M1 Maths

Learning by Thinking

M3-4 Length, Area and Volume 3

- perimeters, areas and volumes of compound shapes
- distinguishing length, area and volume formulae by inspection
- the effect on area and volume of multiplying the dimensions of a shape

Learn Answers

This LbT (Learning by Thinking) module is an alternative to the ‘Learn’ section of the normal module. It is designed to lead the student to work out the maths themselves by solving problems. Thus it contains only minimal explanations. The rationale behind the approach can be read **here**.

Learn

Perimeters of compound shapes

A compound shape is a shape made up of two or more simple shapes.

Q1 Find the perimeter of these compound shapes. Assume all angles are right angles. Some sides are unmarked, but with a bit of detective work you should be able to find their lengths. For these types of problems, showing working is important. Copy the diagram, divide it into simple shapes, mark on it all the relevant lengths, then add them up to get the final answer.

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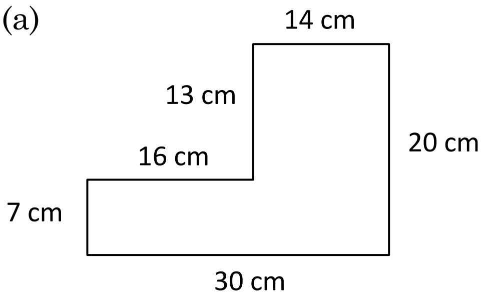

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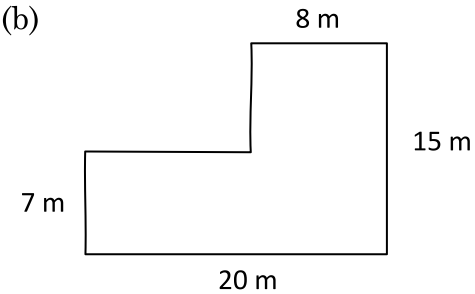

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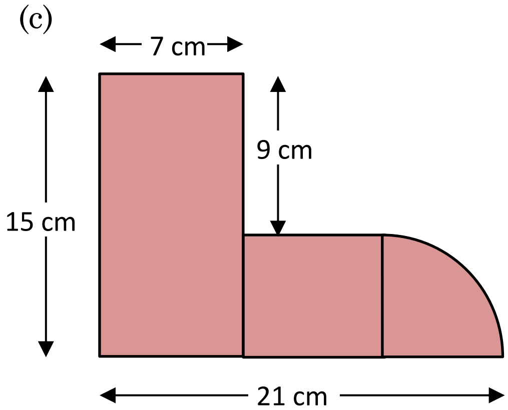

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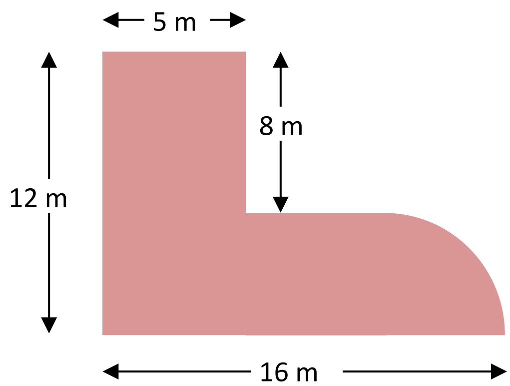

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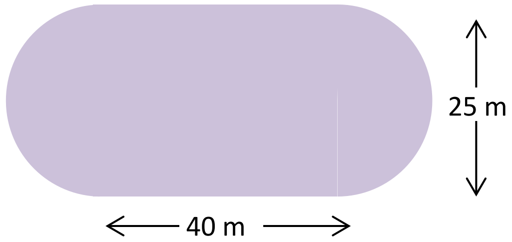

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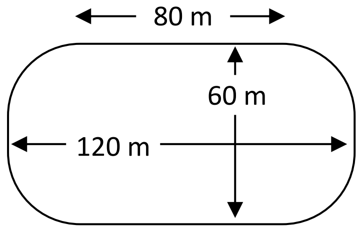

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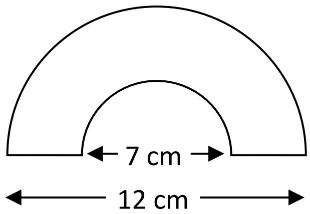

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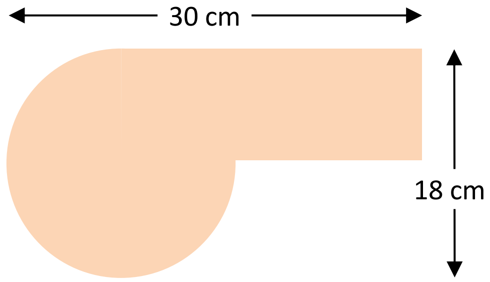

Pythagoras

Sometimes, Pythagoras' theorem has to be used to find side lengths before calculating the perimeter.

