M1 Maths

M3-3 Slope

- slope as an angle
- gradient

Summary Lead-In Learn Solve Revise Answers

Summary

The steepness of a line can be given as its slope angle, the angle between the horizontal and that line.

It can also be given as a gradient, the number of units you would rise for each unit travelled horizontally from left to right if you travelled along the line.

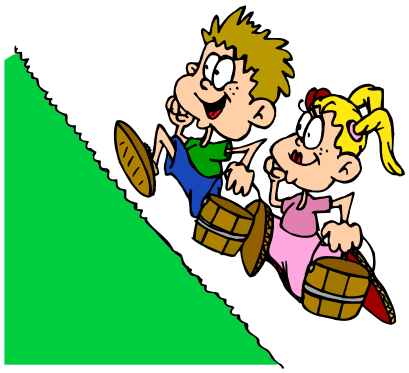
To draw a line with a given gradient m, draw a horizontal line 1 unit long, then, at the right end draw a line going up from it m units (it will go down if m is negative). Connect the ends, then extend the line if necessary.

To determine the gradient of a given line, pick two points on the line then divide the rise (vertical difference between the points) by the run (horizontal difference between the points).

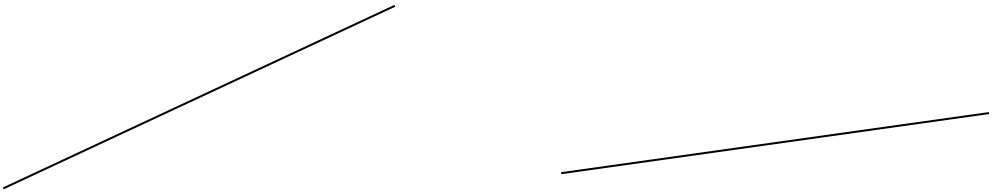
The gradient of a line is the tan of the slope angle and the slope angle is the inverse tan of the gradient.

Lead-In

Find out what is the steepest slope someone can walk up.



Learn

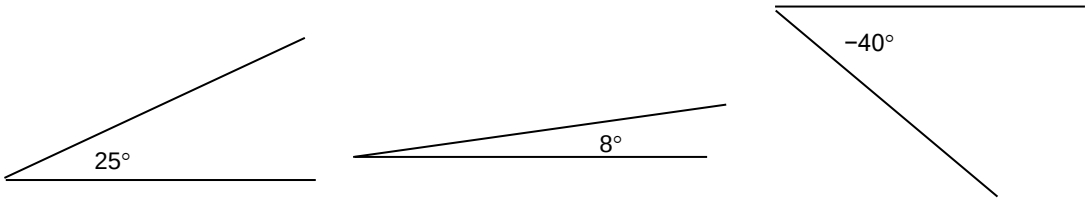


The line on the left is steeper than the line on the right. If the lines were hills looked at from the side, the hill on the left would be the steeper.

There are two ways to express the steepness of a line.

Steepness as slope angle

The first way is by the angle it makes with the horizontal. We can find this angle by drawing the line and the horizontal as shown below, then measuring the angle between the line and the horizontal.



The line on the left makes an angle of 25° and therefore has a slope angle of 25°.

The line in the middle makes an angle of 8° and therefore has a slope angle of 8°.

The line on the right makes an angle of 40°, but goes down as you go to the right. So we say that the steepness is negative, −40°. In maths, we say that a line which goes up from left to right has a positive slope and that a line which goes down from left to right has a negative slope.

