M1 Maths

M1-4 Length, Area and Volume 1

- calculating perimeters of polygons
- calculating areas of rectangles and volumes of rectangular prisms

Summary Learn Solve Revise Answers

Summary

Perimeter means the distance around a 2D shape. A polygon is a 2D shape with all its sides straight. To calculate the perimeter of a polygon, we just add the lengths of all the sides.

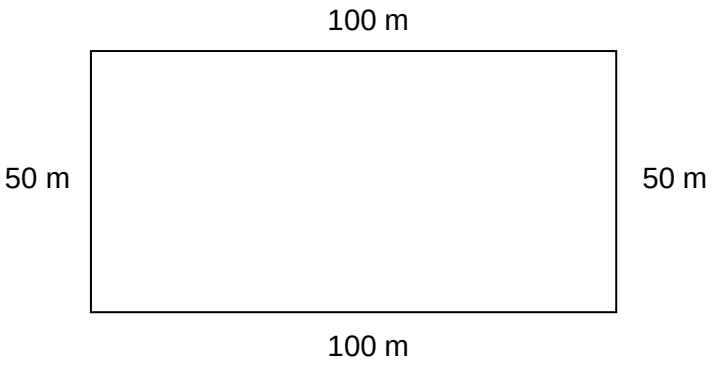
The area of a rectangle can be calculated using *Area* = *length* × *width*.

For the volume of a rectangular prism, use *Volume* = *length* × *width* × *height*.

Learn

Perimeters of polygons

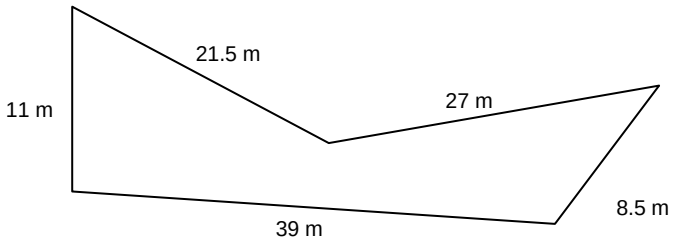
Perimeter means the distance around a shape. For example if a rectangular football field is 100 m long and 50 m wide, to walk around it you would have to walk along each of the four sides. Going round the field shown below clockwise from the top left corner, this would be 100 m, then 50 m, then 100 m, then 50 m. This is a total of 300 m. So the distance around the field is 300 m. In other words the perimeter is 300 m.



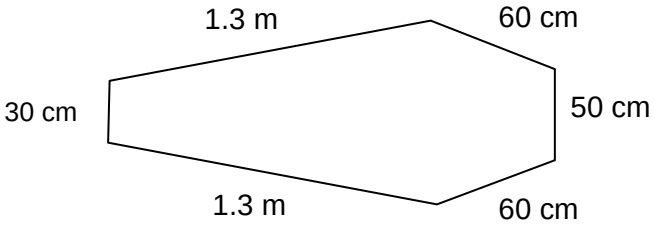
To find the perimeter of any shape, all you have to do is add up the lengths of all the sides. A **polygon** is a shape with straight sides, which makes it easy.

Practice

Q1 Find the distance around the outside of the polygon below.

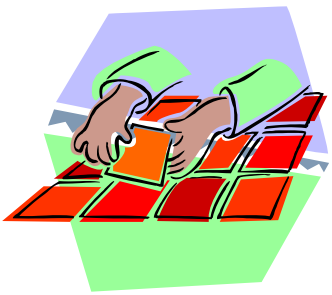


Q2 Find the perimeter of this polygon.



