M1 Maths

M1-1 Dimensions, Size and Mass

- dimensions and measures of size in 1D, 2D and 3D
- measuring and estimating length, area, volume and mass

Summary Learn Solve Revise Answers

Summary

The number of dimensions something has is the number of directions at right angles in which it has size.

A 1D (one dimensional) object, e.g. a line, has size in just one direction. The size of a 1D object is called its length and can be measured for example with a ruler. Length is most commonly measured in metres or in units derived from a metre like millimetres and kilometres.

A 2D object, e.g. a rectangle, has a size in two directions at right angles, e.g. length and width. The size of a 2D object is called its area. Areas can be measured by seeing how many squares of unit area it takes to cover the object. Area is most commonly measured in square metres or in squares of units derived from metres like square centimetres or square kilometres.

A 3D object, e.g. a box, has size in three directions at right angles, e.g. length width and height. The size of a 3D object is called its volume. Volumes can be measured by seeing how many cubes of unit volume it takes to make the object, or, more simply, by immersing the object in water and seeing how much water is displaced. Volume is most commonly measured in cubic metres or in cubes of units derived from metres like cubic centimetres. They can also be measured in litres or units derived from litres, e.g. millilitres or kilolitres.

The mass of an object (sometimes incorrectly called its weight) is not a measure of the object's size, but rather a measure of how much matter it contains. Mass is usually measured using some kind of balance. It can be measured in grams or in units derived from grams like milligrams, kilograms or tonnes.

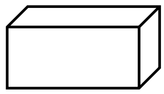
Learn

Dimensions

Something that is **zero-dimensional** (0D) has no size. It is just a point in space, like the corner of a square.

Something that is **one-dimensional** (1D) has a **length** which can be measured in metres, centimetres, millimetres or kilometres, but it has no size in any other direction. A line or a curve is an example.

Something that is **two-dimensional** (2D) has a length, but also a width at right angles to the length. A rectangle is an example. So is a triangle or a circle. The size of a 2D shape is called its **area** and can be measured in square metres, square centimetres, square millimetres, square kilometres or hectares.



Something that is **three-dimensional** (3D) has size in three directions at right angles. It has a length, a width perpendicular to the length and a height or thickness perpendicular to both. A box is a 3D object, so is a sphere and a person. The size of a 3D shape is called its **volume** and can be measured in cubic metres, cubic centimetres, cubic millimetres or cubic kilometres, or in litres, millilitres, kilolitres etc.

Real Dimensions and Practical Dimensions

We would usually measure the size of a sheet of paper in square centimetres. This is because we are interested in how much we can write on it or what size surface it will cover. In actual fact, though, a piece of paper has a length and a width, but also a thickness (even if the thickness is only 0.1 mm or so). So a 30 cm by 20 cm sheet of paper does have a volume: it is about 6 cm^3.

In real terms, the sheet of paper is three dimensional, though in practice we usually treat it as two dimensional. This is because, for practical purposes, like considering how big a picture you could draw on it, we are only interested in the length and the width; the thickness doesn't matter.

Similarly, when we buy a clothes line, we buy it by the metre, not by the cubic centimetre, because we are generally interested in how long it is and not how thick (as long as it is thick enough not to break when we hang clothes on it). In real terms, the clothes line is 3-dimensional; in practical terms it is 1-dimensional.

In most cases we talk in practical terms and consider clothes lines to be 1-dimensional and paper to be 2-dimensional, even though, in real terms, all physical objects are 3-dimensional.

In a similar way, a point is 0-dimensional. The spot we draw on paper to represent the point might in real terms be 3-dimensional, but in practical terms we consider it to be 0-dimensional.

Practice

Q1 What name do we use for the size of things with the following numbers of dimensions?

(a) 1

(b) 2

(c) 3

Q2 How many dimensions do the following have?