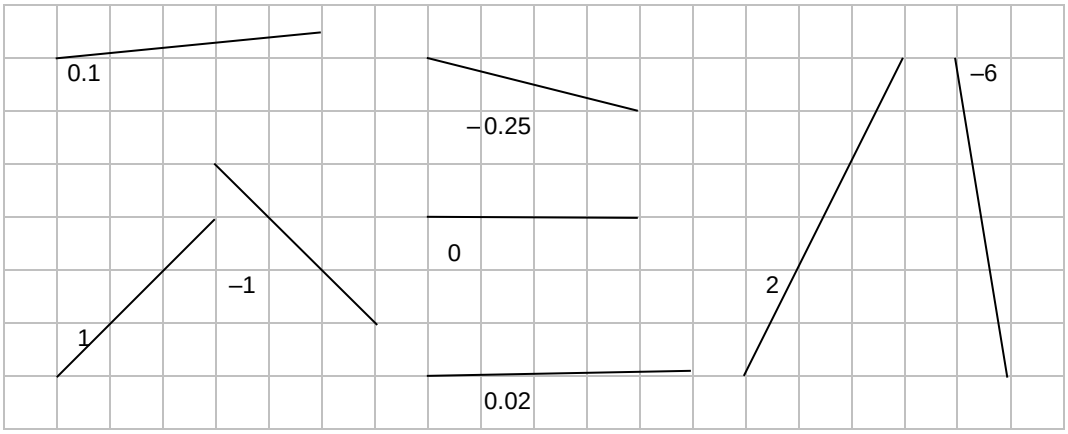
Steepness as gradient

The second way to express the steepness of a line is as a gradient. In maths, we take the gradient of a line as the number of units you would **rise** for every unit you move **horizontally from left to right** if you travelled along the line. It can be given as a percent or a common or decimal fraction.

For example, if a line goes up 0.5 units for each unit we move to the right, then we say it has a gradient of 0.5.

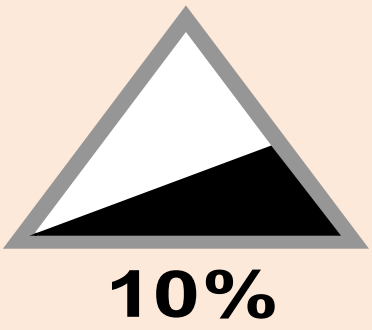
If a line goes **down** 2 units for every unit you move to the right, then we say it has a gradient of −2.

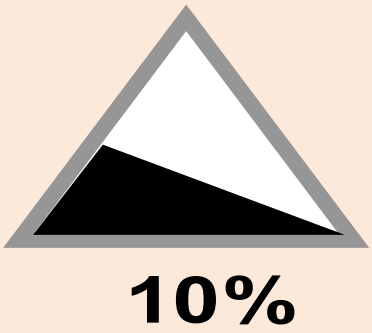
The gradients of the following lines are as shown. Make sure you can see why each number is correct.



If you have ever travelled in a car you might have seen signs like this:

The sign to the right shows that the road ahead goes up hill with a gradient of 10%. The 10% means that every time you travel a metre horizontally, you will ascend 10% of a metre, i.e. 0.1 m. Or that every time you travel a kilometre horizontally, you will ascend 10% of a kilometre, i.e. 0.1 km or 100 m. In fact whatever distance you travel horizontally, you will ascend 10% of that distance.



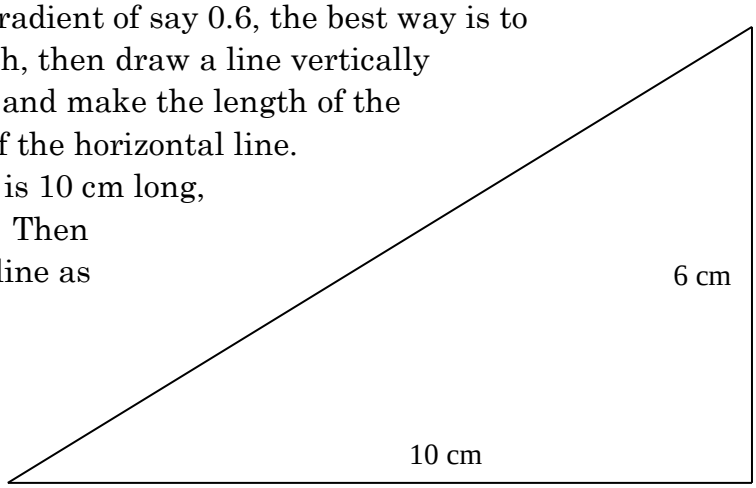


The sign on the left tells you that the road goes downhill with a gradient of 10%.

