

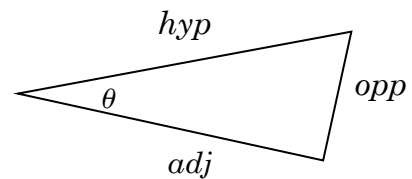
## M3-2 Trigonometry

- using tangent, sine and cosine ratios to find side lengths and angles in right-angle triangles
- solving problems using trigonometry including those involving direction and angles of elevation/depression

[Summary](#) [Learn](#) [Solve](#) [Revise](#) [Answers](#)

### Summary

In a right-angle triangle, relative to a given acute angle,  $\theta$ , the sides are called the hypotenuse, the opposite and the adjacent as shown in the diagram.



Three formulae relate the sides and angles:  $\sin \theta = \frac{opp}{hyp}$ ,  $\cos \theta = \frac{adj}{hyp}$ ,  $\tan \theta = \frac{opp}{adj}$ . The mnemonic *sohcahtoa* can be useful in memorising these.

If two quantities are known (2 sides or a side and an angle), then any third quantity can be found using one of these formulae. We choose the one which involves the 3 quantities involved in the question – 2 knowns and 1 unknown. We sub the known quantities into the formula and solve the resulting equation.

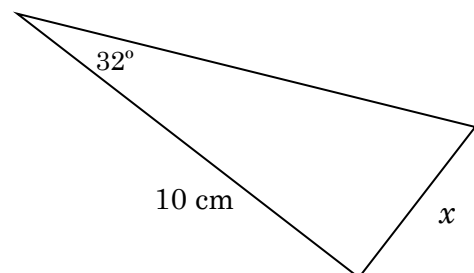
Practical problems can be solved by drawing a diagram, identifying a right-angle triangle in the diagram and applying the above method to it.

### Learn

#### Type 1 Problems - Unknown Side on the Top

Suppose we have the right-angle triangle shown to the right. Knowing one of the side lengths and one of the angles, we can calculate the length another side, like the one marked  $x$ .

We will now go through the steps used to do so.



#### Step 1

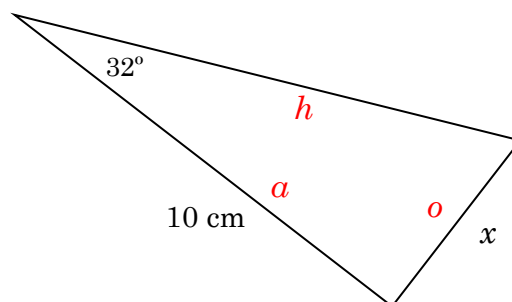
First we have to decide which sides are the hypotenuse, the opposite and the adjacent.

The **hypotenuse** is the longest side, the side opposite the right angle, the side that is not at right angles to any other side.

The **opposite** side is the side which is opposite the known angle, i.e. furthest away from it.

The **adjacent** side is other one, the side which is adjacent to (next to) the known angle.

We often use  $h$ ,  $o$  and  $a$  as abbreviations for hypotenuse, opposite and adjacent. It can be worth writing  $h$ ,  $o$  and  $a$  on your diagram to remind you which is which.



## Step 2

Then we decide which two sides are involved in the calculation: these are the one we know and the one we have to find out. The one we know is the adjacent and the one we have to find out is the opposite. In this case, the **opposite and adjacent** are involved.

## Step 3

Then we decide which trigonometric ratio to use. The choice is  $\sin$  (short for sine),  $\cos$  (short for cosine) and  $\tan$  (short for tangent). You will find three buttons with these names on your calculator.

Now the  $\sin$  of an angle in a right-angle triangle is the length of the opposite side divided by the length of the hypotenuse; the  $\cos$  of the angle is the length of the adjacent side divided by the length of the hypotenuse and the  $\tan$  of the angle is the length of the opposite side divided by the length of the adjacent side.

$$\sin \theta = \frac{o}{h} \qquad \cos \theta = \frac{a}{h} \qquad \tan \theta = \frac{o}{a}$$

[ $\theta$ , pronounced *theta* is a Greek letter which makes a *th* sound. Mathematicians often use it for angles.]

We pick the trigonometric ratio ( $\sin$ ,  $\cos$  or  $\tan$ ) which has in its equation the same sides as the sides we decided were involved in Step 2. They were the opposite and adjacent.  $\tan$  has  $o$  and  $a$  in its equation, so we choose  $\tan$  and we write:

$$\tan \theta = \frac{o}{a}$$

A good way to remember that sin is  $o/h$  etc. is to remember the word

## ***sohcahtoa***

The letters stand for  $\sin = o/h$ ,  $\cos = a/h$ ,  $\tan = o/a$ .

Another way is to use the mnemonic 'some old hags can always hide their old age', or maybe 'some old hippie caught another hippie tripping on acid', or even 'sex on hard concrete always hurts; try other alternatives'.

### **Step 4**

We then substitute the values we know into our equation. The known angle in the triangle is  $32^\circ$  and the adjacent side is 10. So we write

$$\tan 32 = \frac{x}{10}$$

### **Step 5**

We then solve the equation. As the right side is  $x$  divided by 10, we multiply both sides by 10 and get

$$10 \tan 32 = x$$

Then we enter  $10 \tan 32$  into the calculator to get 6.25

$$6.25 = x$$

So the length of the side marked  $x$  is 6.25 cm.

### **Step 6**

Make sure the answer looks about right for the diagram. This step is easily overlooked, but is worth the two seconds it takes to make sure we haven't made a mistake.