(a) a square

(b) a line

(c) a sphere

(d) The top face of a cube

(e) A cardboard box

(f) The vertex of a pyramid

Q3 How many practical dimensions do the following have?

(a) A playground

(b) A tank of water

(c) A clothes line

(d) A cardboard box

(e) The surface of a brick

Length

Length is the size of a 1D object or one dimension of a 2D or 3D object. It is the distance travelled when moving along the object from one end to the other.

Length Units

The basic unit for length is the metre. A metre is one ten-millionth of the distance from the equator to the North Pole. In more helpful terms, it is the length of a metre rule, or about the height of a six-year old, or the height of an adult's belly button above the ground, or the length of a long pace.

We might say that a rope is 2 metres long or that a path is 55 metres long. However, if we are talking about the length of an eye lash, it can be more convenient to use a smaller unit. A centimetre is one hundredth of a metre; a millimetre is one thousandth of a metre and therefore one tenth of a centimetre.

In fact there are a set of prefixes which can be put with any unit, e.g. metre, gram, second, volt, Hertz etc. and the prefixes always mean the same thing. They are as follows.

Prefix	Abbreviation	Meaning	Example
Tera-	T	one trillion (1 000 000 000 000)	e.g. a Terabyte is a trillion bytes
Giga-	G	one billion (1 000 000 000)	e.g. a Gigalitre is a billion litres
Mega-	M	one million (1 000 000)	e.g. a Megabuck is a million bucks
kilo-	k	one thousand (1 000)	e.g. a kilojoule is a thousand Joules
milli-	m	one thousandth of ($^{1}/_{1\,000}$)	e.g. a millimetre is one thousandth of a metre
micro-	µ	one millionth of ($^{1}/_{1\,000\,000}$)	e.g. a microamp is one millionth of an amp
nano-	n	one billionth ($^{1}/_{1\,000\,000\,000}$)	e.g. a nanosecond is one billionth of a second
pico-	p	one trillionth ($^{1}/_{1\,000\,000\,000\,000}$)	e.g. a picofarad is on trillionth of a farad

NOTE the abbreviations for the prefixes. These are standard and should be used as shown, including whether upper or lower case. For example m always means milli and M always means Mega. The standard abbreviation for metre is m. So kilometre is abbreviated to km, millimetre to mm etc.

When writing say 50 millimetres in abbreviated form, we write 50 mm. Note the space between the 50 and the mm.

It is worth taking the time to memorise these prefixes and abbreviations if you don't know them already, because they crop up in so many situations.

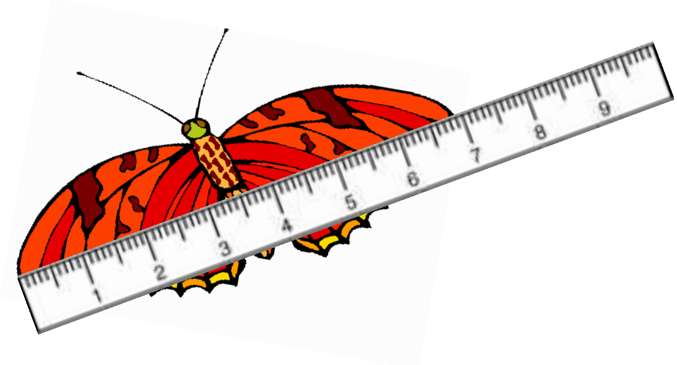
With metres, the commonly used prefixes are milli- and kilo-. But, as you will know, another prefix is sometimes used with metres, though rarely with any other unit. The prefix is centi-, which means hundredth of. The prefix centi- (abbreviation c) is not a standard prefix and so is not used by engineers, builders etc. Builders will give the height of a window as say 1200 mm rather than 120 cm.

So we have the millimetre (which is about the thickness of 10 sheets of paper), the centimetre (10 millimetres or about the thickness of your little finger) and the kilometre (about the distance you would walk in 15 minutes), as well as the metre as commonly used length units. You can see millimetre and centimetre divisions on any ruler.

