M1 Maths

G2-6 Congruence

- the meaning of congruence
- tests for congruence of triangles

Summary Learn Solve Revise Answers

Summary

Two shapes are congruent if they are the same shape and size. Congruence of triangles has useful applications in geometry.

Two triangles are congruent if they pass any of the tests for congruency: SSS, SAS, AAS, RHS.

Learn

Meaning

Two shapes are said to be congruent if they are the same shape and the same size, i.e. if one can be laid on top of the other with no bits of one coming past the edges of the other. If a shape is translated, rotated or reflected, the image will be congruent to the object. But if the shape is dilated (from a line or from a point), it won't be.

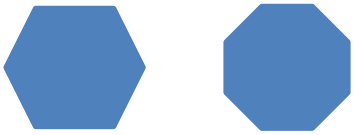
These pairs of shapes are congruent.

These pairs aren't.

Practice

Q1 For each of these pairs of shapes, say whether the shapes are congruent.

(a)



(b)

(c)

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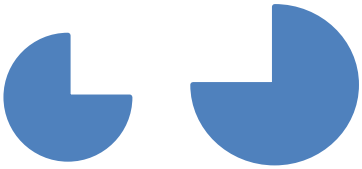


(f)

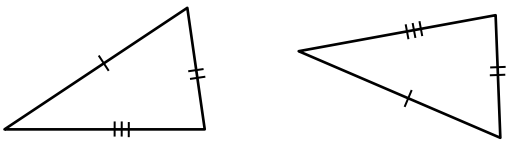
In mathematics, considering the congruence of triangles is quite useful. Other shapes less so. So we only need to consider triangles, and from here on, that's all we will do.

Tests for Congruence

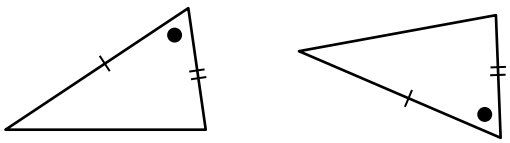
In many situations, we need to decide if triangles are congruent, given some information about their side lengths and angles (though without accurately drawn pictures of them). For example, suppose we know that two triangles both have angles of 50°, 55° and 75°. Does that mean that they are congruent?

There are four rules for picking congruent triangles. The rules are abbreviated to SSS, SAS, AAS and RHS. Two triangles are congruent if any of the following apply:

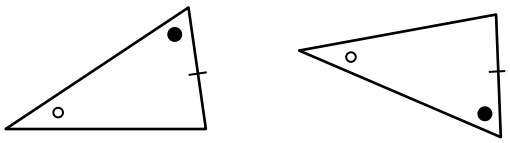
SSS: If the three sides on one triangle are the same length as three sides on the other.



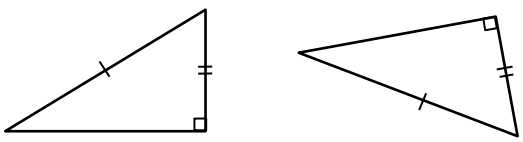
SAS: If two of the sides on one triangle are the same as two of the sides on the other and the angles between those sides are equal.



AAS: If two of the angles on one triangle are the same as two of the angles on the other and two sides in equivalent positions relative to the angles are equal.



RHS: If both triangles are right-angle triangles and the hypotenuse of both are equal and one of the other sides is equal.



The first-principles method

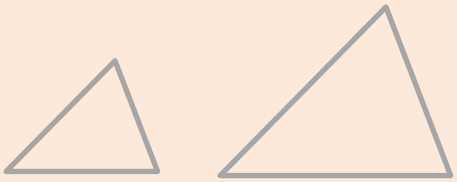
There is another method, the first-principles method, to pick congruent triangles rather than memorising the rules. However, most people prefer to use the rules. You do not need to know the first-principles method or to read this box. It is just here in case you would like to.

You can start to draw a triangle using the given information and see if it is possible to finish it in more than one way with different shapes or sizes. If it is possible, then the triangles are not necessarily congruent. If there is only one possibility, then they must be congruent.

For example, suppose we know that two of the sides are the same length on both triangles and that the angles between them are the same. We can start the triangle like this:

Clearly, there is only one way to finish the triangle, so any triangles with those specifications must be congruent.

On the other hand, suppose you know that the three angles in one are the same as the three angles in the other. You should realise that it is possible to draw different triangles with those specifications.

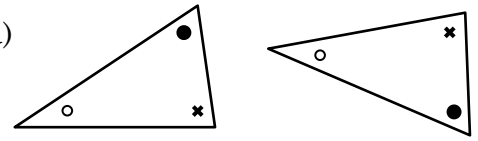


So knowing that the three angles are the same in the two triangles doesn't prove that the triangles are congruent.

Practice

Q2 For each of these pairs of triangles, say whether we can know that they are congruent and which rule is used. (Or explain how you know from first principles.)
Do not go by the appearance of the triangles as they are not necessarily drawn to scale; rely only on the given information about the side lengths and angles.

