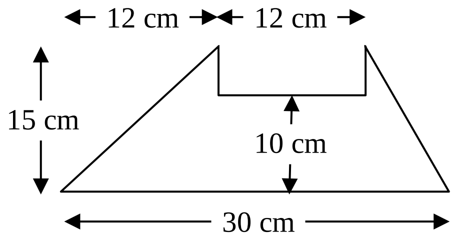

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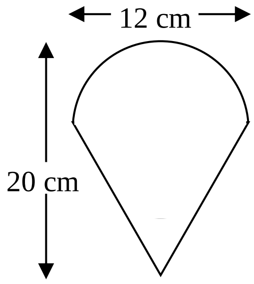

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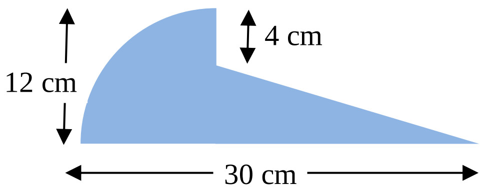


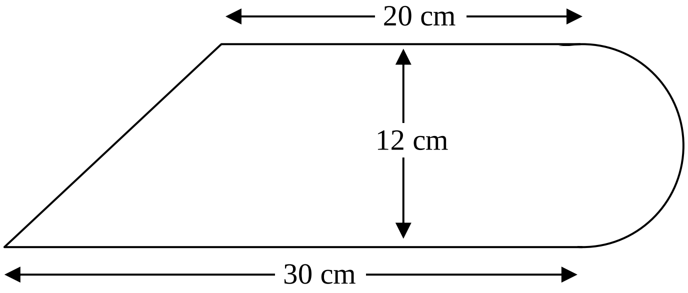
Areas of compound shapes

You already know how to find the areas of rectangles, parallelograms, trapeziums, triangles, circles and sectors. To find the area of a shape made up of these, just

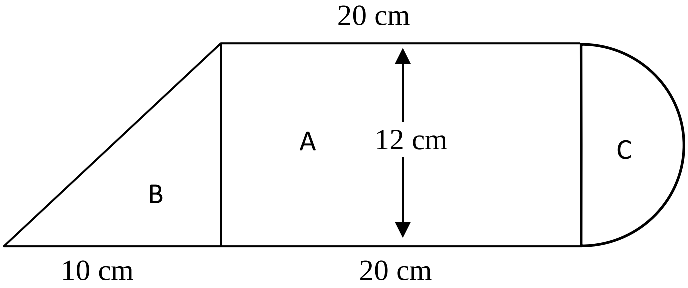
- divide it up into parts that you know how to find the area of and label them A, B, C etc.
- work out any extra measurements you need

- find the area of each part
- add them up.

For example, to find the area of this shape,



we would divide it up and label it like this and add in measurements as shown.



We could then work out the area of each part as follows.

A (rectangle): *Area* = *length* × *width* = 20 × 12 = 240 cm^2

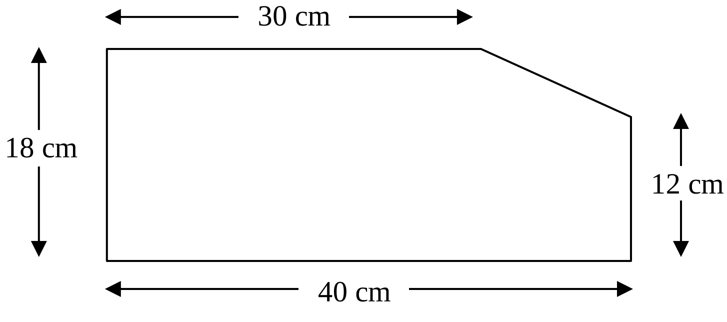
B (triangle): *Area* = ½ *base* × *height* = 10 × 12 ÷ 2 = 60 cm^2

C (semicircle): *Area* = $\pi/4$ × $diameter^2$ × ½ = $\pi/4$ × 12^2 × ½ = 56.5 cm^2

The area of the whole shape is the sum of these individual areas.