### **Working**

Our completed working should look like this:

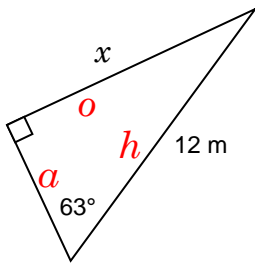
$$\tan \theta = \frac{o}{a}$$

$$\tan 32 = \frac{x}{10}$$

$$10 \tan 32 = x$$

$$6.25 = x$$

Here is another example.



$$\sin \theta = \frac{o}{h}$$

$$\sin 63 = \frac{x}{12}$$

$$12 \sin 63 = x$$

$$10.7 = x$$

One thing you have to check when using the sin, cos or tan button is that the calculator is set to measure angles in degrees rather than radians or grades. Scientific calculators will often have a small DEG, RAD or GRAD at the top of the screen. It needs to show DEG. Changing between these modes is different on different calculators. If you cannot work out how to do it, your manual or your teacher will probably be able to help you.

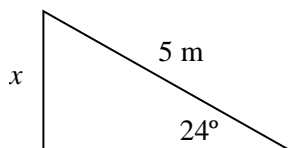
A quick way to check that the calculator is in degrees if you cannot see from the screen is to press tan 45. If you get 1, then it is in degrees. If you get a decimal fraction, then it isn't and needs to be changed.

Check your calculator is in degrees, then do the following practice questions. It is worth doing them all, showing working like above.

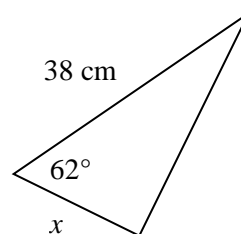
## Practice

Q1 Find the length of the side  $x$  in these right-angle triangles.

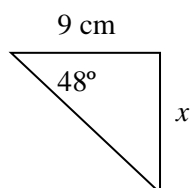
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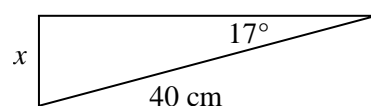
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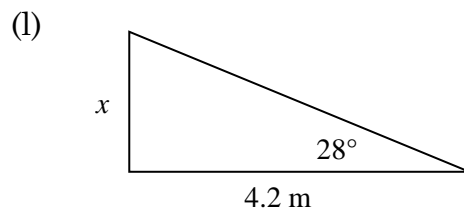
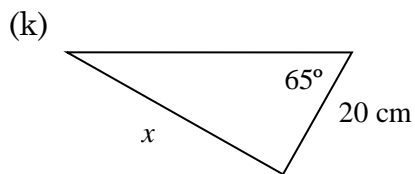
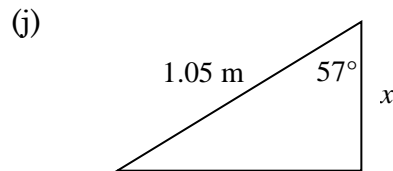
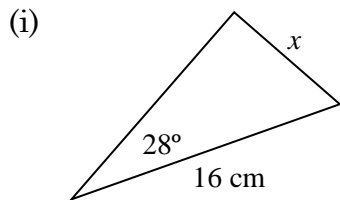
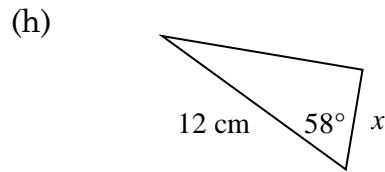
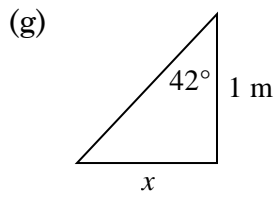
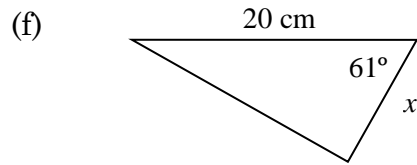
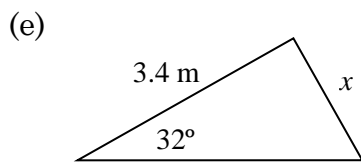


(c)

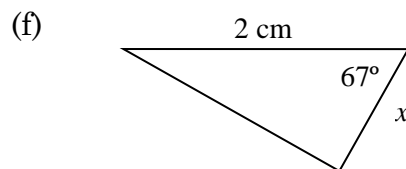
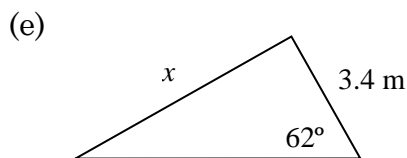
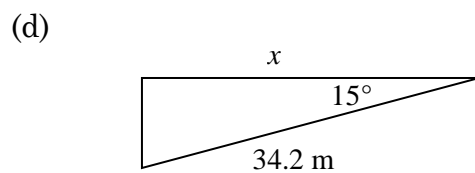
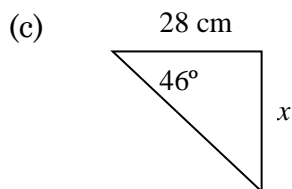
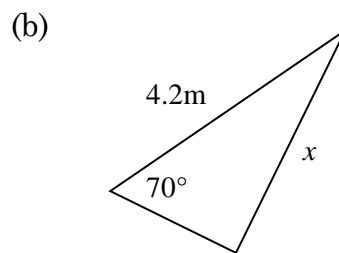
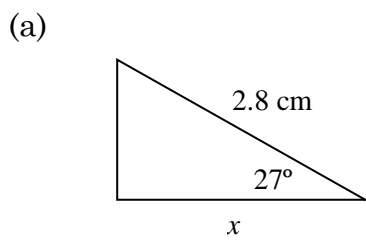


(d)

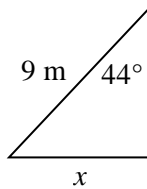




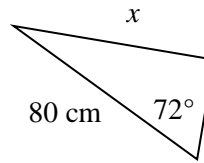
Q2 Find the length of the side  $x$  in these right-angle triangles.