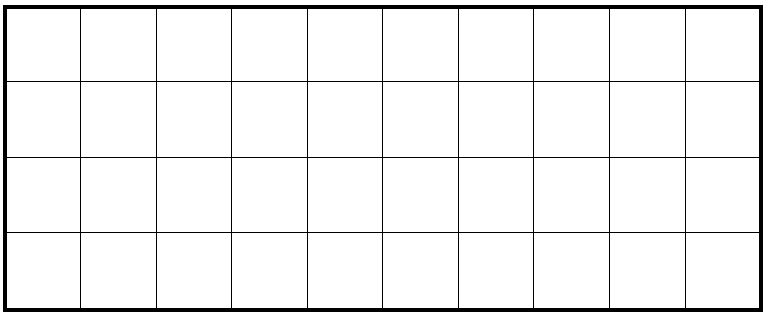
Q3 Find the perimeter of a regular hexagon (6-sides) if all the sides are 2.1 cm long.

Calculating Areas of Rectangles



The area of a rectangle in square centimetres is the number of 1 cm by 1 cm squares needed to cover it.

A 10 cm by 4 cm rectangle can be covered with 1 cm squares like this:



Then we can find the area by counting the squares. There are 40, so the area is 40 cm^2.

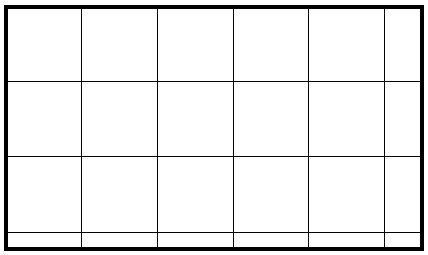
There is a short cut to counting all the squares. That is to count the number in one row and multiply by the number of rows. In this example there are 10 in a row and 4 rows. 4 rows of 10 makes 40 squares.

The number of squares in a row is the same as the length of the rectangle in centimetres. The number of rows is the same as the width of the rectangle in centimetres. So all we really have to do is multiply the length by the width.

Area = length × width

A rectangle 10 cm long and 4 cm wide has an area of 10×4 or 40 cm². A rectangle 15 cm by 9 cm has an area of 15×9 = 135 cm². It would take a long time to draw and count 135 squares, so being able to multiply the length by the width can make things much quicker.

What if the length and width are not whole numbers? For example, a rectangle 5.5 cm by 3.2 cm.



We can add up all the squares and fractions of squares. In this case we get 15 whole squares, 3 half squares (down the right side), 5 smaller bits (0.2) of a square (along the bottom) and one bit which is half of 0.2 of a square, i.e. 0.1 of a square (bottom right corner). This adds up to 15 + 3×0.5 + 5×0.2 + 0.1 = 17.6 cm².

Now if we had just multiplied the length by the width, we would have got 5.5×3.2 = 17.6 cm². So, in this case the *length×width* method would have worked. In fact the *length×width* method will always work, whatever the length and width are. So the area of a 14.279 cm by 6.48 cm rectangle is 14.279×6.48 = 92.52792 cm².

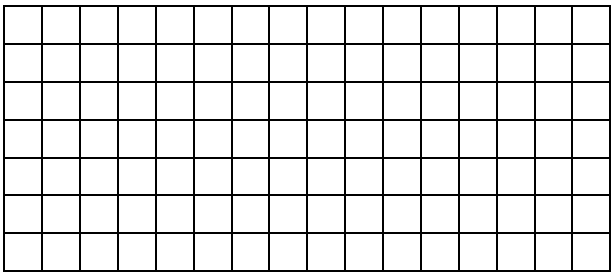
Exactly the same method can be used if the length and width are in metres, except of course that the area is then in square metres. Likewise for any other unit. So the area of a 17 m by 8 m rectangle is 136 m²; the area of a 2.6 km by 4 km rectangle is 10.4 km².

If a rectangle's length and width are given in different units, we just convert one of the units first, so they are both the same. For example if the rectangle is 1 m by 60 cm, we convert the 1 m to 100 cm. Then multiply 100 by 60 to get 6000 cm². (Or we could convert the 60 cm to 0.6 m and get 1 × 0.6 = 0.6 m².)

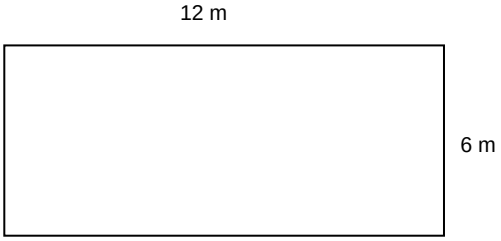
You will note that, although there are 100 cm in a metre, there are not 100 cm² in a m². There are 10 000. Can you see why?