240 + 60 + 56.5 = 356.5 cm^2.

Sometimes it is easier to think of a compound shape as a shape with other shapes cut out of it. For example, look at the shape below.



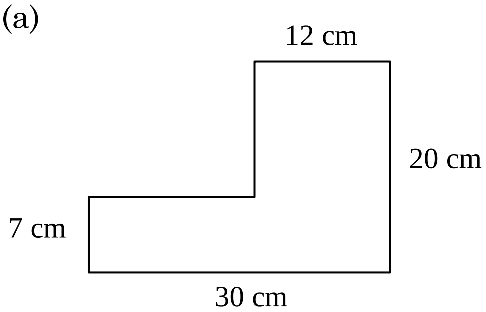
It could be divided into two rectangles and a triangle. Or it could be thought of as a rectangle with a triangle cut off one corner. The area is then the area of the rectangle minus the area of the triangle.

This is $40 \times 18 - 10 \times 6 \div 2$, which is 690 cm^2.

Practice

Q3 Find the areas of the following shapes.

(a)



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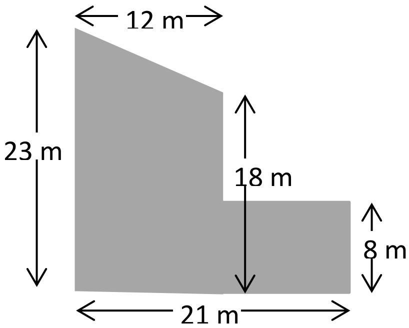

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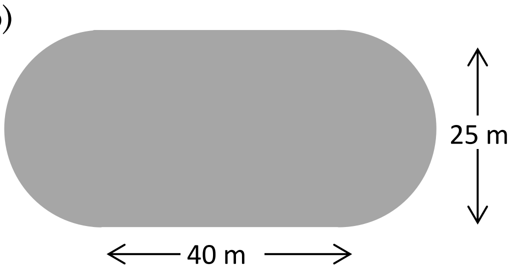

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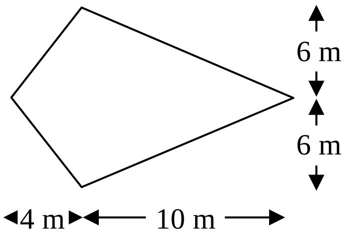

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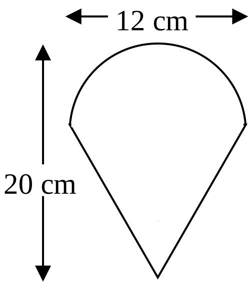

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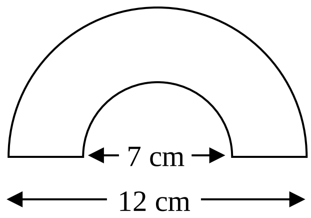

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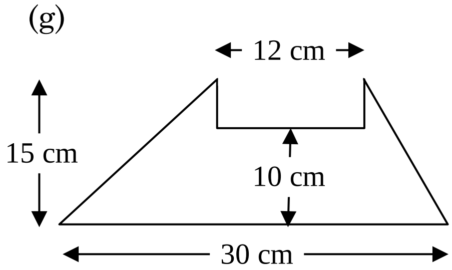

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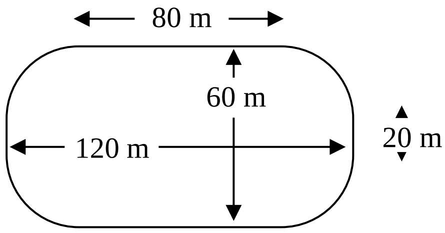
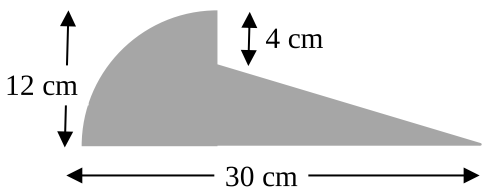


(i)