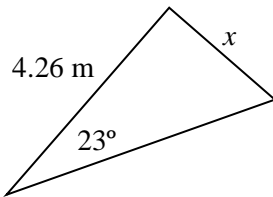
(g)



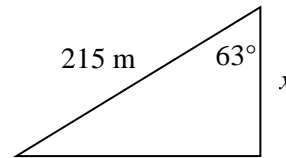
(h)



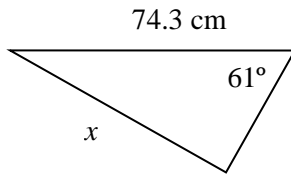
(i)



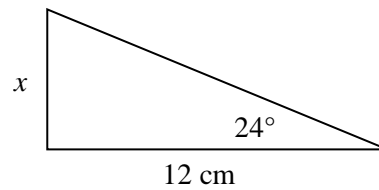
(j)



(k)



(l)



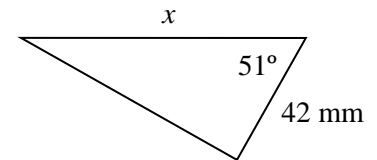
## Type 2 Problems - Unknown Side on the Bottom

In Questions 1 and 2, the unknown side was always on the top of the fraction in the equation. For example, for Q2 part (k), the finished equation would have been

$$\sin 61 = \frac{x}{74.3}$$

But look at the triangle to the right. The equation from this would be

$$\cos 51 = \frac{42}{x}$$



Here the unknown side is on the bottom of the fraction.

This can still be solved, though the steps are a bit different. First we multiply both sides by  $x$  to get

$$x \cos 51 = 42$$

Then we divide both sides by  $\cos 51$  to get

$$x = \frac{42}{\cos 51}$$

Then we put  $42 \div \cos 51$  into the calculator and get 66.7.

This two-step solution is actually equivalent to just swapping over the  $\cos 55$  and the  $x$  in the original equation. You can use this as a shortcut if you prefer. Your working will then look like this:

$$\cos \theta = \frac{a}{h}$$

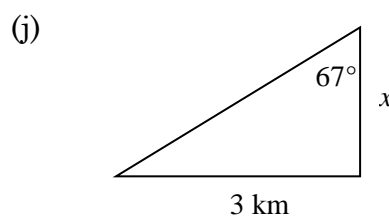
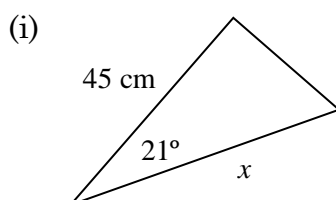
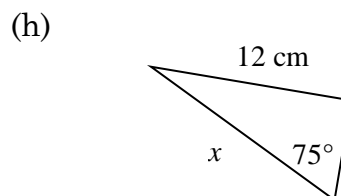
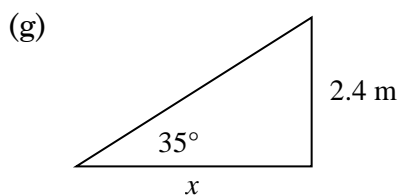
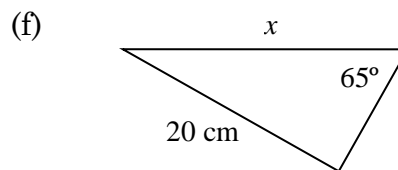
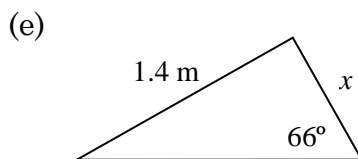
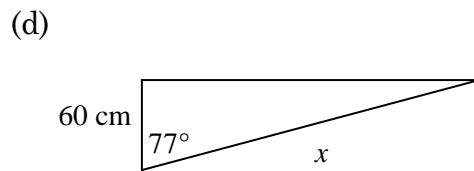
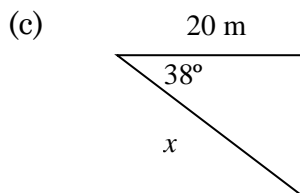
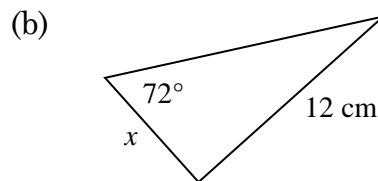
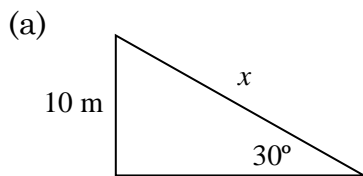
$$\cos 51 = \frac{42}{x}$$

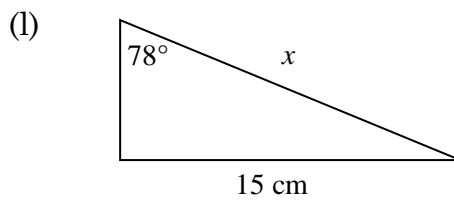
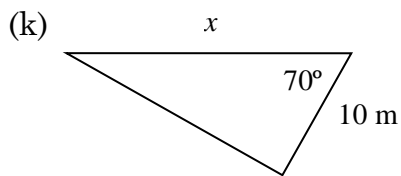
$$x = \frac{42}{\cos 51}$$

$$x = 66.7$$

## Practice

Q3 Find the length of the side  $x$  in these right-angle triangles. The unknown will be on the bottom of the equation.