Road signs don't use negative numbers: they always give the gradient as a positive fraction, but show you that the road goes downhill by the fact that the line on the sign goes down as you go to the right.

Drawing lines with a given gradient

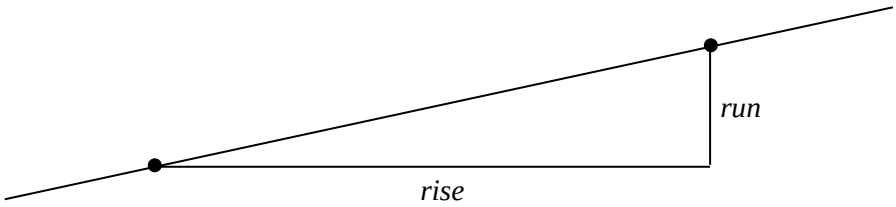
If you need to draw a line with a gradient of say 0.6, the best way is to draw a horizontal line of any length, then draw a line vertically upwards from the right hand end, and make the length of the vertical line 0.6 times the length of the horizontal line. For example, if the horizontal line is 10 cm long, then the vertical line will be 6 cm. Then connect the ends with the sloping line as shown below.



If the gradient is negative, draw the vertical line downwards instead.

Finding the gradient of a given line

A good way to find the gradient of a line is to pick two points on the line (maybe the two ends). Then draw a horizontal line from the left-hand point and a vertical line from the right-hand point until they meet. The length of the horizontal line is the amount the line moves horizontally. This is called the run. The length of the vertical line is how far the line rises. This is called the rise.



The gradient is the *rise* divided by the *run*. This is often written as:

$$gradient = \frac{rise}{run}$$

For example, if the run is 20 cm and the rise is 5 cm, this means that for each 1cm you move to the right, you will rise $^1/_{20}$ of 5 cm upwards. This is the same as $^5/_{20}$ or ¼. So the gradient is ¼.

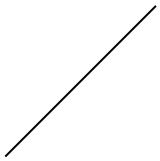
If the right hand-point is lower than the left-hand point, then the rise is negative, so the gradient is negative.

The slope angle or gradient of a surface is the slope angle or gradient of a line drawn on the surface which runs directly up and down the hill.

Practice

Q1 Estimate the slope angle and the gradient of each of these lines.

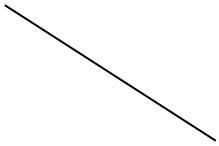
(a)



(b)

(c)

(d)

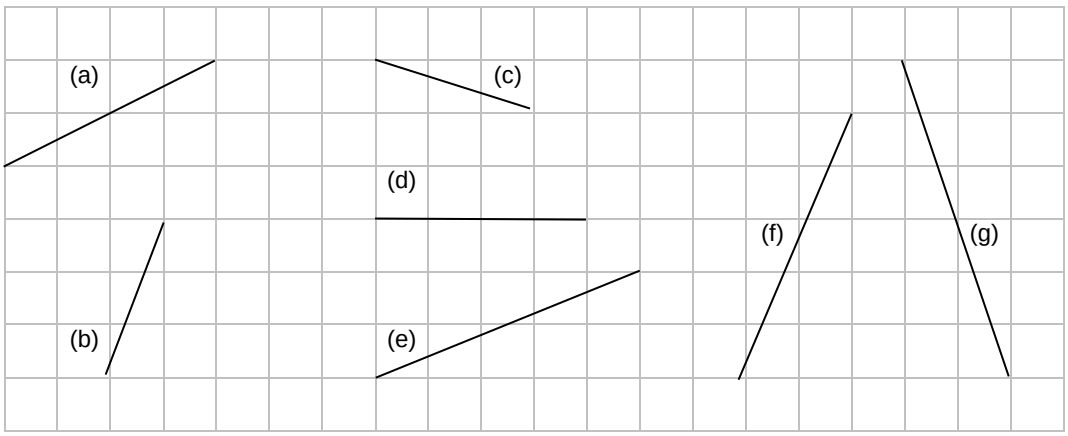


Q2 On grid paper, draw lines with the following gradients.

