M1 Maths

G3-1 Similarity

- the meaning of similarity
- tests for similarity
- applications of similar triangles

Summary Learn Solve Revise Answers

Summary

Two shapes are similar if they are the same shape. [Congruent means same shape **and** size.] Similarity of triangles has useful applications in geometry, particularly in measuring inaccessible distances.

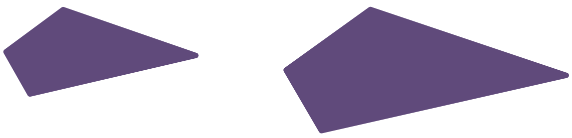
Two triangles are similar if they pass any of the tests for similarity: AA, SSS, SAS.

Learn

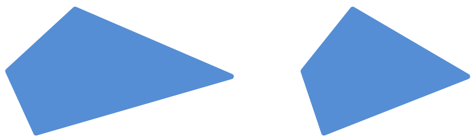
Meaning

Two shapes are said to be similar if they are the same shape. They don't have to be the same size, but they can be.

These purple shapes are similar.



These blue ones aren't.



These pink ones are similar. Note, it doesn't matter if they aren't round the same way (i.e. one is rotated).

So are these green ones. Note, it doesn't matter if one is flipped (reflected) or that they are also congruent.

So, if a shape is translated, dilated from a point, rotated and/or reflected, the image will be similar to the object.

Note we are using the word 'similar' here in a different sense from the common English sense, where it means 'somewhat alike, though a bit different'.

Practice

Q1 For each of these pairs of shapes, say whether the shapes are similar.

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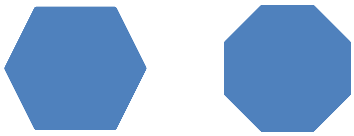

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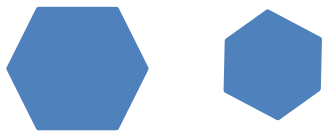

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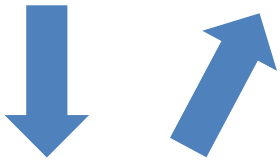

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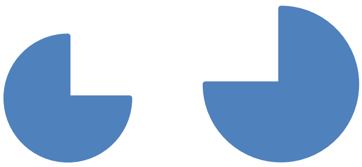


(f)

In mathematics, considering the similarity of triangles is quite useful. Other shapes much less so. So we only need to consider triangles, and from here on, that's all we will do.

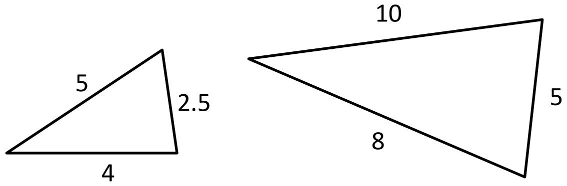
Tests for Similarity

In many situations, we need to decide if triangles are similar, given some information about their side lengths and angles (though without accurately drawn pictures of them). For example, suppose we know that two triangles both have angles of 50°, 55° and 75°. Does that mean that they are similar?

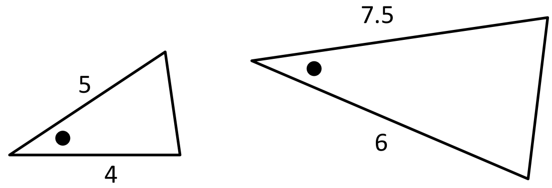
There are three rules for picking similar triangles: SSS, SAS and AA. As with congruence, you can use a first-principles approach instead, though you should still know the rules.

Triangles are similar if any of the following apply.

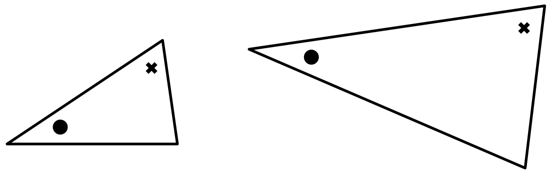
SSS: If two triangles are similar, then one is an enlargement of the other and all sides on one are n times as long as the corresponding sides on the other, where n is a constant called the enlargement factor. In other words, the lengths of three sides on one triangle will be proportional to the lengths of the three sides on the other.