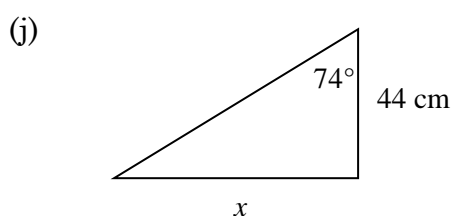
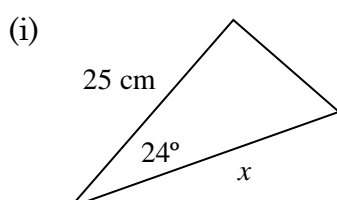
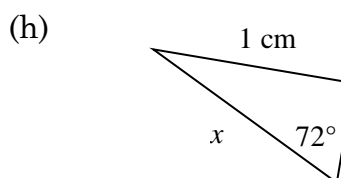
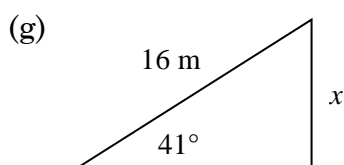
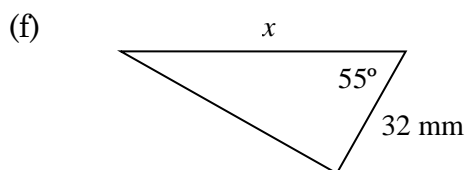
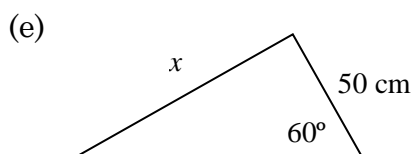
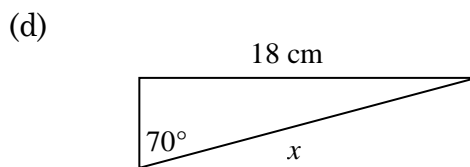
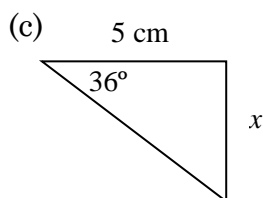
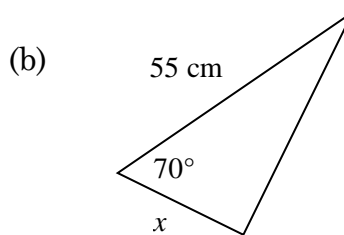
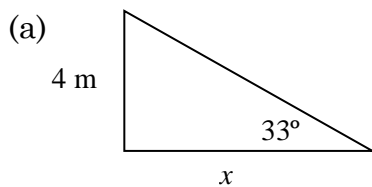




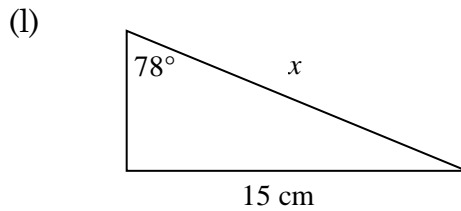
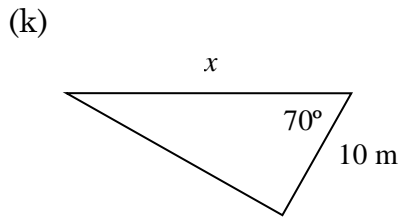
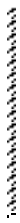
The next practice set has some questions that will have the unknown on the top of the fraction and some that will have it on the bottom.

### Practice

Q4 Find the length of the side  $x$  in these right-angle triangles. The unknown may be on the top or bottom of the equation.



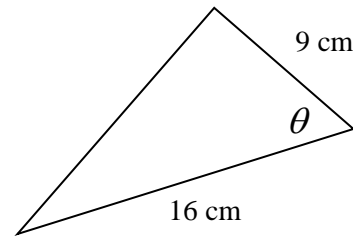




### Type 3 Problems - Unknown Angle

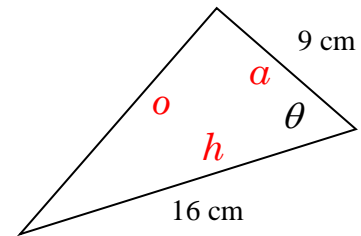
It is also possible to find an angle in a right-angle triangle if we know the lengths of two of the sides.

Consider the triangle to the right. We have to find the angle marked  $\theta$ . The method is fairly similar to that used for the Type 1 and Type 2 problems.



#### Step 1

Label the sides as hypotenuse, opposite and adjacent relative to the angle  $\theta$ .



#### Step 2

Decide which sides are involved. This time it is two known sides,  $a$  and  $h$ .

#### Step 3

Decide which trig ratio to use:  $\cos$ . So write:

$$\cos \theta = \frac{o}{a}$$

#### Step 4

Sub in the known values to get:

$$\cos \theta = \frac{9}{16}$$

#### Step 5

Solve the equation. In the previous problems we used the calculator to give us the  $\cos$  or whatever of an angle and we used the  $\cos$  button. Now we know the  $\cos$  and want the calculator to give us the angle. This is the inverse operation. So we press the inverse or shift or 2nd function button on the calculator, then the  $\cos$  button. The inverse is called  $\cos^{-1}$  (pronounced *cos minus 1* or *inverse cos* or occasionally *arc cos*) and this will be written above the  $\cos$  button.

So we press Inverse cos (  $9 \div 16$  ) = (or EXE)

This should give 55.8. So the required angle is 55.8°.

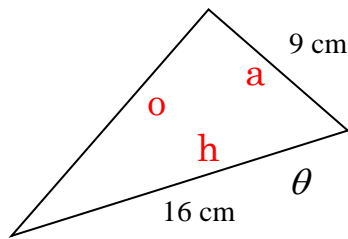
Note that the  $9 \div 16$  has to be in brackets. This is because the calculator will do operations like sin, cos and tan before multiplication and division. So, if we omitted the bracket it would find the inverse cos of 9 and then divide that by 16. Actually, it would give ERROR because no angle has a cos of 9.