Q2 Find the perimeters of the following shapes. Show working as before, but also show the Pythagoras calculation.

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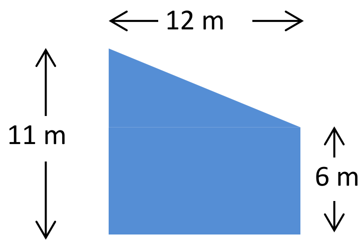

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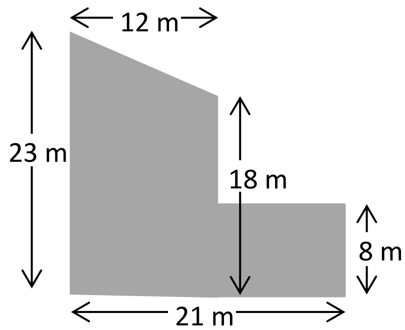

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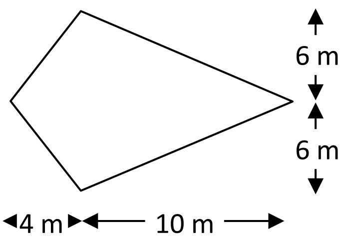

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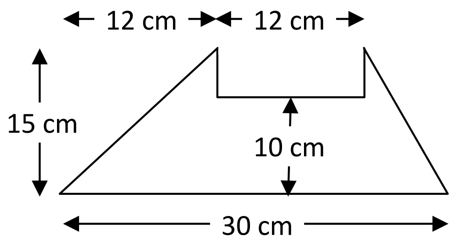

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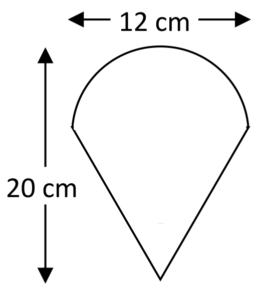

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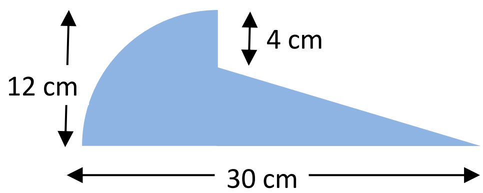
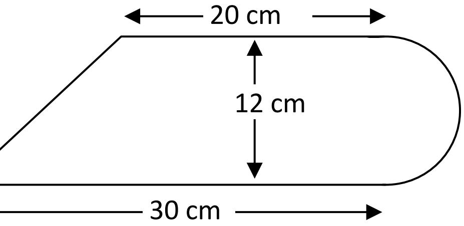


Areas of compound shapes

Q3 Find the areas of the following shapes. Again, working is important. Copy the diagram, divide it into simple shapes and mark on all relevant measurements. Then show the working for the area of each of the simple shapes. Finally, add to get the total area.

(a)



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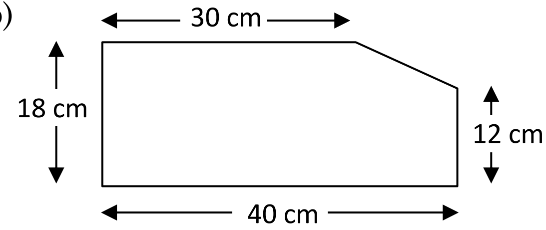
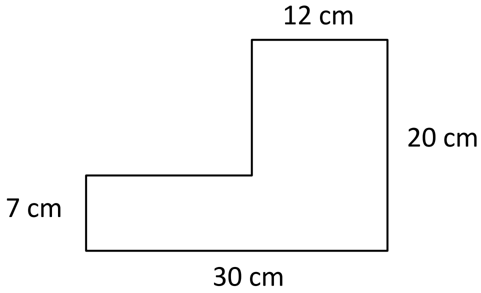


Note that for (b) it can be easier to treat the shape as a rectangle with a triangle removed than as two rectangles plus a triangle.