We usually use the unit which will give the size of what we are measuring as a small number of the units – not a small fraction of a unit and not thousands of the units.

Measuring Length

A common way to measure the length of an object is to take a ruler or other measuring device with centimetres, millimetres or whatever marked on it. We lay it along the thing to be measured. We place the zero of the scale at one end of the object. (Be careful here because, often the zero is not at the end of the ruler, but a few millimetres from the end.) Then we read the scale at the other end of the object.

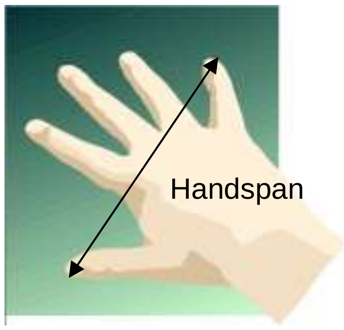


The wing span of this butterfly is about 7 cm. You may be able to read it more accurately as 6.9 cm.

If we are measuring a length which is not straight, e.g. the distance along a curved line, we can lay a tape measure along the curve. If we only have a straight ruler, we can lay a piece of string along the curve, mark on the string where the curve starts and ends, then straighten out the string and measure between the points with the ruler.

We can estimate lengths in a couple of ways if we don't have a measuring device. One is to measure it with parts of our body that we know the lengths of. It is worth knowing a few of these lengths, for example

- our hand span (for most people about 20 cm)
- the length of a finger joint (for most people about 2.5 cm)
- our height (for most people about 1.6 – 1.8 m)
- our pace length (for most people about 80 cm)



If we cannot reach the object we need to estimate the length of, then we just imagine things of known length lined up along it and estimate how many it would take.

Practice

Q4 What is the basic unit for length?

Q5 Write out the table of prefixes and abbreviation without copying. You can leave out the example if you like. If you don't get it all right, learn it better, then redo it.

Q6 Measure the lengths of these lines:

(a) ____

(b) _____

(c) ___________________

(d) ____

(e) –

(f) ________________________________

Q7 Estimate the following lengths. Give your estimates in appropriate units.

(a) the length of your nose

(b) the thickness of a piece of paper

(c) the distance from Brisbane to Cairns

(d) the distance from Sydney to London

(e) the length of your classroom

Area

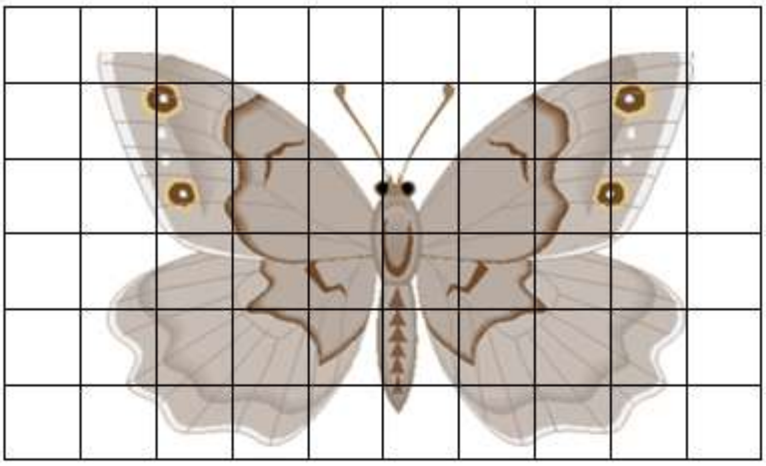
Area is the size of a 2D object or the surface of a 3D object.

Area Units

The units for area are squares. A 1 m by 1 m square is called a square metre (or occasionally a metre squared). A 1 cm by 1 cm square is called a square centimetre. A 1 km by 1 km square is called a square kilometre and so on. The abbreviations for the area units are just the abbreviation for the corresponding length unit followed by a superscripted 2 (as in *squared*). So square metre is m^2, square centimetre is cm^2, square kilometre is km^2 etc. We might say that the area of a sheet of A4 paper is about 600 cm^2.