### Step 6

Make sure the answer looks about right for the diagram.

### Working

Your working should look something like this:



$$\cos \theta = \frac{o}{a}$$

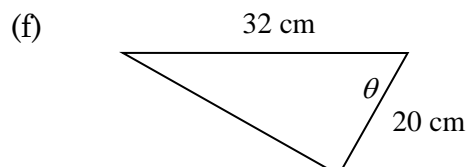
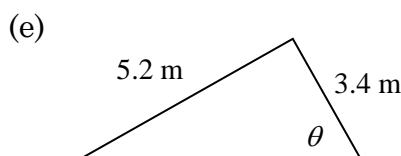
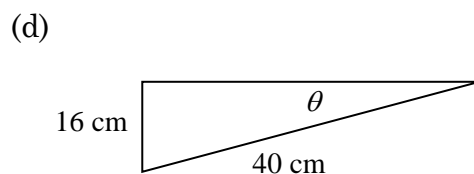
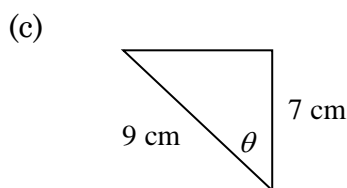
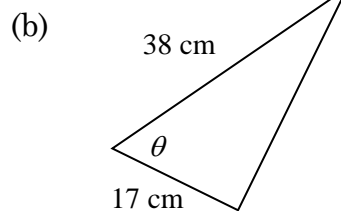
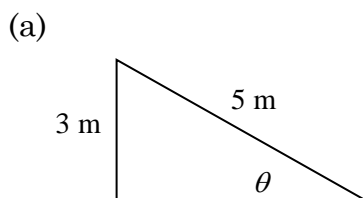
$$\cos \theta = \frac{9}{16}$$

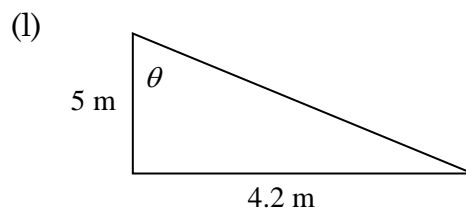
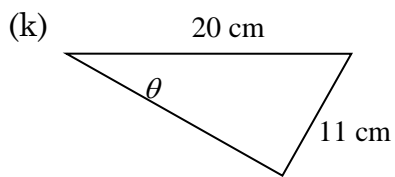
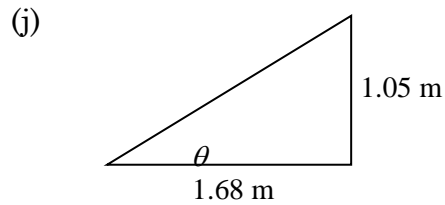
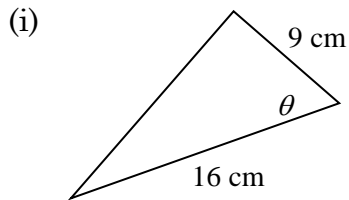
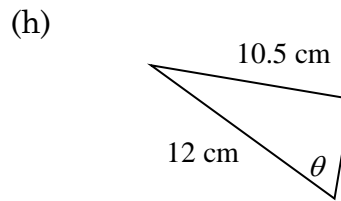
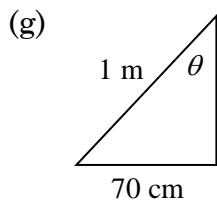
$$\theta = \cos^{-1} \frac{9}{16}$$

$$\theta = 55.8^\circ$$

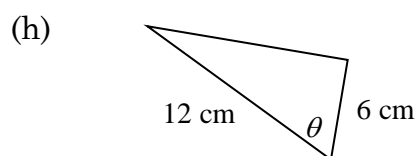
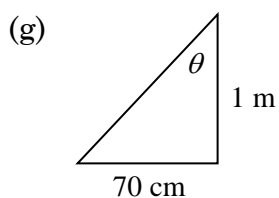
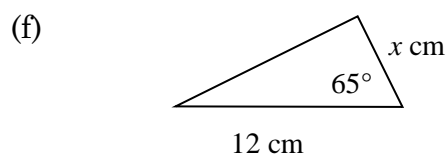
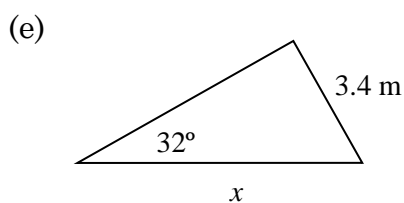
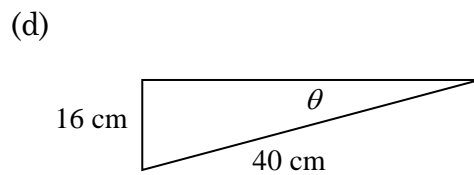
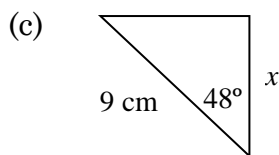
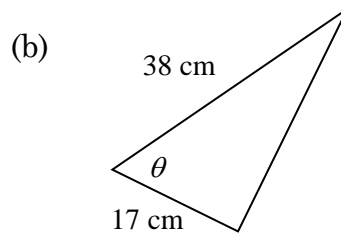
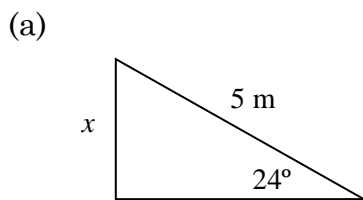
### Practice

Q5 Find the angle labelled  $\theta$  in each of these right-angle triangles.