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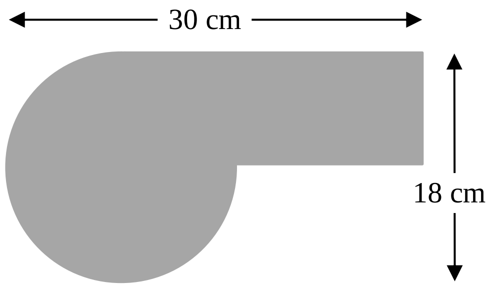


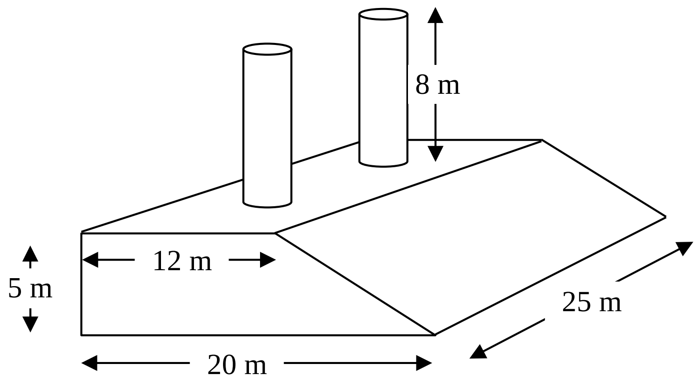
Volumes of compound shapes

To find the volume of a compound shape, you can use an approach very similar to that for finding areas of compound shapes. You

- divide it up into simpler shapes that you know how to find the volume of,
- work out the measurements you need,
- find the volumes of the simple shapes and
- add them up

It's not always easy to divide up the shape on the diagram, so sometimes, just describing the parts is better.

For example, suppose we needed to find the volume of this shape where the chimneys are 1.5 m in diameter.



Our working might look like this.

The base is a trapezoidal prism with base area $\frac{20+12}{2} \times 5 = 80$ m^2.

The volume of the prism is $80 \times 25 = 2000$ m^3.

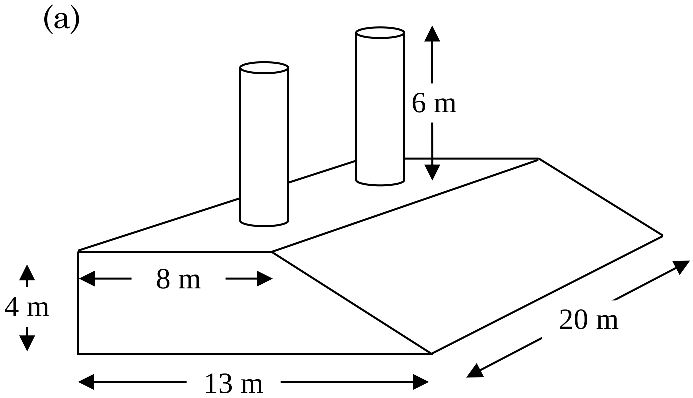
The volume of one cylindrical chimney is $\pi/4 \times 1.5^2 \times 8 = 14.14$ m^3.

Therefore the total volume = $2000 + 2 \times 14.14 = 2028$ m^3.

Practice

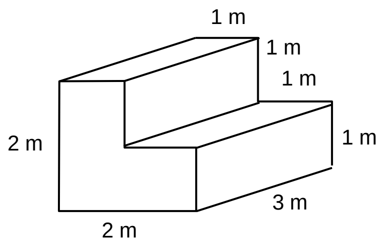
Q4 Find the volume of each of the following shapes.

(a)



The chimneys have a diameter of 1.2 m.