(a) 1 (b) 2 (c) 3 (d) 0.5 (e) 0.1

(f) 1.5 (g) –1 (h) –2 (i) –0.2 (j) 0

Q3 Find the gradient of each of the following lines.



Converting between slope angle and gradient

The steepness of any line can be given as either a slope angle or as a gradient. A scientific calculator has a button for converting from slope angle to gradient and vice versa. It is the 'tan' button that you have used in trigonometry.

Suppose a line has a slope angle of 12°. We can find its gradient by pressing tan, then 12. We should get roughly 0.213.

The inverse function of the tan button ($\tan^{-1}$) will convert the other way, i.e. it will change a gradient to a slope angle. If we press $\tan^{-1}$ 0.213, we get roughly 12, as we would expect.

Try to convince yourself of why this works.

The importance of gradient

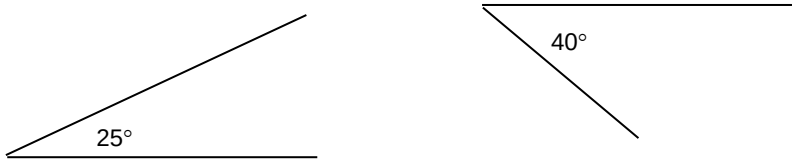
Gradient probably doesn't seem like a terribly important idea, but it is a lot more important than it might seem at first sight. It is used a lot in algebra where it is used to describe the steepness of a graph. It is also the basis of calculus which you will study if you do Level 6. As such, it is essential to much of higher mathematics. So it is important to get a good feel for it.

Practice

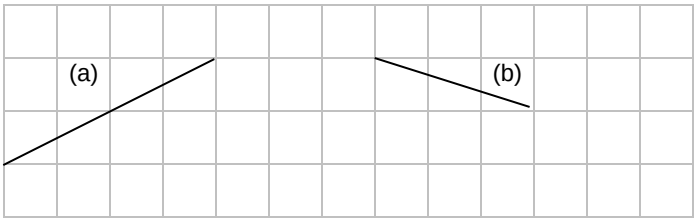
Q4 Copy and complete the following table showing corresponding slope angles and gradients

Slope angle	gradient
15º	0.268
30º	
–10º	
	0.4
	1.9
	5
	–0.8
29.3º	
	0
72 º	
	–4.17
49.28 º	
89º	
90º	
105 º	

Q5 Use your calculator to find the gradient of these sloping lines.



Q6 Use your calculator to find the slope angles of these lines.



Q7 Does the tan button convert between slope angle and gradient properly if the gradient is negative?

Q8 What does the calculator give as the gradient for a slope angle of 90°? Why?

Q9 How about slope angles greater than 95°? Explain.

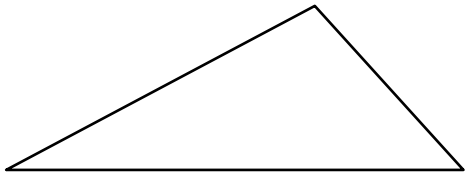
Solve

Q51 A plank is 3 m long. One end is on the ground. How high would the other end need to be lifted for the plank to have:

(a) a slope angle of 50°?

(b) a gradient of 0.8?

Q52 The base of this shape is horizontal and 1.2 m long. The other two sides have gradients of 0.5 and −1. Find the lengths of the other two sides.



Q53 A line with gradient ½ passes through the points (−4, −1) and (6, a). What is the value of a?

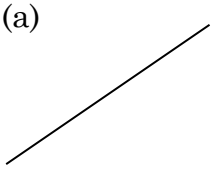
Q54 A line with gradient m passes through the points (a, b) and (c, d). Write a formula for d in terms of the other variables.

Revise

Revision Set 1

Q61 Estimate the slope angle and the gradient of each of these lines.