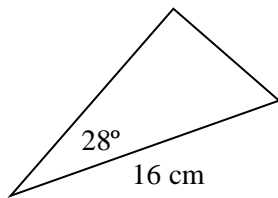


Q6 Find the side labelled  $x$  or the angle labelled  $\theta$  in each of these right-angle triangles.

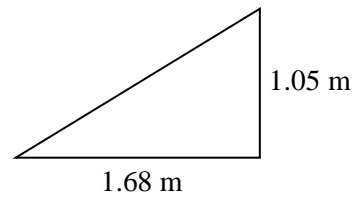


Q7 Find all the unmarked sides and angles in these right-angle triangles.

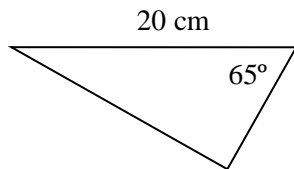
(a)



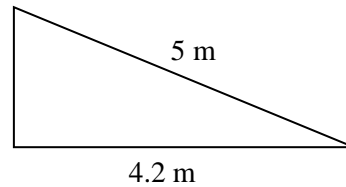
(b)



(c)



(d)



## Word Problems

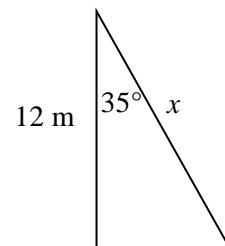
Now you have mastered the techniques of trigonometry, we will learn to apply them to practical situations described in word problems. Here is an example:

A  $12\text{ m}$  high aerial is held up by wires attached to the top of the aerial and to the ground some distance from the base of the aerial. If the angle between the wires and the pole is  $35^\circ$ , how long is the wire?

This is a solution. The same procedure should be applied to all trig word problems.

### First

Draw a diagram. You must draw a diagram, firstly so you can see the right-angle triangle you are dealing with, and secondly because not doing so is very poor communication. The diagram only needs to show the features necessary for getting a solution. It might look like this.



### Then

Find  $x$  by one of the methods you have learnt.

$$\cos \theta = \frac{o}{a}$$

$$\cos 35 = \frac{12}{x}$$

$$x = \frac{12}{\cos 35}$$

$$x = 14.65\text{ m}$$

## Then

Answer the question in words:

*The length of the wire is 14.65 m.*

## Finally

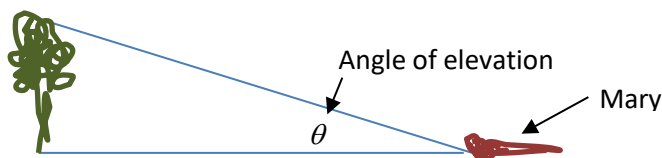
Make sure the answer looks about right in the diagram and seems about right for the question and you're doing.

Note that we assumed that the ground was level even though it wasn't specified. When solving maths problems with incomplete data, it is normal to assume the simplest case. Level ground was simpler than sloping ground. And besides, if the ground was sloping, we wouldn't be able to get an answer.

## Practice

Q8 Solve the following trigonometric word problems. Don't forget to draw a diagram for each question and to make sure your answer seems about right.

- A 15 m high vertical pole is held up by wires attached to the top of the pole and to the ground some distance from the base of the pole. If the angle between the wires and the pole is  $40^\circ$ , how long are the wires?
- A 10 m high aerial is held up by wires attached to the top of the aerial and to the ground some distance from the base of the aerial. If the angle between the wires and the pole is  $30^\circ$ , how far is it from the base of the aerial to where the wires meet the ground?
- An 8 m aerial is held up by tight 11 m wires. What is the angle between the wires and the ground?
- Mary lies on the ground and uses a clinometer to find the angle of elevation from the ground to the top of a tree. She finds that it is  $28^\circ$ . She then measures the distance from where she is lying to the tree and finds it to be 44 m. How tall is the tree? [Note that angle of elevation is the angle between the horizontal and the line of sight.]



- Sophie uses a clinometer to find the height of a different tree. But she does it standing up. She stands 30 m from the tree and finds the angle of

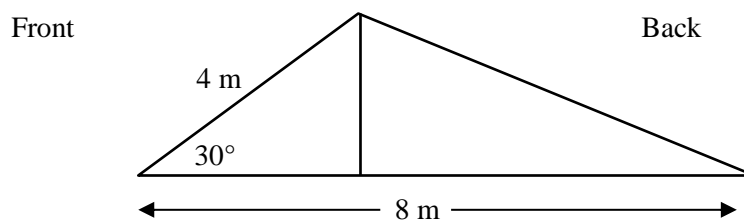
elevation of the top to be  $35^\circ$ . Her eyes and clinometer are 1.5 m above the ground. How tall is her tree?

- (f) A rectangular block of land is 42 m by 20 m. What is the angle between the diagonal and the long side?
- (g) A plane is 4500 m above the ground. Norbert sees the plane at an angle of elevation of  $31^\circ$ . What is the direct distance between Norbert and the plane? How far from Norbert is the point on the ground directly under the plane?
- (h) Nula stood on the top of a 65 m high building and looked at her brother on the street below. The angle of depression was  $52^\circ$ . How far from the base of the building was her brother? [Note that angle of depression is also the angle between the horizontal and the line of sight, but it is called depression rather than elevation when looking downwards.]
- (i) Harriet stood 450 m from a cliff. The top of the cliff was 80 m higher than her eyes. What would be the angle of elevation of the top?
- (j) Jonno was walking along a straight north-south road. He saw a hill on a bearing of  $40^\circ$  (i.e. it was  $40^\circ$  clockwise from north or to the right of north). After walking another 3.6 km north, the hill was due east of him. How far was the hill from the road?
- (k) A 4 m ladder is leaning against a wall with its base 1.8 m from the wall. What is the angle between the ladder and the ground?
- (l) Another ladder is resting against a wall with its top 2.2 m up the wall. The angle between the ladder and the ground is  $52^\circ$ . How long is the ladder?