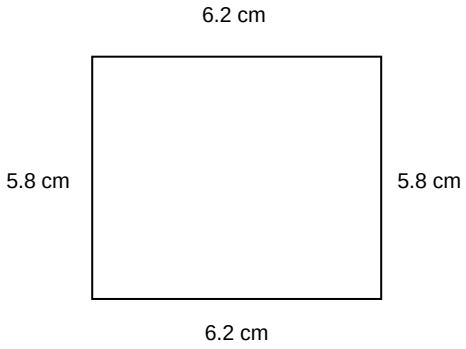
Practice

Q4 Work out how many squares in this rectangle without counting them all.



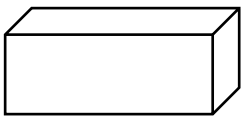
Q5 Calculate the areas of these rectangles.





Calculating Volumes of Rectangular Prisms

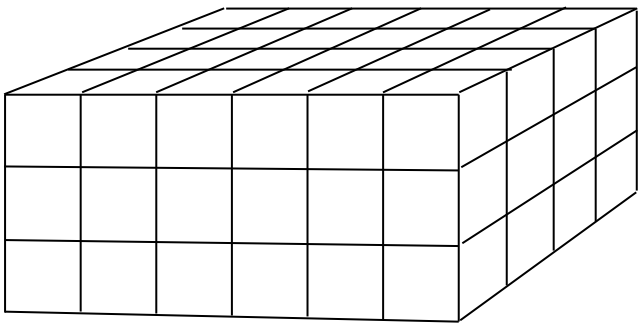
A rectangular prism is a box shape, like this.



The method for finding volumes of rectangular prisms is very similar to the method for finding areas of rectangles.

The volume of a rectangular prism in cm^3 is the number of 1 cm by 1 cm by 1 cm cubes needed to make it.

A 6 cm by 4 cm by 3 cm rectangular prism can be made from 1 cm cubes like this:



In the top layer there are 4 rows of 6 cubes, i.e. 24 cubes. The other two layers also have 24 cubes each, so there are 3×24 i.e. 72 cubes altogether.

Let's call the size of the rectangular prism in the ⟷ direction the length, the size in the ↗↙ direction the width and the size in the ↕ direction the height.

Then we can see that the number of cubes in the top layer is the length × the width and the number of layers is the height. So all we have to do to find the volume is multiply the length by the width by the height.

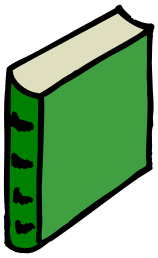
Volume = length × width × height.

It wouldn't matter if we called the ↗↙ size the length and the ⟷ size the width, the volume would still work out the same.

All we have to do is multiply the three dimensions together.

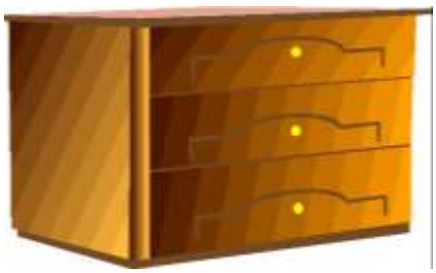
Volume = length × width × height

So a 20 cm by 12 cm by 7 cm rectangular prism has a volume of 20×12×7 = 1680 cm^3. It doesn't matter which of the three measurements is the height, which is the length and which is the width.



As with areas of rectangles, it doesn't matter if some of the dimensions are not whole numbers. The method still works. So the volume of a 3.5 cm by 6 cm by 11.4 cm rectangular prism is 3.5×6×11.4 = 239.4 cm^3.