SAS: Two sides on one triangle are proportional (same enlargement factor) to two on the other and the angle between them is the same.



AA Two angles on one triangle are equal to two angles on the other. This of course implies that all three are equal.



Practice

Q2 For each of these pairs of triangles, say whether we can know that they are similar and which rule is used. (Or explain how you know from first principles.)
Do not go by the appearance of the triangles as they are not necessarily drawn to scale; rely only on the given information about the side lengths and angles.

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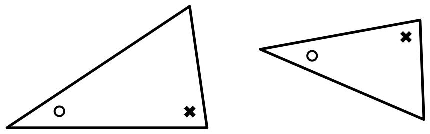

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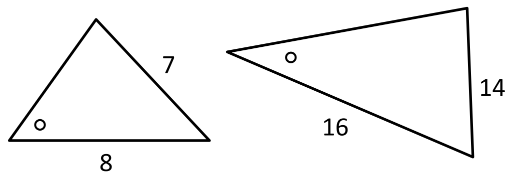

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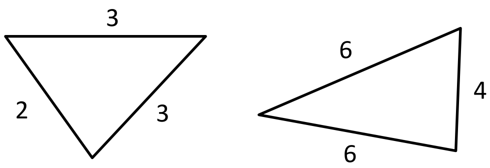

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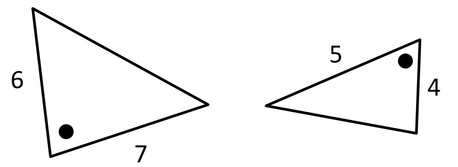

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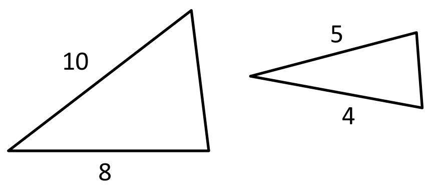

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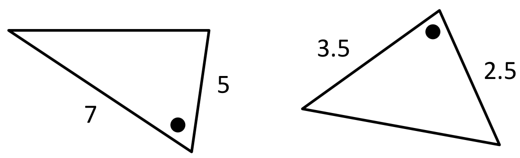

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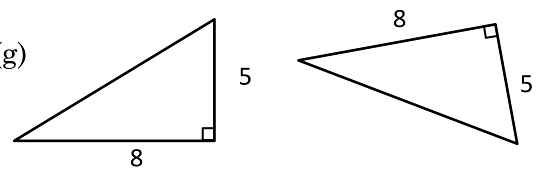

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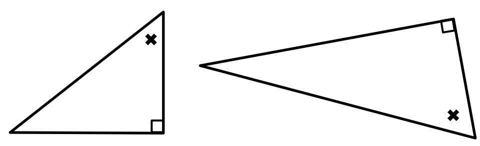
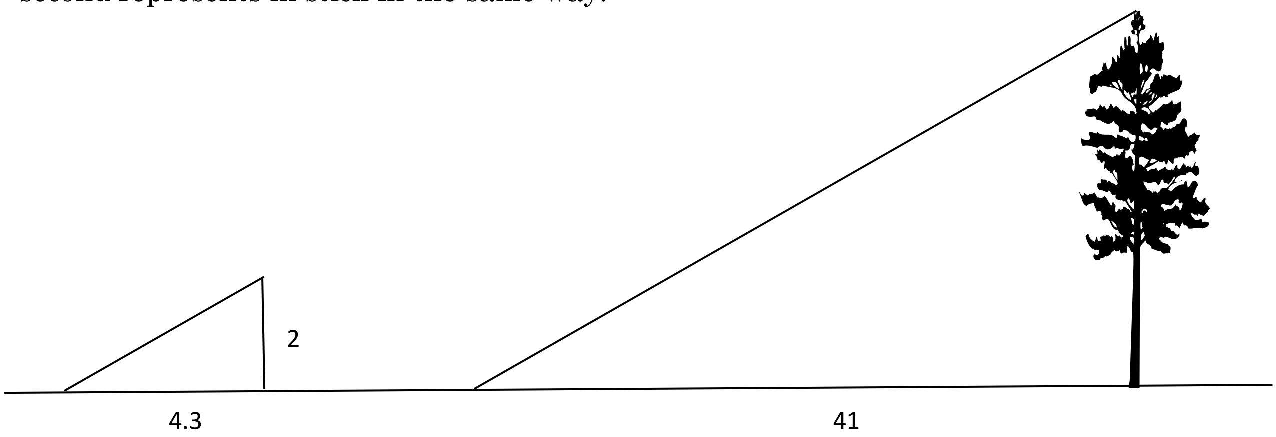