(a)



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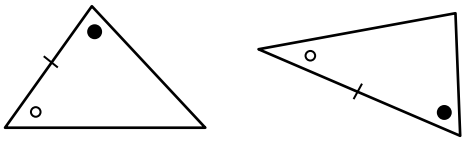

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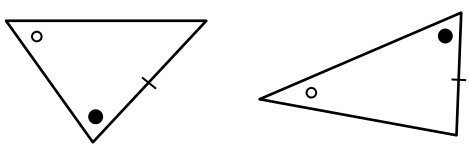

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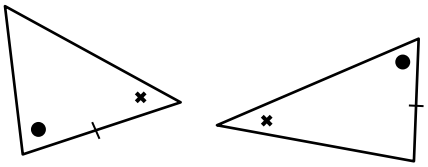

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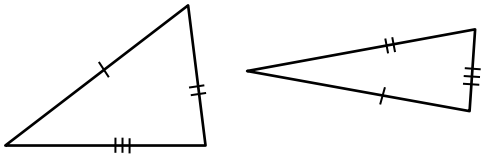

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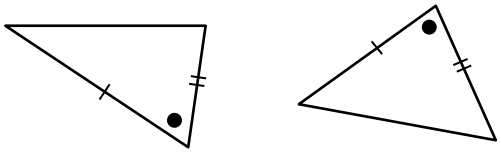
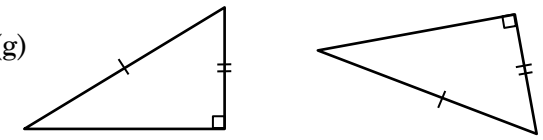
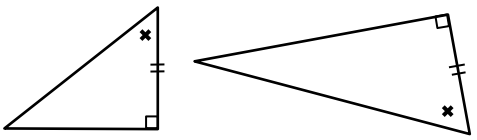


(g) 

(h) 

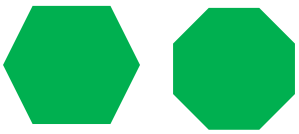
Solve

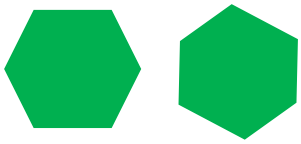
Q51 Any 4 congruent triangles can be put together to make a larger triangle the same shape as the originals, but larger. 5 congruent triangles can't. What other numbers can?

Revise

Revision Set 1

Q61 For each of these pairs of shapes, say whether the shapes are congruent.

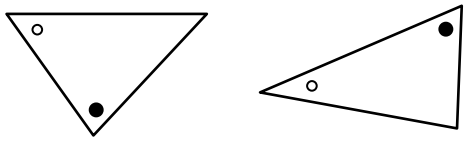
(a) 

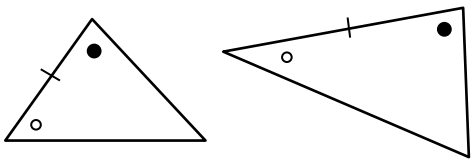
(b) 

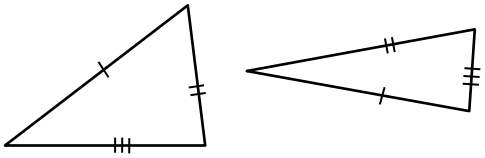
(c) 

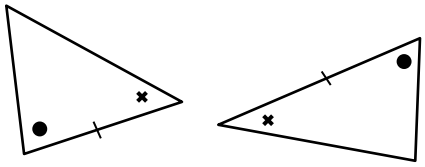
Q62 State the four rules for congruence of triangles and give a diagrammatic example of each.

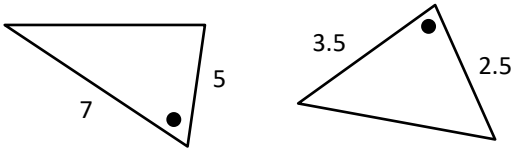
Q63 For each of these pairs of triangles, say whether they are congruent. Give the rule for those which are congruent.

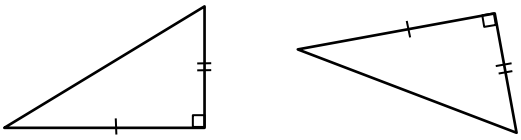
(a) 

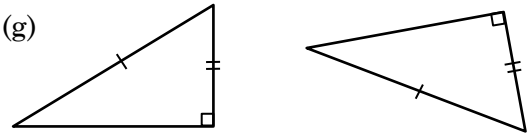
(b) 

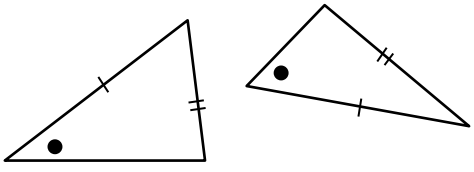
(c) 

(d) 

(e) 

(f) 

(g) 

(h) 

Answers

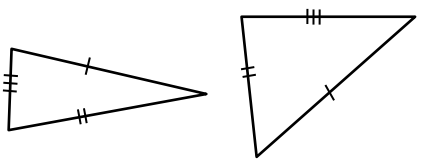
Q1 (a) no (b) no
(c) yes (d) yes
(e) no (f) no

Q2 (a) no (b) yes (AAS) (c) no (d) no
(e) yes (SSS) (f) yes (SAS) (g) yes (RHS) (h) yes (AAS)

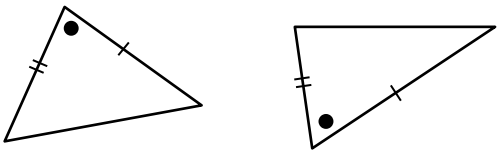
Q51 Any square number.

Q61 (a) not congruent (b) congruent (c) congruent

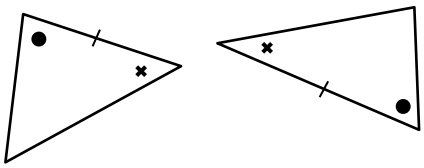
Q62 SSS



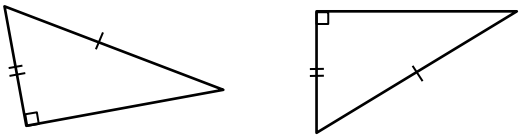
SAS



ASA



RHS



Q63 (a) no (b) yes (AAS)
(c) yes (SSS) (d) yes (AAS)
(e) no (f) yes (SAS)
(g) yes (RHS) (h) no