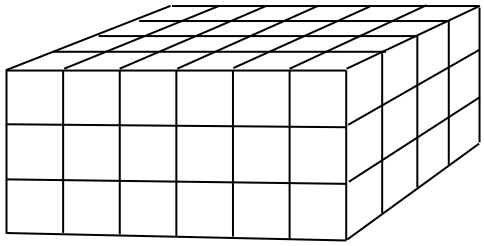
Also the same method can be used if the measurements are in metres or any other unit, except of course that the volume will be in cubic metres or whatever. So the volume of a 3 mm by 5 mm by 6.6 mm rectangular prism is 3×5×6.6 = 99 mm^3.



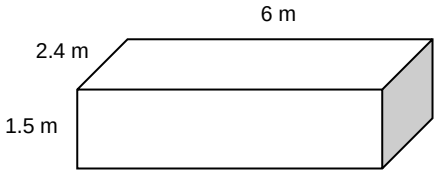
If a rectangular prism's dimensions are given in different units, we just convert them first, so they are all in the same units. For example if the rectangular prism is 2 m by 1.2 m by 75 cm, we convert the 75 cm to 0.75 m (or of course we could convert the metres to centimetres). Then multiply 2 by 1.2 by 0.75 to get 1.8 m^3.

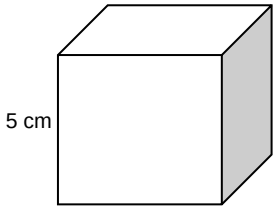
Practice

Q6 How many small cubes are needed to make this rectangular prism?



Q7 Calculate the volumes of this rectangular prism and this cube.



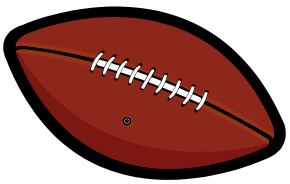


Solve

Q51 In the shape in Q6, how many of the small cubes are completely hidden within the large cube?

Q52 If the shapes in Q7 were made from cardboard, what area of cardboard would be needed for each?

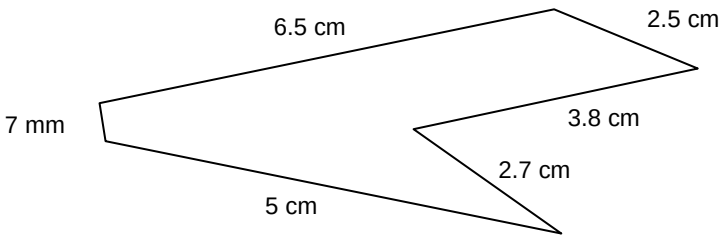
Q53 A football is 30 cm long and 20 cm wide. Estimate its volume.



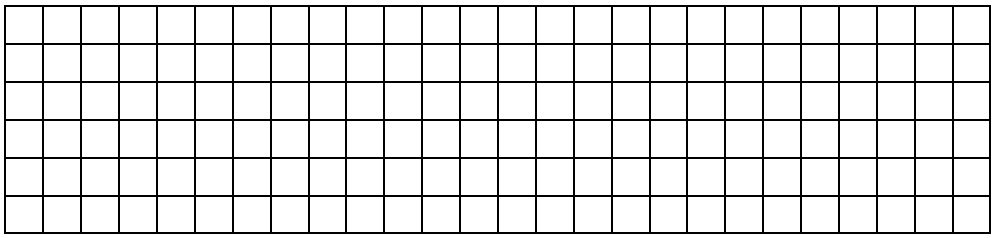
Revise

Revision Set 1

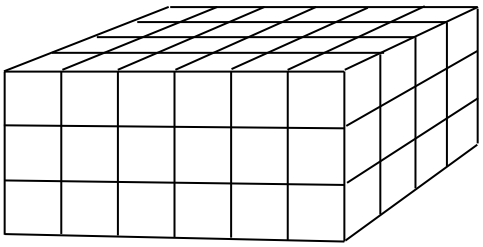
Q61 Calculate the perimeter of this polygon.