Measuring Area

One way to measure the area of a shape is to cover it with 1 cm^2 squares or 1 m^2 squares. Making all the squares can be laborious though. A similar method is to use a transparent grid marked with 1 cm^2 squares. We then count the squares which are taken up by the shape.



There are various ways to deal with the part squares around the edge. We can estimate the fraction of each part square taken up, then add them up. But this is laborious. It is usually as accurate just to count all the squares which are more than half taken up and to ignore all the squares which are less than half taken up.

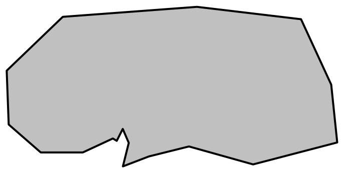
The area of the butterfly worked out this way is 29 cm^2.

Areas can be estimated by picturing in your mind how many square centimetres or square metres it will take to cover it.

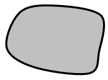
Practice

Q8 Use a grid to measure the areas of the shapes below

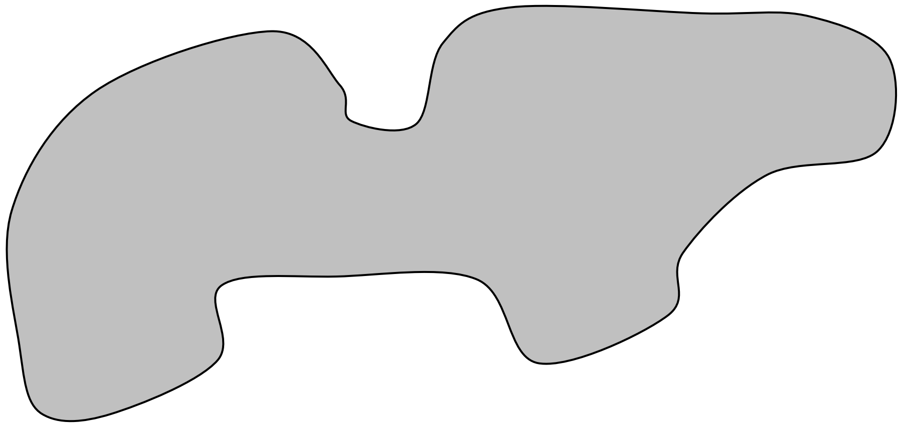
(a)



(b)



(c)



Q9 Estimate the areas of the following. Give your estimates in appropriate units.

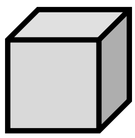
(a) the classroom floor
(b) a postage stamp
(c) the school
(c) Australia
(e) your desk
(f) the top of your head

Volume

Volume is the size of a 3D object.

Volume Units

The units for volume are cubes. A 1 m by 1 m by 1 m cube is called a cubic metre (or occasionally a metre cubed); a 1 cm by 1 cm by 1 cm cube is called a cubic centimetre; and so on.



The abbreviations for the volume units are just the abbreviation for the corresponding length unit followed by a superscripted 3 (as in *cubed*). So cubic metre is m^3, cubic centimetre is cm^3, cubic kilometre is km^3 etc. We might say that the volume of a brick is about 2000 cm^3.

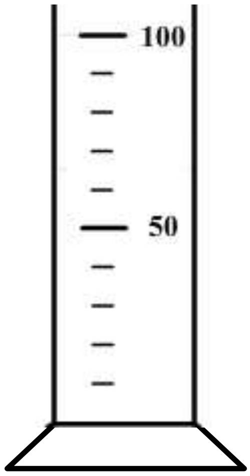
There are actually 1 000 000 cubic centimetres in a cubic metre. [If that doesn't seem right, think of a box 1 m by 1 m by 1 m and imagine filling it with little cubes each 1 cm by 1 cm by 1 cm.] So to avoid using fractions, we would have to write half a cubic metre as 500 000 cm^3.