(c)



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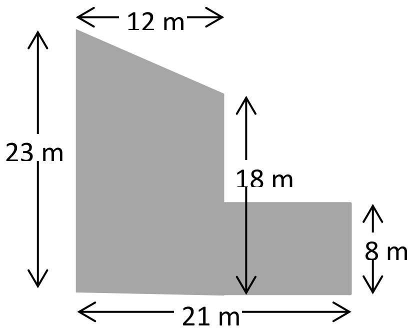

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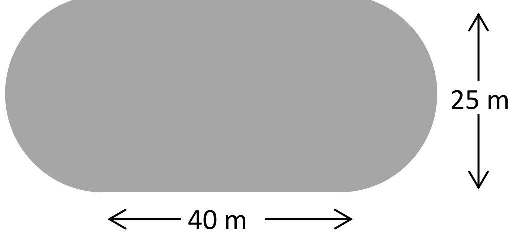

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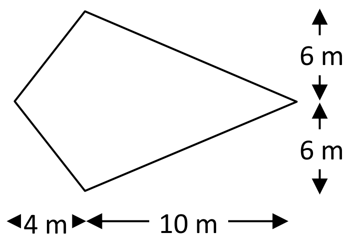

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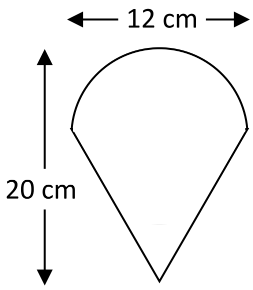

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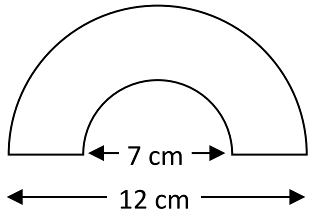

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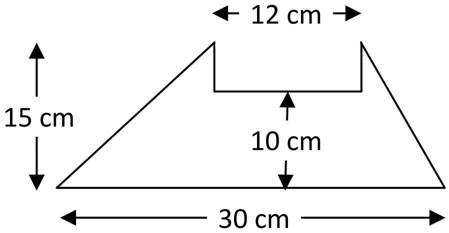

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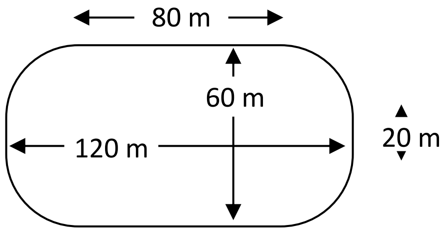
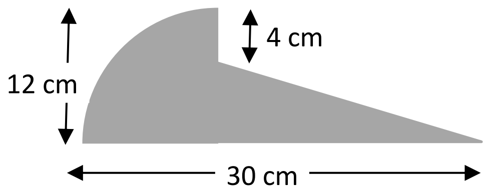
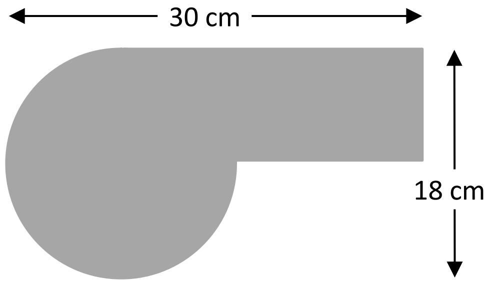


(k)

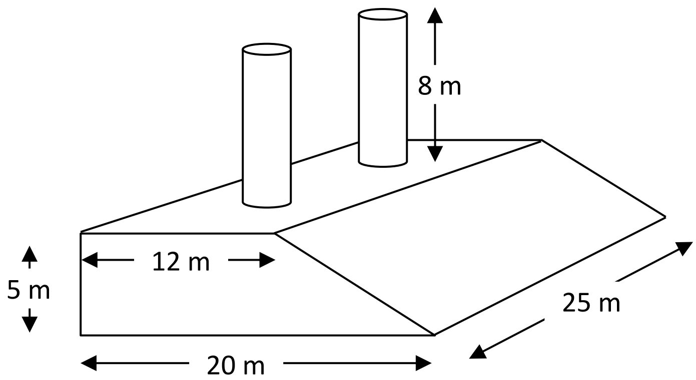


(l)



Volumes of compound shapes

As with areas, the volume of a compound shape can be found by dividing it into simple shapes and adding their volumes. Because it is harder to show how the shape is divided up on a 3D diagram, you should give a fuller explanation, something like the following.



The chimneys have a diameter of 1.5 m.

The base is a trapezoidal prism with base area $\frac{20+12}{2} \times 5 = 80$ m^2.