Applications of Similar Triangles

Similar triangles find uses in determining lengths which cannot be measured directly. An example is finding the height of a tree.

Let's say that the sun is shining and a tree makes a shadow on horizontal ground 41 m long. At the same time, a 2 m stick standing vertically on the ground makes a shadow 4.3 m long. How tall is the tree?

To answer this, we can draw two triangles. The first represents the tree with the height of the tree as the height of the triangle and the shadow of the tree as the base of the triangle. The second represents in stick in the same way.



The bottom right-hand angle in both triangles is 90°. As the sun is at the same elevation for the tree and for the stick, the bottom left-hand angles are the same. Thus the two triangles are similar (AA).

We now work out how many times bigger one triangle is than the other. This is called the enlargement factor. To do this we need to know the lengths of the same sides on both triangles. We know the base lengths of both.

We can see that the base of the larger triangle is about 10 times the base of the smaller triangle. So the enlargement factor is about 10. To be more precise, we divide: 41 ÷ 4.3 = 9.5. So the enlargement factor is 9.5 – the larger triangle is 9.5 times as big as the smaller one.

So, the height of the larger triangle will also be 9.5 times the height of the smaller one.
2 × 9.5 = 19. So the height of the tree is 19 m.

We might lay this out like this:

Base of larger triangle = 41

Base of smaller triangle = 4.3

Enlargement factor = 41 ÷ 4.3 = 9.5

Height of smaller triangle = 2

Height of larger triangle = 2 × 9.5 = 19

So the height of the tree is 19 m.

When solving problems like this, always draw a diagram.

Practice

Q3 Jimbo found that the shadow of a flag pole on level ground was 11.7 m long. At the same time, the shadow of a 1.5 m vertical stick was 1.3 m long. Draw a suitable diagram and find the height of the flag pole.

Q4 A building known to be 72 m tall casts a shadow 114 m long. At the same time, a tree casts a shadow 23 m long. Use a suitable diagram to find how tall the tree is.

Q5 Jethro wanted to find the height of a fibre-glass Tyrannosaurus at a dinosaur park. It had rained and there was a small puddle on the ground 52 m from the Tyrannosaurus. When he stood 4.2 m from the puddle, he could see the top of the Tyrannosaurus reflected in it. If his eyes were 1.6 m above the ground, how tall was the dinosaur?
[You may know from science that when light is reflected in a puddle, the reflected light leaves at the same angle to the horizontal as the incident light arrives.]

Solve

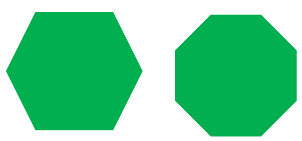
Q51 Emma wants to measure the width of a river without crossing it. She notices a tree on the far bank. She puts a stake in the ground on the near bank opposite the tree, then walks 15 m downstream along the bank and puts a second stake there. She then goes back to the first stake, walks 20 m perpendicularly away from the river, then walks 26 m downstream parallel to the river to where her second stake is in line with the tree. How wide is the river?

Revise

Revision Set 1

Q71 For each of these pairs of shapes, say whether the shapes are similar.

(a)



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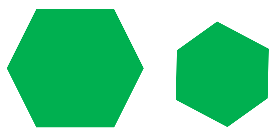

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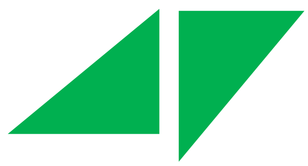
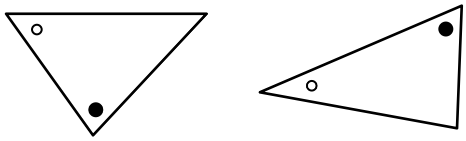


Q72 State the three rules for similarity of triangles and give a diagrammatic example of each.