To get around this, we use another volume unit, the litre (abbreviation L – note always an upper case L). A litre is 1000 cm^3 and 1000 L makes 1 m^3. A litre is the size of a carton of milk.

The prefixes milli-, kilo-, Mega- and Giga- are often used with litre (mL, kL, ML, GL). So we might say that a can of Coke is 375 mL or that a tub of ice cream is 2 L or that the amount of water in a lake is 300 GL.

You might notice that 1 mL is the same as 1 cm^3. The two units are thus interchangeable. In the same way, 1 kL is the same as 1 m^3. Try to remember these two facts – remembering them will be very handy later.

Measuring Volume



The volume of a liquid can be measured by pouring it into a container marked off in cm^3, L or whatever (e.g. a measuring cylinder or graduated bucket).

The volume of a solid object can be measured by filling a container with water so that it is just overflowing, then immersing the object in the water and measuring the amount of water that spills out using a measuring cylinder or other graduated container.

Volumes can be estimated by comparing them in one's mind's eye to a 1 cm^3 or 1 m^3 cube or a litre of milk.

Capacity

Capacity is the volume of fluid a container will hold. Being a volume, it is measured in the units for volume, e.g. cubic centimetres, litres etc.

Note: some people will say that volume should be measured in cm^3, m^3 etc. while capacity should be measured in mL, L, kL etc. This is ***not*** true: both types of units can be used for volume and for capacity.

Capacity is best measured by filling the container with fluid, then pouring the fluid into a graduated container to see how much there is.

NOTE: The word ‘capacity’ can be used in a less mathematical way to mean the amount of anything that something can hold. So the capacity of a stadium might be 20 000 people because there are 20 000 seats (even though you could fit more people in if you piled them up to the roof). The context should tell you in which sense the word is being used. It can even be used in a totally non-mathematical way as in ‘Helen has a high capacity for hard work’, which means that she is very capable of hard work. In this module, we will stick to the more mathematical meaning – the volume of fluid a container will hold.

Practice

Q10 Explain how you would measure the volume of

(a) the milk in a glass

(b) an apple.

Q11 Estimate the volumes of the following. Use appropriate units.

(a) a matchbox

(b) your maths teacher

(c) your classroom

(d) a pea

(e) a large tree

(f) a car

Mass

Mass is not a measure of the size of an object. Instead it is a measure of the amount of matter in it (basically the number of protons, electrons and neutrons in it). We can feel how much mass something has by how much gravity pulls it downwards when we try to hold it.

Small things can have large mass (like a rock or a hunk of metal) and large things can have small mass (like a pillow or a balloon).

Mass Units

The basic unit for mass is the gram. The abbreviation is g. A gram is about the mass of a bean.

The usual prefixes can be used with the unit to get µg, mg, kg, Mg etc.

However, a Megagram is generally called a tonne (abbreviation t). A tonne is 1 000 000 g or 1000 kg.

kt and Mt are used for explosive powers of bombs (a 50 Mt bomb has the same explosive energy as 50 000 000 tonnes of TNT), but for masses of say ore bodies we generally just say so many million tonnes or whatever.

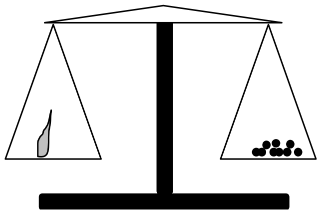
Measuring Mass

There are various devices for measuring the mass of an object.

With spring balances, we place or hang the object on the balance and read the graduated scale.

Electronic balances are similar except that the scale is usually digital which makes reading it easier.

With an old-fashioned balance scale we place the object on the pan on one side of the scale, then put known weights on the other pan until the two pans balance, i.e. sit at the same level. The weight of the object is then the same as the total of the weights on the other pan.