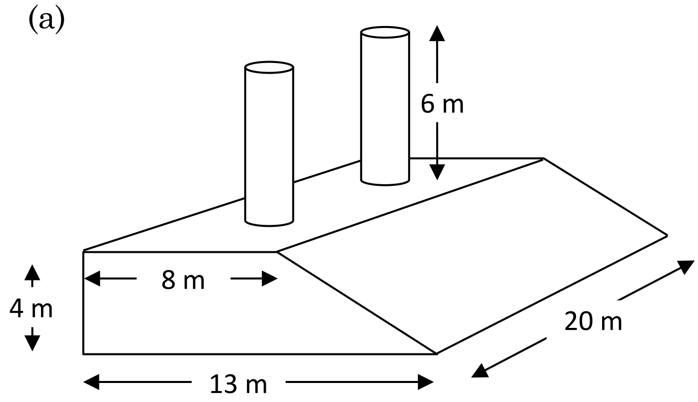
The volume of the prism is $80 \times 25 = 2000$ m^3.

The volume of one cylindrical chimney is $\pi/4 \times 1.5^2 \times 8 = 14.14$ m^3.

Therefore the total volume = $2000 + 2 \times 14.14 = 2028$ m^3.

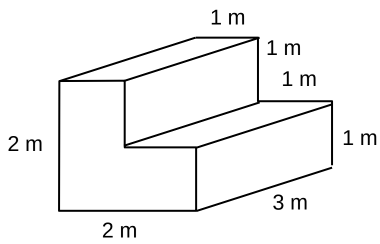
Q4 Find the volume of each of the following shapes.

(a)



The chimneys have a diameter of 1.2 m.

(b)



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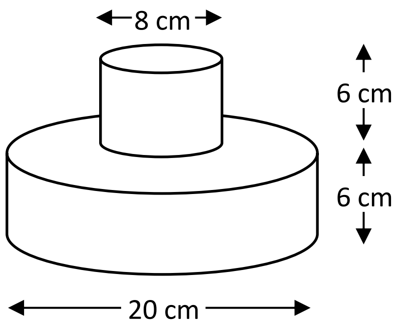
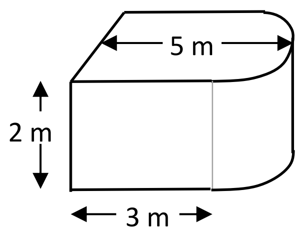
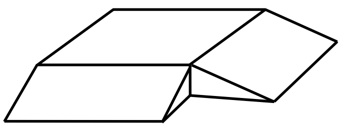


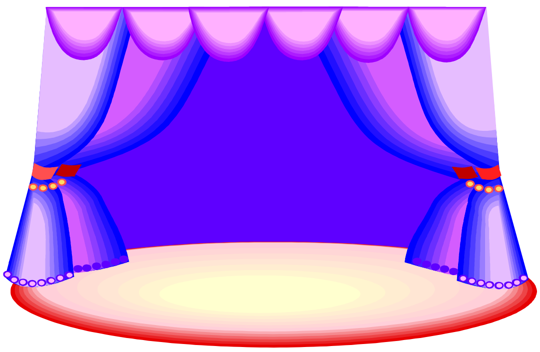
(d)



(e)*



* The stage in (e) is 5 m by 5 m and 1.2 m high. Each ramp extends 3 m horizontally from the stage.

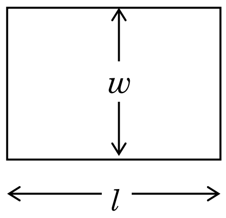


Distinguishing length, area and volume formulae

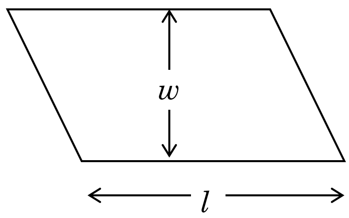
Area formulae

An area is always worked out by multiplying two lengths at right angles to each other and multiplying by the fill factor.

