M1 Maths

M3-1 Pythagoras

- using Pythagoras' theorem to find any side of a right-angle triangle in 2- and 3-dimensional situations

Summary Learn Solve Revise Answers

Summary

The longest side of a right-angle triangle is called the hypotenuse. Pythagoras' Theorem states that the square of the hypotenuse is equal to the sum of the squares of the other two sides. This is often written as $h^2 = a^2 + b^2$.

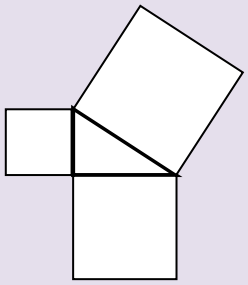
Substituting into this formula and solving allows us to find any side of a right angle triangle if we know the lengths of the other two sides.

In the 3D equivalent, if a rectangular prism has dimensions a, b and c and diagonal length h, then $h^2 = a^2 + b^2 + c^2$.

Learn

Optional Lead-in Activity

Draw a right-angle triangle. Then construct a square on each side of the triangle as in the diagram to the right. Find the area of each square and record them.



Then do the same with some other right-angle triangles and look for a pattern in your results.

Pythagoras' theorem allows us to find the length of the third side of a right-angle triangle if we know the lengths of the other two sides.

The theorem states that, if we draw squares on the three sides of a right-angle triangle like in the diagram in the Lead-in Activity, then the area of the largest square is the same as the areas of the two smaller ones put together.

If we call the side lengths a, b and h, so that h is the longest side (we chose h because the longest side is called the *hypotenuse*), then the areas of the squares are a^2, b^2 and h^2 and we can express the theorem as

$$h^2 = a^2 + b^2.$$

We can then substitute for two of the variables in the formula and solve for the third one.

If the hypotenuse is the unknown and if $a = 5$ and $b = 8$, then

$$h^2 = 5^2 + 8^2$$

We solve like this:

$$h^2 = 25 + 64$$

$$h^2 = 89$$

$$h = \sqrt{89}$$

$$h = 9.43$$

Alternatively, if one of the shorter sides is the unknown and if $a = 3$ and $h = 7$, then

$$7^2 = 3^2 + b^2$$

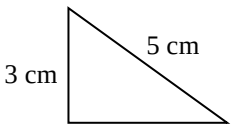
$$49 = 9 + b^2$$

$$40 = b^2$$

$$6.32 = b$$

Q1 Find the length of the third side in each of these right-angle triangles.

(a)



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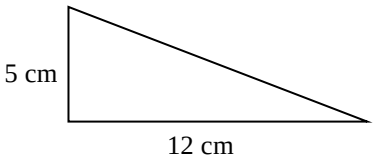

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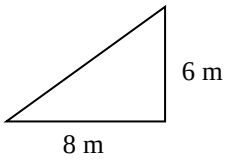

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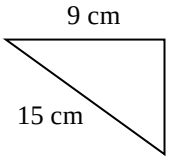

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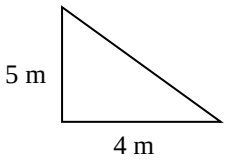

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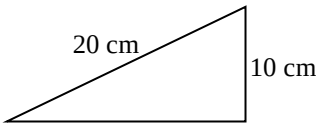

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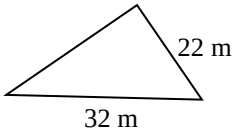
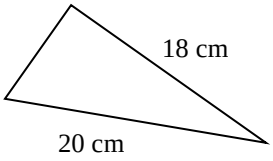


(h)



Q2 Answer these questions by drawing a diagram of the situation, identifying a right-angle triangle, and applying Pythagoras theorem.

(a) How long is the diagonal of a 20 cm by 15 cm rectangle?

(b) The diagonal of a rectangle is 6.2 m. If its length is 5.1 m, what is its width?

(c) A 4-metre ladder is leaning against a wall with its base 1.5 m from the wall. How far up the wall does it reach?

(d) A support wire is tied to the top of a vertical 6 m tall aerial. The other end of the wire is attached to the ground 5 m from the base of the aerial. How long is the wire?

(e) What is the distance between the points (4, 2) and (1, 6)?

(f) What is the distance between the points (– 3, 1) and (5, 0)

(g) An isosceles right-angle triangle has the shorter sides each 12-cm long. How long is the hypotenuse?

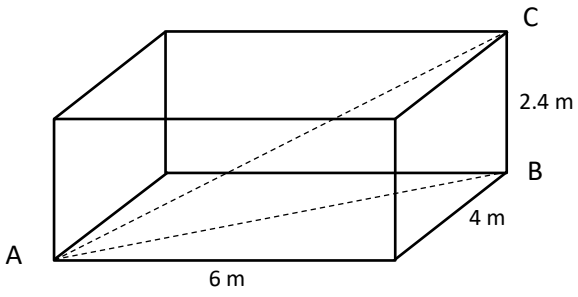
(h) Another isosceles right-angle triangle has a 30-cm hypotenuse. How long are the short sides?

(i) Yet another right-angle isosceles triangle has an area of 200 cm^2. What is its perimeter?

(j) The distance between the points (1, 3) and (4, a) is 5 units. Given that a is positive, find its value.