(a)

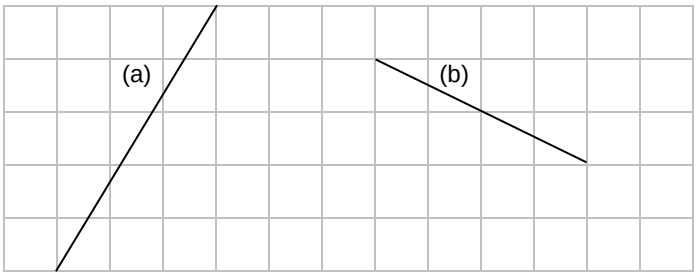


(b)

Q62 On grid paper, draw lines with the following gradients.

(a) 1.5 (b) –0.6 (c) 0

Q63 Find the gradient of each of the following lines.



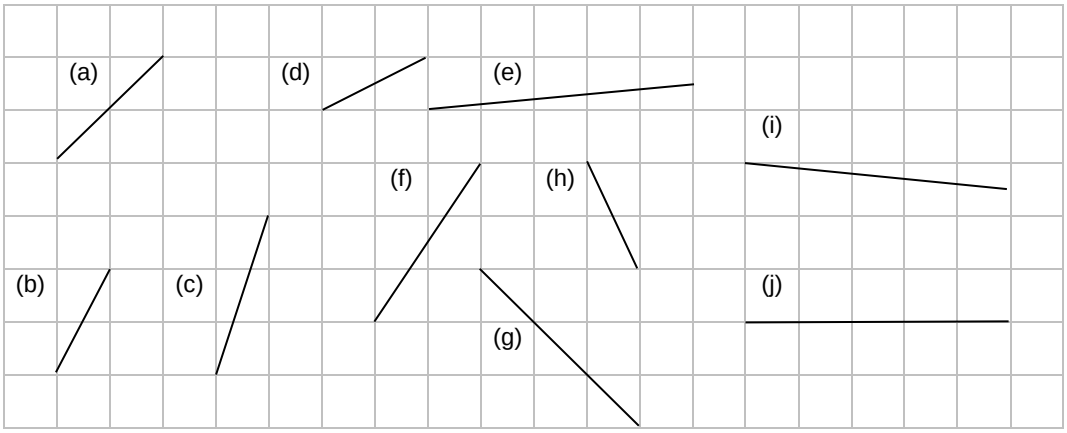
Q64 What is the slope angle of a line with gradient 0.5?

Q65 What is the gradient of a line with slope angle 78.2°

Answers

Q1 (a) 43°, 0.93 (b) 6°, 0.10 (c) 76°, 4.0 (d) –31°, –0.60

Q2



Q3 (a) ½ (b) 3 (c) $-\frac{1}{3}$ (d) 0 (e) 0.4
(f) 2.5 (g) –3

Q4

Slope angle	gradient
15º	0.268
30º	0.577
–10º	–0.176
21.8 º	0.4
62.2 º	1.9
78.7 º	5
–38.7 º	–0.8
29.3º	0.561
0 º	0
72 º	3.078
–76.4 º	–4.17
49.28 º	1.162
89º	57.3
90º	–
105 º	–3.73

Q5 0.466, –0.893

Q6 (a) 26.6º (b) –18.4º

Q7 Yes

Q8 Error, because it is infinite (not defined).

Q9 It gives a negative gradient because after going past 90 º, the line slopes the other way.

Q51 (a) 2.30 m (b) 1.08 m

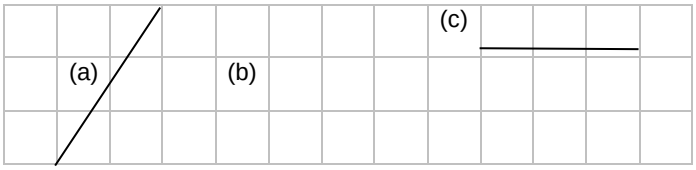
Q52 89.4 cm, 56.6 cm

Q53 4

Q54 $d = b + m(c - a)$

Q61 (a) 34º, 0.67 (b) –8º, –0.14

Q62



Q63 (a) $^5/_3$ (b) $-^1/_2$

Q64 26.6°

Q65 4.79