(b)



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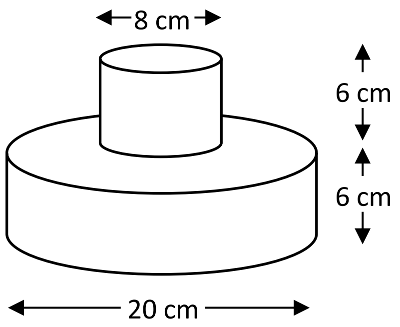

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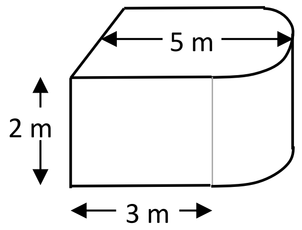

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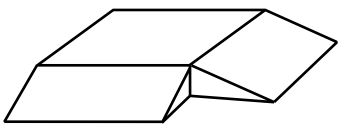
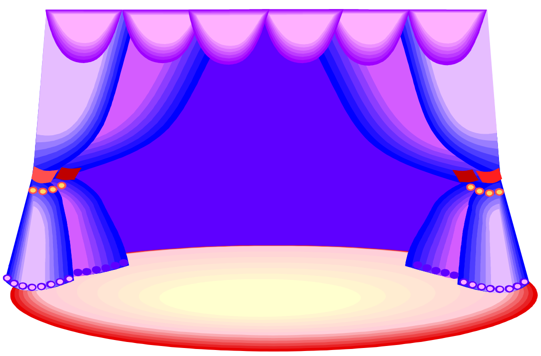


The platform is 5 m by 5 m and 1.2 m high. Each ramp extends 3 m horizontally from the platform.

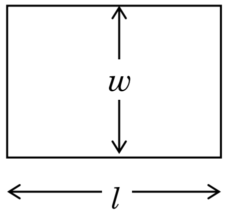


Distinguishing length, area and volume formulae

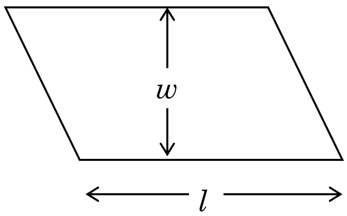
Area formulae

An area is always worked out by multiplying two lengths at right angles to each other and maybe multiplying by a constant.

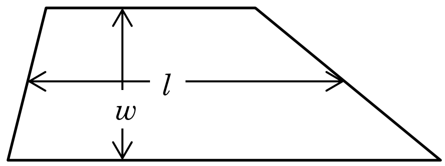
Rectangle
Area = $l \times w$



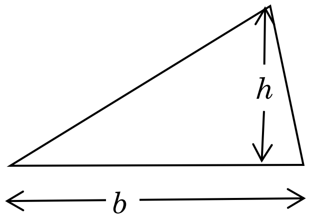
Parallelogram
Area = $l \times w$



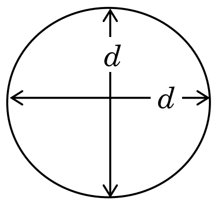
Trapezium
Area = $l \times w$



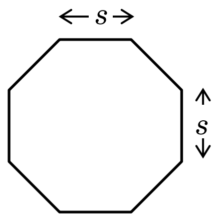
Triangle
Area = $b \times h \times ½$



Circle
Area = $d \times d \times {}^{\pi}/_{4}$

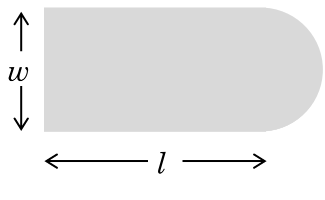


Regular Octagon
Area = $s \times s \times (2 + 2\sqrt{2})$



Thus an area formula will always have two lengths multiplied together.

If the formula has two or more terms (because the shape has two or more parts), each term will have two lengths multiplied together. For instance, the formula for this shape



would be $A = l \times w \;+\; w \times w \times {}^{\pi}/_{8}$

Be careful to distinguish lengths from constants (simple numbers). l and w are lengths; 2, ½ and ${}^{\pi}/_{8}$ are constants. There could actually be any number of constants, but there will always be two lengths multiplied in each term.

Volume formulae

In the same way, a volume is always worked out by multiplying three lengths together. Again, the lengths will all be at right angles to each other. So a volume formula will always have three lengths multiplied (and maybe a constant).

For example, the volume of a rectangular prism is *Area* = *length* × *width* × *height*.

The volume of a cylinder is *base area* × *height*. This looks at first glance like two things multiplied, but we have to realise that base area is an area, not a length and it will be calculated by multiplying two lengths. So, to get the volume, three lengths will be multiplied.