We can estimate the mass of light objects by picking them up. This is called hefting. We compare the feel with the feel of objects whose mass we know, for example with a litre of milk which has a mass of 1 kg or with a bean which has a mass of about a gram.

Objects that we can't pick up, we just have to estimate by looking at them and again deciding whether it is likely to weigh about the same as or more or less than objects whose mass we know. Some useful masses to know are:

an average adult: around 70 kg
a small car: about a tonne
a large truck or bus: around 20 tonnes

It is a useful thing to know that for water.

1 L has a mass of 1 kg
1 mL (or 1 cm^3) has a mass of 1 g
1 kL (or 1 m^3) has a mass of 1 t

The same is approximately true for most common liquids like drinks, blood etc. and also for living things like animals and plants.

Note on Mass and Weight

People often confuse mass and weight. Mass is the amount of matter in an object; **weight** is the amount of gravitational force on the object. If you take a spanner into space, it may become weightless, but its mass doesn't change because the number of protons, electrons and neutrons in it doesn't change.

People commonly talk about the weight of an object when they mean the mass. To confuse things more, finding the mass of an object can legitimately be called ‘weighing it’.

In general, people will know what you mean if you use the wrong word, but you’ll do better on maths and science tests if you get in the habit of using the right word.

Practice

Q12 Estimate the mass of the following. Use appropriate units.

(a) a budgie
(b) a cow
(c) a car tyre
(d) your maths teacher
(e) a bus
(f) a flea

Q13 What is the mass of 300 mL of water?

Q14 Would a metal bar have the same mass if you sent it into space?

Solve

Q51 How many millimetres in a kilometre?

Q52 How many grams in 50 kg?

Q53 How many centimetres in half a kilometre?

Q54 If a computer takes 1 ns to do an operation, how many operations will it be able to do in 5 minutes?

Q55 How many square centimetres in 1 m^2? Careful . . . the answer isn’t 100.

Q56 As mentioned, a metre is one ten-millionth of the distance from the North Pole to the equator. What is the distance right around the world in kilometres?

Q57 Weight is measured in Newtons, a unit of force. A 12 kg object on Earth weighs about 120 Newtons. If gravity is only $^1/_6$ as strong on the moon, what would its mass and weight be on the moon?

Revise

Revision Set 1

Q61 How many practical dimensions are there in a block of land?

Q62 Measure the length of this line:

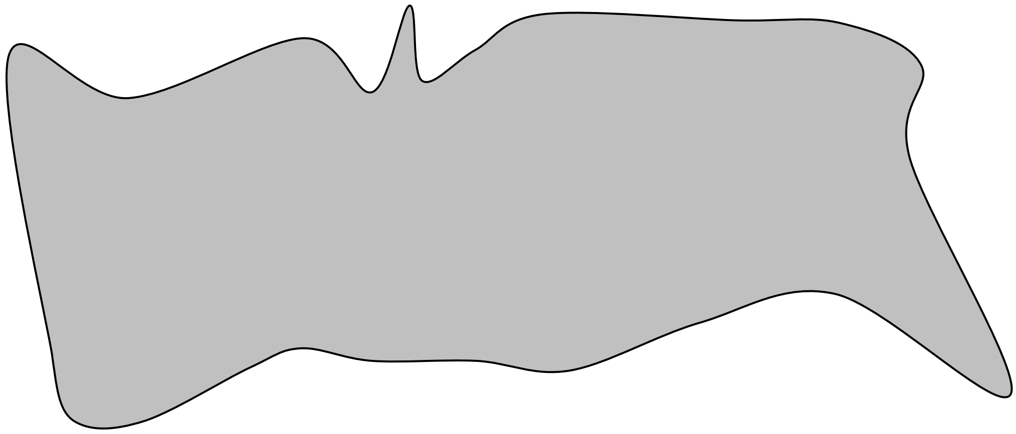
Q63 Estimate the following lengths:

(a) the height of a door

(b) the thickness of your thumb nail

(c) the distance to India

Q64 Use a grid to measure the area of the shape below



Q65 Estimate the areas of the following:

(a) the floor of an average classroom

(b) this letter: O

(c) the palm of your hand (not including the fingers)

Q66 Explain how you would measure the volume of your head.