Pythagoras in 3D

Pythagoras Theorem works in any number of dimensions, not just in 2D. As an example of its use in 3D, consider a room in the shape of a rectangular prism 6 m by 4 m and 2.4 m high. Suppose we wanted to know the distance from the bottom-left-front corner to the top-right-back corner, i.e. from A to C in this diagram.



We can do this two ways.

The first is to use 2D Pythagoras to find the distance AB. This is $\sqrt{6^2 + 4^2}$.

Then we can use 2D Pythagoras again on the triangle ABC to find AC.

$$AC^2 = AB^2 + BC^2$$
$$= 6^2 + 4^2 + 2.4^2$$
$$= 57.76$$

$$AC = \sqrt{57.76} = 7.6$$

The second way is to use the 3D version of Pythagoras which in this case would be

$$h^2 = a^2 + b^2 + c^2$$

$$AC^2 = a^2 + b^2 + c^2$$

$$= 6^2 + 4^2 + 2.4^2$$

$$= 57.76$$

$$AC = \sqrt{57.76} = 7.6$$

You can probably see that these are really the same thing.

Q3 (a) Find the distance between opposite corners of a rectangular room 5 m by 3.2 m by 2.4 m.

(b) What is the longest pencil that can be contained in a rectangular box 20 cm by 15 cm by 10 cm?

(c) In a 3D coordinate system, find the distance between the point (0, 0, 0) and the point (3, 4, 7).

(d) Find the distance between (3, −1, 6) and (5, 2, −1).

(e) A plane is at a position which is 14 km east of the airport, 9 km north of the airport and 4 km above the ground. What is the direct distance from the airport to the plane?

(f) Kerrie walks 12 m west from where Lara is standing, then 18 m north, then climbs a tower. She is then 23.1 m from Lara. How far up the tower is she?

Solve

Q51 Find the area of a regular octagon with 10 cm sides.

Q52 Find the area of an equilateral triangle with sides each 10 cm long.

Q53 Find the surface area of a square-based pyramid with base area 400 cm^2 and height 10 cm.

Q54 Three coins, each with a diameter of 3 cm are laid on a table so that each is touching both the others. Find the area of the space between them.

Q55 How many right-angle triangles with hypotenuse less than 18 have all sides with whole number lengths?

Q56 A room is a rectangular prism 5 m by 4 m by 2 m. A spider is in the corner of the floor and a fly is in the middle of the ceiling. What is the shortest distance the spider must walk to get to the fly?

Q57 A pole is 2 m tall and 50 cm in diameter. One end of a 5-m string is tied to the top, then the string is wrapped around the pole in a spiral and the other end is attached to the bottom. How many times around the pole does it go?

Q58 When you travel 1 minute (1') of latitude, you travel 1 nautical mile which is 1.852 km. How far apart (in kilometres) are two bomb shelters if one is at 28° 47' S 153° 33' E and the other is at 29° 9' S 153° 26' E? [1' is $^1/_{60}$ of 1°]

Q59 (a) In a 4D world, we could use a 4D coordinate system. What would be the distance between the two points (2, 2, 2, 0) and (0, 4, −5, −6)?

(b) Find a formula for the distance between the points (x_1, y_1) and (x_2, y_2) on the Cartesian plane.

(c) Find a formula for the distance between the points (x_1, y_1, z_1) and (x_2, y_2, z_2) in a 3D coordinate system.

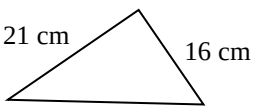
(d) Find a formula for the distance between (x_1, y_1, z_1, t_1) and (x_2, y_2, z_2, t_2) in a 4D coordinate system.

Revise

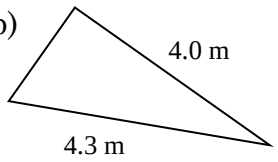
Revision Set 1

Q61 Find the length of the third side of these two right-angle triangles.

(a)



(b)



Q62 What is the length of the diagonal of a rectangle 12 cm by 5 cm?

Q63 What is the distance between the points (3, 1) and (6, −3)?

Q64 A 5-m wire is pulled tight between the top of a pole and the ground 2.8 m from the base of the pole. How tall is the pole?

Q65 What is the longest pole that could be contained inside a rectangular room 4 m by 3 m by 2.4 m?

Answers

Q1.	(a) 4 cm	(b) 13 cm	(c) 10 m	(d) 12 cm
	(e) 6.4 m	(f) 17.3 cm	(g) 23.2 m	(h) 8.7 cm
Q2.	(a) 25 cm	(b) 3.5 m	(c) 3.71 m	(d) 7.81 m
	(e) 5	(f) 8.1	(g) 16.97 cm	(h) 21.2 cm

(i) 68.3 cm (j) 7

Q3. (a) 6.40 m (b) 26.9 cm (c) 8.60 (d) 7.87
(e) 17.1 km (f) 8.1 m

Q51. 483 cm² Q52. 35.4 cm² Q53. 966 cm³ Q54. 1.497 cm²

Q55. 4 Q56. 4.92 m Q57. 2.92 Q58. 42.8 km

Q59. (a) 9.64 (b) $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(c) $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ (d) $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 + (t_2 - t_1)^2}$

Q61. (a) 26.4 cm (b) 1.58 m

Q62 5 cm Q63 5 units Q64 4.14 m Q65 5.55 m