The same goes if there is more than one term, each term must have three lengths multiplied together. A formula like $l \times w + l \times w \times h$ cannot be a formula for area or volume or anything else.

Perimeter formulae

A perimeter is just a length and so a perimeter formula will have just one length in each term: it will never contain two or more lengths multiplied together. The lengths will often be multiplied by a constant or added to other lengths, but will never multiplied by each other.

Units

The number of lengths multiplied together is reflected in the units used:

Perimeter	1 length	m
Area	2 lengths multiplied	m^2
Volume	3 lengths multiplied	m^3

Practice

Q5 For each of these formulae, say whether it is for perimeter (P), area (A), volume (V) or none of the above (N). Assume that letter abbreviations are lengths, but that π is a constant.

(a) $l \times w \times h \times 3$	(b) $l \times h \times ½$	(c) $n \times 2$
(d) $3dr + dh$	(e) $5\pi r$	(f) $3v^2h$
(g) $^{\pi}/_{6}r^3 + 2r^2h$	(h) $4dh + 2\pi d$	(i) πr^2
(j) $2\pi r$	(k) $2(rh + rs)$	(l) $4r + r^2$
(m) $3hr - h^2$	(n) $p + 3r + \pi d$	(o) $5t^2 + 2$

We have mostly used $P = \pi d$ for the perimeter of a circle and $A = {}^{\pi}/_{4}d^2$ for the area. These use the diameter, d. Many people, however, prefer the formulae in terms of the radius, r. These are $P = 2\pi r$ and $A = \pi r^2$. These look quite alike and many people confuse them. If you use the method we have learnt here, you can see that $2\pi r$ must be a perimeter and πr^2 must be an area.

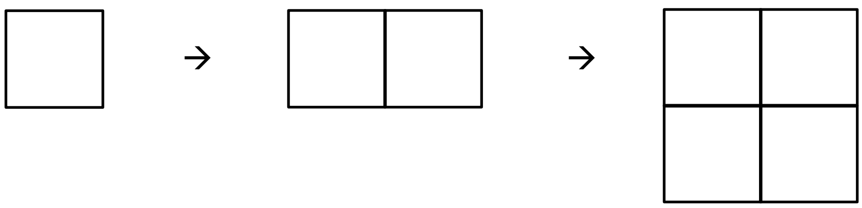
Multiplying the dimensions of a shape

2D shapes

It is a common mistake to think that if you double the dimensions of a 2D shape, you will double the area. For example, if you double the side lengths of a square, the area will be doubled.

However, think about a 5 cm by 5 cm square. Its area is 25 cm^2. Double the side lengths to 10 cm by 10 cm and its area won't double to 50 cm^2, but will quadruple to 100 cm^2.

This is because, we have actually doubled its size twice. First from 5 cm by 5 cm to 10 cm by 5 cm, then from 10 cm by 5 cm to 10 cm by 10 cm.



Doubling the width makes the square twice as big – doubles the area; then doubling the height makes it twice as big again – doubling the area again. So it ends up four times the area.

In the same way, if you multiplied the side lengths by 3, you would multiply the area by 3, then by 3 again – by 9 altogether.

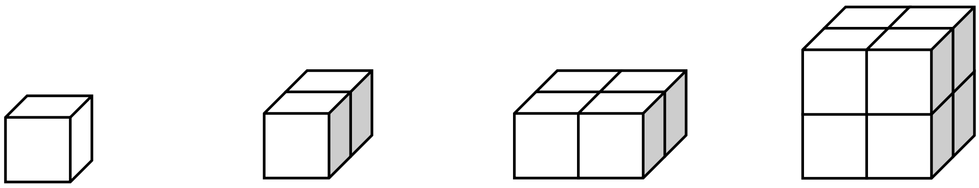
If you multiplied the side lengths by 4, you would multiply the area by 4, then by 4 again – by 16 altogether.

Whatever you multiply the dimensions (side lengths) by, you multiply the area by that number twice – or by that number squared. In other words, if you multiplied the side lengths by n, you would multiply the area by n, then by n again – by n^2 altogether.