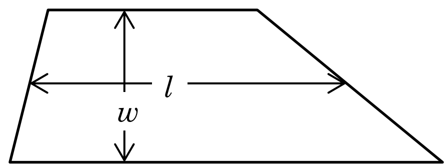
Rectangle
Area = $l \times w \times 1$



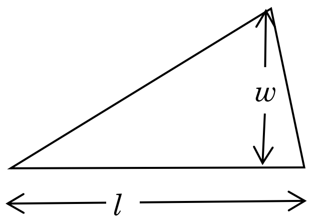
Parallelogram
Area = $l \times w$



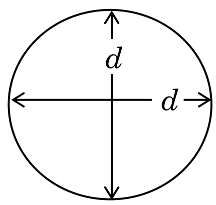
Trapezium
Area = $l \times w$



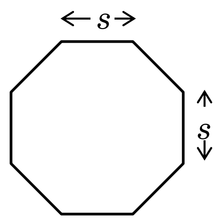
Triangle
Area = $l \times w \times ½$



Circle
Area = $d \times d \times {}^{\pi}/_4$

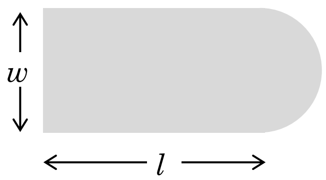


Regular Octagon
Area = $s \times s \times (2 + 2\sqrt{2})$



Thus an area formula will always have two lengths multiplied together. (The fill factor is not a length.)

If the formula has two or more terms (because the shape has two or more parts), each term will have two lengths multiplied together. For instance, the formula for this shape



would be $A = l \times w \ + \ w \times w \times {}^{\pi}/_4 \div 2$

Be careful to distinguish lengths from constants (simple numbers). l and w are lengths; 2, ½ and ${}^{\pi}/_4$ are constants. There could actually be any number of constants, but there will always be two lengths multiplied in each term.

Volume formulae

In the same way, a volume is always worked out by multiplying three lengths together. Again, the lengths will all be at right angles to each other. So a volume formula will always have three lengths multiplied (and maybe a constant).

For example, the volume of a rectangular prism is *Area* = *length* × *width* × *height*.

The volume of any prism is *end area* × *length*. This looks at first glance like two things multiplied, but we have to realise that end area is an area, not a length and it will be calculated by multiplying two lengths. So, to get the volume, three lengths will be multiplied.

The same goes if there is more than one term: each term must have three lengths multiplied together. $l \times w \times 2d + l \times w \times h$ would be a volume formula. A formula like $l \times w + l \times w \times h$ cannot be a formula for area or volume or anything else.

Perimeter formulae

A perimeter is just a length and so a perimeter formula will have just one length in each term: it will never contain two or more lengths multiplied together. The lengths will often be multiplied by a constant or added to other lengths, but will never multiplied by each other.

Units

The number of lengths multiplied together is reflected in the units used:

Perimeter	1 length	m
Area	2 lengths multiplied	m^2
Volume	3 lengths multiplied	m^3

Q5 For each of these formulae, say whether it is for perimeter (P), area (A), volume (V) or none of the above (N). Assume that letter abbreviations are lengths, but that π is a constant.

(a) $l \times w \times h \times 3$	(b) $l \times h \times \frac{1}{2}$	(c) $n \times 2$
(d) $3dr + dh$	(e) $5\pi r$	(f) $3v^2h$
(g) $\frac{\pi}{6}d^3 + 2d^2h$	(h) $4dh + 2\pi d$	(i) πr^2
(j) $2\pi r$	(k) $2r(h + s)$	(l) $4r + r^2$
(m) $3hr - h^2$	(n) $p + 3r + \pi d$	(o) $5t^2 + 2$

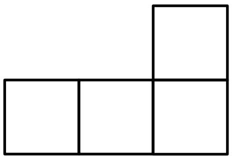
We have mostly used $P = \pi d$ for the perimeter of a circle and $A = \frac{\pi}{4}d^2$ for the area. These use the diameter, d. Some people, however, prefer the formulae in terms of the radius, r. These are $P = 2\pi r$ and $A = \pi r^2$. These look quite alike and many people confuse them. If you use the method we have learnt here, you can see that $2\pi r$ must be a perimeter and πr^2 must be an area.

Multiplying the dimensions of a shape

2D shapes

It is a common mistake to think that if you double the dimensions of a 2D shape, you will double the area. For example, if you double the side lengths of a square, the area will be doubled. This is **not** the case. Repeat: this is **not** the case!

Q6 Draw 4 1-cm squares to make this shape.

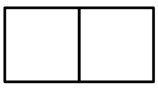


Then think about drawing the same shape but twice as big, i.e. twice as long and twice as wide. How many 1 cm squares do you think it would take?

Now draw it and check your answer.

Q7 Draw another shape using 6 1-cm squares. Then predict how many squares it would take to make the same shape twice as big. Then draw it to check.