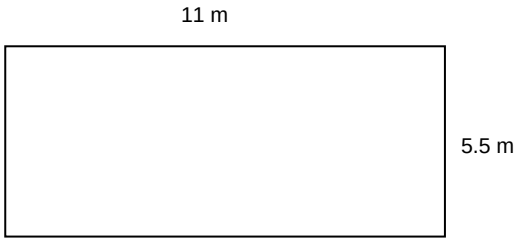
Q62 Work out how many squares in this rectangle.



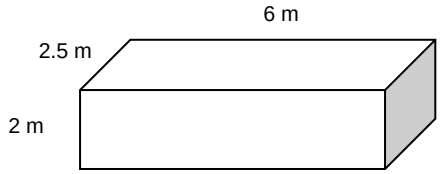
Q63 How many small cubes are needed to make this rectangular prism?



Q64 Calculate the area of this rectangle.

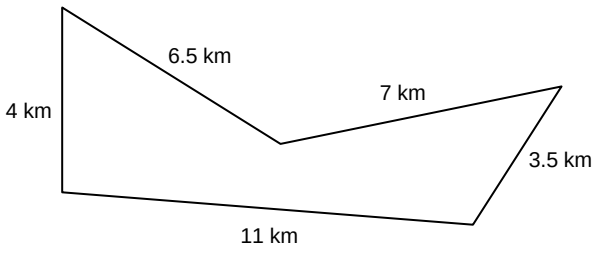


Q65 Calculate the volume of this rectangular prism.



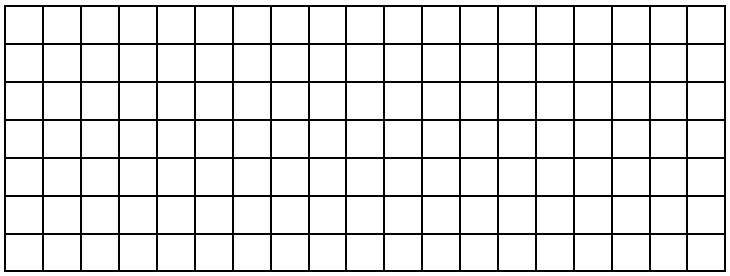
Revision Set 2

Q71 Find the distance around the outside of a pentagon with side lengths as shown on the diagram.

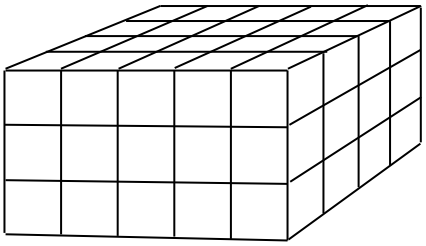


Q72 Find the perimeter of a regular octagon (8 sides) if each side is 0.9 m long.

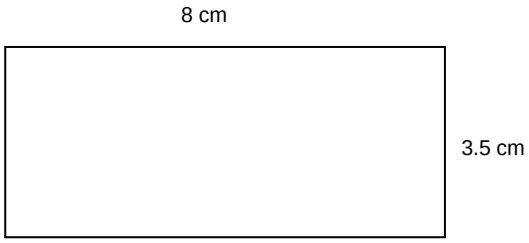
Q73 Work out how many squares in this rectangle.



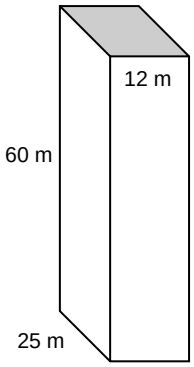
Q74 How many small cubes are needed to make this rectangular prism?



Q75 Calculate the area of this rectangle.



Q76 Calculate the volume of this rectangular prism.



Answers

Q1	107 m	Q2	4.6 m or 460 cm	Q3	12.6 cm		
Q4	112	Q5	72 m^2, 35.96 cm^2	Q6	72	Q7	21.6 m^3, 125 cm^3

Q51	8	Q52	54 m^2, 150 cm^2	Q53	About 6000 cm^3

Q61	21.2 cm	Q62	156	Q63	72
Q64	60.5 m^2	Q65	30 m^3		
Q71	32 km	Q72	7.2 m	Q73	133
Q74	60	Q75	28 cm^2	Q76	18 000 m^3