The same goes for changing the dimensions of any shape. If you doubled the base and height of a triangle, the area would be multiplied by 4. If you multiplied the diameter of a circle by 6, the area would be multiplied by 36. If you halved the radius of a circle, the area would be multiplied by $(½)^2$, i.e. ¼.

3D shapes

Now think about a 3D shape like a cube. If you double the edge lengths of a cube, you double the length (this will double the volume), then you double the width (this will double the volume again, so it's now 4 times what it started at), then you double the height (this will double the volume again, so it's now 8 times what it started at).



If you double the dimensions of any 3D object, you will double its volume 3 times or multiply it by 8. In general, if you multiply the dimensions of a 3D object by n, then you will multiply the volume by n^3.

Many people find this a difficult and counter-intuitive concept. Think about it until it is obvious.

Practice

Q6 What would the area and volume of a shape be multiplied by if you multiplied its dimensions by:

(a) 2 (b) 3 (c) 10 (d) ½ (e) 2.6 (f) 0.8

Q7 A spherical Balloon hold 1 L of air. If it is inflated until its diameter doubles, how much air will it hold then?

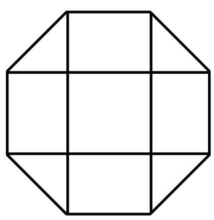
Q8 A piece of material in the shape of an ellipse shrunk in the wash so its length and width both deceased by 10%. If it as originally 0.8 m^2, what would be its area after it shrunk?

Q9 A 4 m rope shrunk so that all its dimensions decreased by 5%. What would its length be then?

Solve

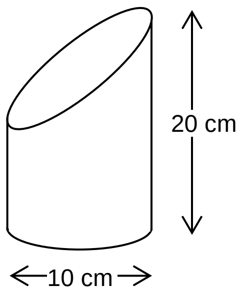
Q51 Ruth walks around the edge of the football field to the opposite corner. If the field is 100 m by 50 m and she walks at 1.2 m/s, how much time would she have saved by walking straight across it?

Q52 Find the area of a regular octagon with side length 4 cm. [Hint: divide it up like this and use Pythagoras to find the area of each part.]



Q53 Show that the formula for the area of a regular hexagon of side length s is $A = \frac{3\sqrt{3}}{2}s^2$. [Hint: divide the hexagon into 6 equilateral triangles and use Pythagoras to get the height of each.]

Q54 Find the volume of this shape which is a cylinder cut off at a 45° angle.



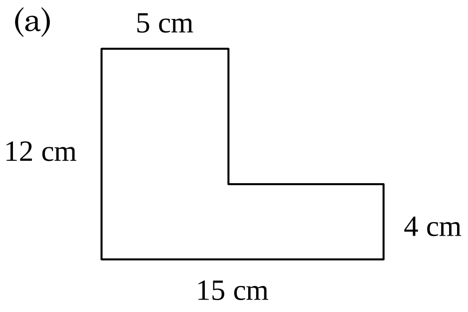
Q55 When dry sand is poured, it forms a conical pile. The slope of the side of the cone is 34°. One truck load of sand forms a pile 1.45 m tall. How tall a pile would 2 truck loads make?

Revise

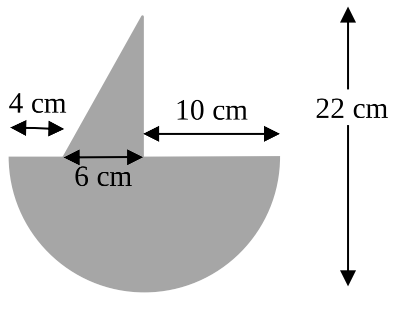
Revision Set 1

Q61 Find the perimeters of these shapes.