**Q9** In the following problems you may have to find another angle or length before you can find the one you need. In some cases you can use Pythagoras instead of trigonometry if you wish.

- (a) A vertical pole has two wires running from the top to the ground to hold it up. One makes an angle of  $30^\circ$  with the ground, the other  $40^\circ$ . If the shorter wire is 18.6 m long, how long is the longer wire?
- (b) Jonno is walking along another north-south road. He sees a water tower on a bearing of  $60^\circ$ . After walking another 3.2 km, the tower is due east. What will its bearing be after he walks a further 1.3 km?
- (c) Two 4 m ladders are leant against each other to make an isosceles triangle with the ground. If the bases are 2.4 m apart, what is the angle at the top of the triangle?

- (d) Sezia is standing on top of a building. She sees Dolores on the ground at an angle of depression of  $55^\circ$  and Malady further away on the ground at an angle of depression of  $38^\circ$ . If Dolores is 60 m from the base of the building, how far away from the building is Malady? [The Z rule (alternate angles) might be useful for this one.]
- (e) An isosceles triangle has angles of  $80^\circ$ ,  $50^\circ$  and  $50^\circ$ . If the longest side is 16 cm, how long are the other sides? What is its area?
- (f) Find the area of an equilateral triangle with sides 1 m long.
- (g) A triangular roof truss is 8 m across the base. The sloping side that will go on the front of the house is 4 m long and makes an angle of  $30^\circ$  with the base. How long is the sloping side that will go to the back? What is its angle with the base?



Q Why did the students not believe their maths teacher had been to the beach?

A cos there was no sin of a tan.

## Solve

- Q51 A cylinder with diameter 10 cm is lying on the floor. A rod is laid across the cylinder, perpendicular to its length, with one of its ends on the floor. The rod touches the cylinder 40 cm from where it touches the floor. What angle does the rod make with the floor?
- Q52 A square-based pyramid with a base 20 cm by 20 cm and a height of 8 cm is made from cardboard. The pyramid was then taken apart into a square base and four identical isosceles triangular faces. Find the angles at the corners of the triangular faces.
- Q53 Dean is in a rectangular room 6 m by 4 m and 2.4 m high. He tied a piece of string along the bottom of one of the long walls and another from one end of the first string to the furthest point in the room from that end (where the two far

walls and the ceiling meet). Find the length of the second string and the angle between the two strings.

- Q54 Two sides of a triangle are 22 cm and 19 cm and the angle between them is  $31^\circ$ . Find the length of the third side, the other two angles and the area.
- Q55 Find the area of a regular pentagon with 12 cm sides.
- Q56 Find a formula for the area of a regular polygon with  $n$  sides of length  $s$ .

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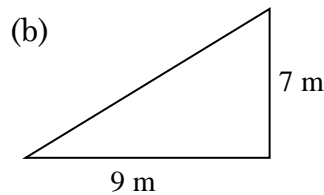
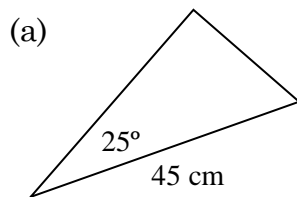
## Revise

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### Revision Set 1

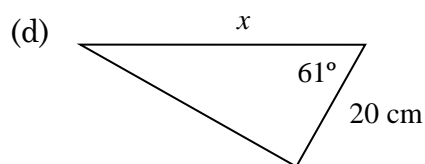
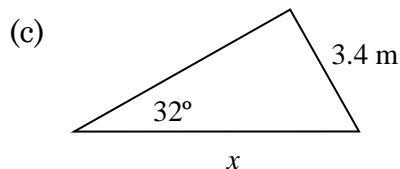
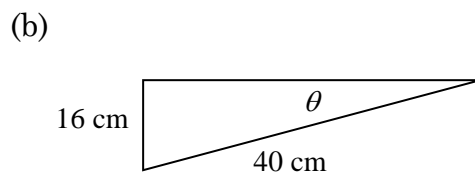
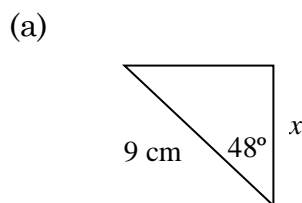
- Q61 Find all the unmarked sides and angles in these right-angle triangles.



- Q62 Garry finds that the angle of elevation of the top of a wall is  $25^\circ$  when he is 15 m out from the wall. His eyes are 1.7 m above the ground. How tall is the wall?
- Q63 Two wires are attached to a vertical pole from the same point on the ground. The first wire makes an angle of  $52^\circ$  with the ground and is attached to the pole 9 m above the ground. The second makes an angle of  $67^\circ$  with the ground. How far above the ground is it attached to the pole?

### Revision Set 2

- Q71 Find the sides labelled  $x$  and the angles labelled  $\theta$  in these right-angle triangles.

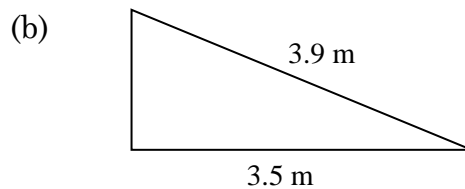
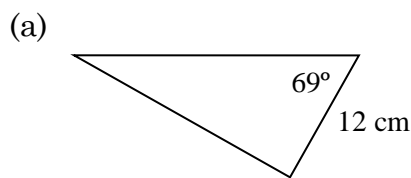




- Q72 A rectangular playground has a short side of 12 m and a diagonal of 22 m. Find the angle between the long edge and the diagonal and the length of the long side.
- Q73 A wire runs from the ground to the top of a 3 m pole, then to the top of a 7 m pole which is 12 m from the 3 m pole. If the total length of the wire is 17.5 m, what angle does the wire from the ground to the shorter pole make with the ground?