Q67 Estimate the volumes of the following:

(a) a can of Coke (b) Arnold Schwarzenegger

(c) your classroom (d) an eyelash

Q68 Estimate the mass of the following:

(a) a rabbit (b) a horse

(c) a bus (d) an eyelash

Answers

Q1	(a) length	(b) area	(c) volume		
Q2	(a) 2	(b) 1	(c) 3		
	(d) 2	(e) 3	(f) 0		
Q3	(a) 2	(b) 3	(c) 1	(d) 3	(e) 2
Q4	metre				
Q5	Check against the table in the text				
Q6	If printed, the lengths will be as follows. If measured on a computer screen, they may be different.				
	(a) 10.4 – 10.6 cm	(b) 1.3 – 1.4 cm	(c) 4.9 – 5.0 cm		
	(d) 12.6 – 12.7 cm	(e) 2 – 3 cm	(f) 8.5 – 8.6 cm		
Q7	(a) 3 – 10 cm	(b) 0.02 – 0.2 mm	(c) 1200-2000 km		
	(d) 8 000 – 20 000 km	(e) 5 – 12 m			

Q8 If printed, the areas will be as follows. If measured on a computer screen, they may be different.

(a) 6 – 9 cm^2 (b) 0.8 – 1.2 cm^2 (c) 27 – 37 cm^2

Q9 (a) 40 – 90 m^2 (b) 2 – 10 cm^2 (c) 5 – 50 ha

(d) 100 000 – 10 000 000 km^2 (e) 0.2 – 0.9 m^2

(f) 200 – 600 cm^2

Q10 (a) Pour the milk into a graduated container and read off the volume

(b) Fill a container with water; immerse the apple and catch the water that spills in a graduated container; read off the volume.

Q11 (a) 5 – 30 cm^3 (b) 40 – 120 L (c) 80 – 400 m^3

(d) 0.2 – 1 cm^3 (e) 5 – 50 cm^3 (f) 3 – 20 m^3

Q12 (a) 5 g – 50 g (b) 200 kg – 2 t (c) 5 kg – 30 kg

(d) 40 kg – 120 kg (e) 2 t – 20 t (f) 0.000 01 g – 0.01 g

Q13 300 g

Q14 Yes. Its weight would be less because weight is a measure of the gravitational force on an object. But its mass would stay the same because the number of protons, electrons and neutrons wouldn't change.

Q51 1 000 000

Q52 50 000

Q53 50 000

Q54 300 000 000 000 (3 hundred billion)

Q55 10 000

Q56 40 000 km

Q57 mass: 12 kg, weight: 20 Newtons

Q61 2

Q62 If printed, the length will be 8.8 – 9.0 cm. If measured on a computer screen, it may be different.

Q63 (a) 1.8 – 3 m (b) 0.1 – 1 mm (c) 5 000 – 15 000 km

Q64 About 35 cm^2

Q65 (a) 40 – 90 m^2 (b) 5 – 25 mm^2 or 0.05 – 0.25 cm^2 (c) 50 – 140 cm^2

Q66 Fill a container with water, immerse your head in it and catch the water the spills out in a graduated container and measure the spilt water.

Q67 (a) 0.2 – 0.5 L or 200 – 500 mL or 200 – 500 cm^3

(b) 80 – 250 L or 0.08 – 0.25 m^3

(c) 80 – 400 m^3

(d) about 1 mm^3 or 0.001 cm^3

Q68 (a) 0.5 – 5 kg (b) 300 kg – 1 t (c) 2 – 20 t (d) about 1 mg