Q8 Draw this shape using 2 1-cm squares.



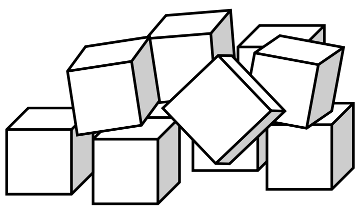
(a) Then predict how many 1-cm squares it would take to make the same shape 3 times as big, i.e. 3 times as long and 3 times as wide.

(b) Do the same again, but this time 4 times as big.

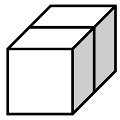
Q9 Find a general rule for predicting the areas of enlarged shapes. Test your rule with some new shapes and different enlargement factors. If it tests ok, good; if it doesn't, rethink your rule and test again until you get one that always works.

3D shapes

If you can, get hold of a bunch of 1-cm cubes, e.g. small MAB blocks or centi-cubes. If you can't, try drawing or imagining the shapes.



Q10 Make this shape.



(a) Then think about making the same shape but twice as big: twice as long, twice as wide and twice as high. How many cubes would it take?

Try it and check.

(b) Do the same again 3 times as big.

Q11 Make other shapes and see how many cubes it takes to enlarge them by different factors.

Q12 Find a general rule for predicting the volumes of enlarged shapes. Test your rule with some new shapes and different enlargement factors. If it tests ok, good; if it doesn't, rethink your rule and test again until you get one that always works.

Q13 Do you think your rules will work for shapes other than squares and cubes?

Q14 The footprint of a baby dinosaur is 240 cm^2. The footprint of its parent is the same shape but 3 times as long. What will its area be?

Q15 (a) The diameter of the Sun is about 100 times that of the Earth. How many times bigger will the volume of the Sun be than the volume of the Earth?

(b) What about the surface area?

Q16 A machine makes small ice spheres 2 cm in diameter and large ice spheres 10 cm in diameter. If you melted 3 large ice spheres and used the water to make small ice spheres, how many could you make?

Q17 What would the area and volume of a shape be multiplied by if you multiplied its dimensions by:

(a) 2 (b) 3 (c) 10 (d) ½ (e) 2.6 (f) 0.8

Q18 A spherical Balloon holds 1 L of air. If it is inflated until its diameter doubles, how much air will it hold then?

Q19 A piece of material in the shape of an ellipse shrunk in the wash so its length and width both deceased by 10%. If it as originally 0.8 m^2, what would be its area after it shrunk?

Q20 A 4 m rope shrunk so that all its dimensions decreased by 5%. What would its length be then?

Answers

Q1	(a) 100 cm	(b) 70 m	(c) 69.42 cm	(d) 54.28 m		
	(e) 158.6 m	(f) 326 m	(g) 34.8 cm	(h) 84.4 cm		
Q2	(a) 42 m	(b) 84 m	(c) 37.7 m	(d) 87.4 cm	(e) 49.3 cm	(e) 54.5 cm
Q3	(a) 356.6 cm^2	(b) 690 cm^2	(c) 366 cm^2	(d) 318 m^2	(e) 1491 m^2	(f) 84 m^2
	(g) 141 cm^2	(h) 37.3 cm^2	(i) 255 cm^2	(j) 6857 m^2	(k) 524 cm^2	(l) 380 cm^2
Q4	(a) 854 m^3	(b) 36 m^3	(c) 2187 cm^3	(d) 36.6 m^3	(e) 48 m^3	
Q5	(a) V	(b) A	(c) P			
	(d) A	(e) P	(f) V			
	(g) V	(h) N	(i) A			
	(j) P	(k) A	(l) N			
	(m) A	(n) P	(o) N			

Q6 16 Q7 24 Q8 (a) 18 (b) 32

Q9 If you enlarge the dimensions of a 2D shape by a factor n, then you increase the area by a factor n^2.

Q10 (a) 16 (b) 54

Q12 If you enlarge the dimensions of a 3D shape by a factor n, then you increase the volume by a factor n^3.

Q13 It will Q14 2160 cm2 Q15 (a) 1 000 000 (b) 10 000 Q16 375

Q17 (a) area ×4, volume ×8 (b) area ×9, volume ×27 (c) area ×100, volume ×1000
(d) area ×¼ , volume ×$^1/_8$ (e) area ×6.76, volume ×17.576 (f) area ×0.64, volume ×0.512

Q18 8 L Q19 0.648 m^2 Q20 3.8 m