Q73 For each of these pairs of triangles, say whether they are similar. Give the rule for those which are similar.

(a)



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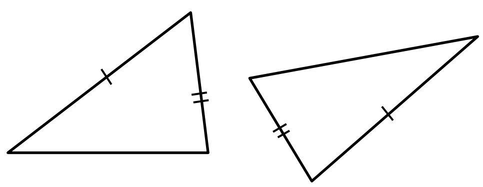

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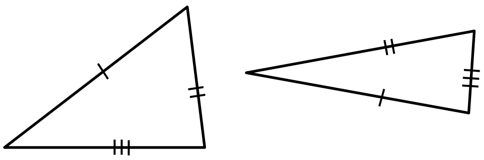

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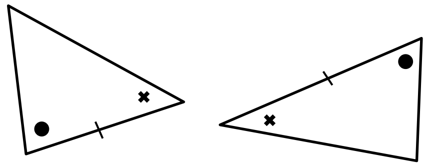

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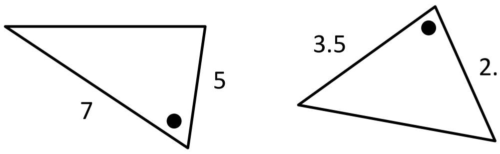

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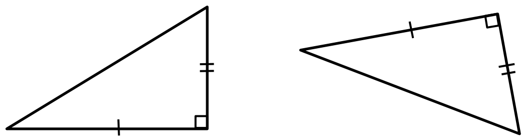

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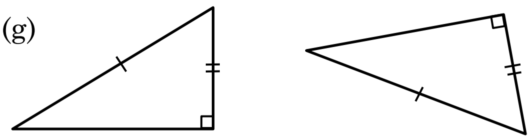

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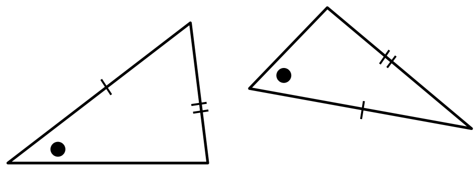


Q74 The shadow of a flag pole on level ground is 8.1m long. At the same time, the shadow of a 2 m vertical stick is 1.44 m long. Find the height of the flag pole.

Answers

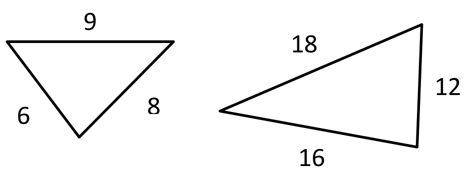
Q1 (a) no (b) yes
(c) yess (d) yes
(e) yes (f) no

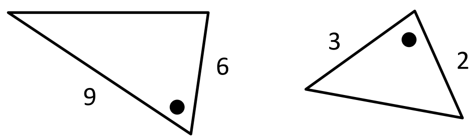
Q2 (a) yes (AA) (b) no (c) yes (SSS) (d) no
(e) no (f) yes (SAS) (g) yes (SAS) (h) yes (AA)

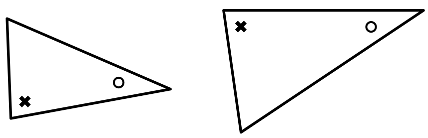
Q3 13.5 m Q4 14.5 m Q5 19.8 m

Q51 27.3 m (You need to write an equation and solve it.)

Q71 (a) no (b) yes (c) yes

Q72 SSS


SAS


AA


Q73 (a) yes (AA) (b) no
(c) yes (SSS) (d) yes (ASA)
(e) yes (SAS) (f) yes (SAS)
(g) yes (RHS) (h) no

Q74 Base of larger triangle = 8.1
Base of smaller triangle = 1.44
Enlargement factor = 8.1 ÷ 1.44 = 5.625
Height of smaller triangle = 2
Height of larger triangle = 2 × 5.625 = 11.25
So the height of the pole is 11.3 m.