### Revision Set 3

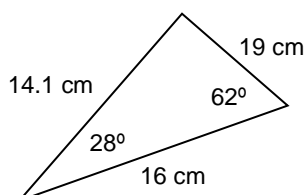
- Q81 Find all the unmarked sides and angles in these right-angle triangles.



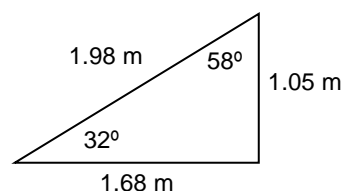
- Q82 Bazza sees a bird at an angle of elevation of  $15^\circ$ . If the bird is 9 m above the ground and his eyes are 1.6 m above the ground, what is the direct distance from his eyes to the bird?
- Q83 Carla is walking along a north-south road. She sees a tree on a bearing of  $52^\circ$ . After walking another 700 m, the tree is due east. What will its bearing be after she walks a further 1100 m?

## Answers

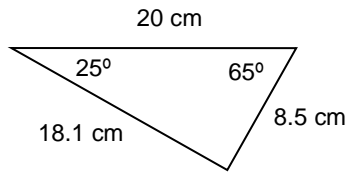
- Q1 (a) 2.033 m (b) 17.84 cm (c) 9.995 cm (d) 11.69 cm (e) 2.124 m (f) 9.696 cm  
 (g) 0.900 m (h) 6.359 cm (i) 7.511 cm (j) 0.571 m (k) 42.89 cm (l) 2.233 m
- Q2 (a) 2.494 cm (b) 3.946 m (c) 28.99 cm (d) 33.03 m (e) 6.394 m (f) 0.781 cm  
 (g) 6.251 m (h) 76.08 cm (i) 1.808 m (j) 97.60 m (k) 64.98 cm (l) 5.342 cm
- Q3 (a) 20.00 m (b) 3.899 cm (c) 25.38 m (d) 266.7 cm (e) 0.6233 m (f) 22.07 cm  
 (g) 3.428 m (h) 12.42 cm (i) 48.20 cm (j) 1.273 km (k) 29.24 m (l) 15.34 cm
- Q4 (a) 6.159 m (b) 18.81 cm (c) 3.633 cm (d) 16.91 cm (e) 86.60 cm (f) 55.79 mm  
 (g) 10.50 m (h) 1.051 cm (i) 27.37 cm (j) 153.4 cm (k) 29.24 m (l) 14.67 cm
- Q5 (a)  $36.9^\circ$  (b)  $63.4^\circ$  (c)  $38.9^\circ$  (d)  $23.6^\circ$  (e)  $56.8^\circ$  (f)  $51.3^\circ$   
 (g)  $44.4^\circ$  (h)  $61.0^\circ$  (i)  $55.8^\circ$  (j)  $32.0^\circ$  (k)  $33.3^\circ$  (l)  $40.0^\circ$
- Q6 (a) 2.03 m (b)  $63.4^\circ$  (c) 6.02 cm (d)  $23.6^\circ$  (e) 6.42 m (f) 5.07 cm  
 (g)  $35^\circ$  (h)  $60^\circ$
- Q7 (a)



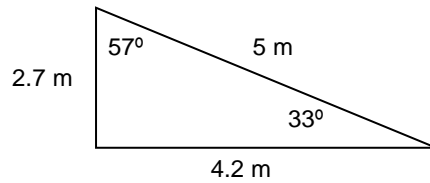
(b)



(c)



(d)



- Q8 (a) 19.6 m (b) 5.77 m (c) 43.3° (d) 23.4 m (e) 22.5 m (f) 25.5°  
 (g) 8737 m, 7489 m (h) 50.8 m (i) 10.1° (j) 3.02 km (k) 63.3°  
 (l) 2.79 m
- Q9 (a) 23.9 m (b) 103° (c) 34.9° (d) 110 m (e) 12.4 m, 76.3 m<sup>2</sup>  
 (f) 0.433 m<sup>2</sup> (g) 4.96 m, 23.8°

Q51 28.1°

Q52 52.0°, 52.0°, 76.0°

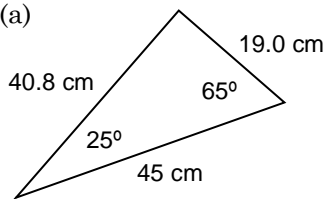
Q53 7.6 m, 37.9°

Q54 11.3 m, 59.7°, 89.3°, 108 cm<sup>2</sup>

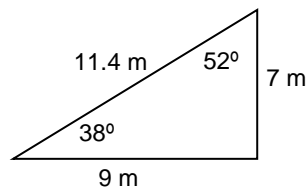
Q55 248 cm<sup>2</sup>

Q56  $\frac{ns}{2 \tan^{-1}(180/n)}$

Q61 (a)



(b)



Q62 8.69 m

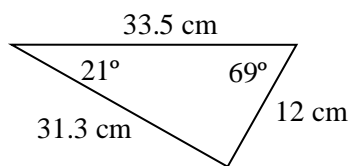
Q63 16.6 m

Q71 (a) 6.02 cm (b) 23.6° (c) 6.42 m (d) 41.3 cm

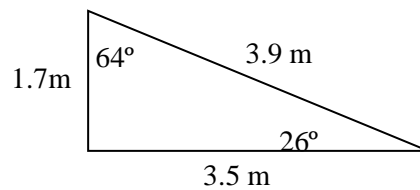
Q72 28.6°, 18.4 m

Q73 31.7°

Q81 (a)



(b)



Q82 32.5 m

Q83 129°