(a)

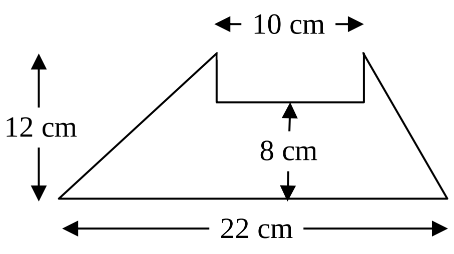


(b)

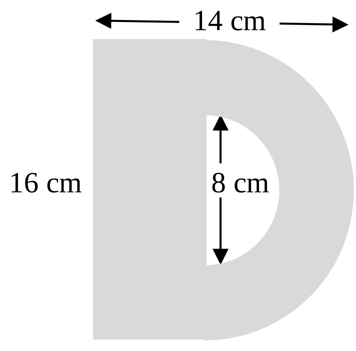


Q62 Find the areas of these shapes.

(a)

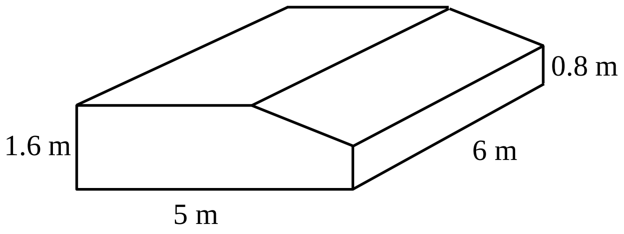


(b)

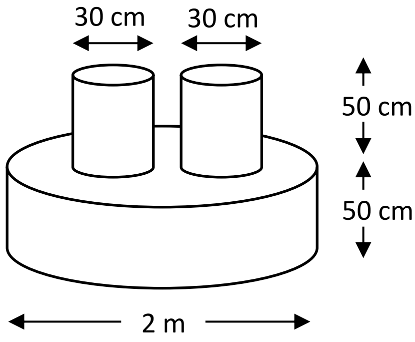


Q63 Find the volumes of these shapes.

(a)



(b)

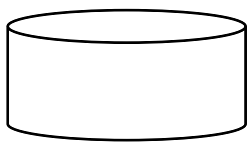


Q64 For each of these formulae, say whether it is for perimeter (P), area (A), volume (V) or none of the above (N). Assume that letter abbreviations are lengths, but that π is a constant.

(a) $6hw$ (b) $(s + 2h) \times \frac{1}{2}$ (c) $\pi n t^2$

(d) $3dr + d^2h$ (e) $2\pi r$ (f) $\pi r^2 h$

Q65 If you increased the dimensions of this shape by a factor of 5, by what factor would the area and volume increase?



Answers

Q1	(a) 100 cm	(b) 158.6 m	(c) 34.8 cm	(d) 326 m	(e) 84.4 cm	
Q2	(a) 84 m	(b) 37.7 m	(c) 87.4 cm	(e) 49.3 cm	(e) 54.5 cm	
Q3	(a) 366 cm^2	(b) 318 m^2	(c) 1491 m^2	(d) 84 m^2	(e) 141 cm^2	
	(f) 37.3 cm^2	(g) 255 cm^2	(h) 6857 m^2	(i) 524 cm^2	(j) 380 cm^2	
Q4	(a) 854 m^3	(b) 36 m^3	(c) 2187 cm^3	(d) 36.6 m^3	(e) 48 m^3	
Q5	(a) V	(b) A	(c) P			
	(d) A	(e) P	(f) V			
	(g) V	(h) N	(i) A			
	(j) P	(k) A	(l) N			
	(m) A	(n) P	(o) N			

Q6 (a) area ×4, volume ×8 (b) area ×9, volume ×27 (c) area ×100, volume ×1000
(d) area ×¼ , volume ×$^1/_8$ (e) area ×6.76, volume ×17.576 (f) area ×0.64, volume ×0.512

Q7 8 L Q8 0.648 m^2 Q9 3.8 m

Q51 32 seconds
Q52 77.3 cm^2
Q54 1963 cm^3
Q55 1.83 m

Q61 (a) 54 cm (b) 70.8 cm
Q62 (a) 152 cm^2 (b) 171 cm^2
Q63 (a) 43.2 m^3 (b) 1.64 m^3 or 16 415 cm^3
Q64 (a) A (b) P (c) V (d) N (e) P (f) V
Q65 area by 25, volume by 125