M1 Maths

G2-5 Transformations and Symmetry

- translations, reflections, rotations and dilations of points and shapes
- reflectional and rotational symmetry

Summary Learn Solve Revise Answers

Summary

A transformation is a change. In this unit, we look at five types of transformations of points and 2D shapes in a plane.

A translation is a movement of a point or shape by a specific distance in a specific direction.

In a reflection, every point in a shape moves perpendicularly to the reflection line, then continues the same distance past it.

In a rotation, every point in the shape moves around the centre of rotation by the same angle, keeping the same distance from it.

In a dilation from a line, for all points in the shape, the distance from a given line of dilation is multiplied by a certain factor called the dilation factor.

In a dilation from a point, for all points in the shape, the distance from a given centre of dilation is multiplied by a certain factor called the dilation factor.

A shape, object or pattern has symmetry if it still looks the same after a transformation.

In reflectional symmetry, the object looks the same after reflection in the line of symmetry.

In rotational symmetry, an object looks the same after rotation about the point of rotational symmetry. In n–fold rotational symmetry, the object looks the same n times per revolution.

Learn

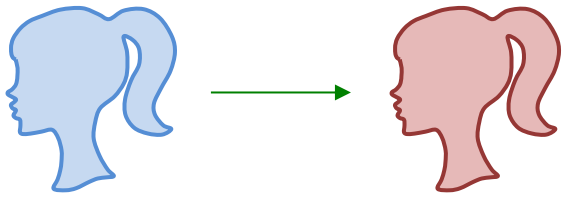
Transformations

'**Transformation**' means 'change'. A caterpillar is transformed into a butterfly. In this module we will be using the word in a rather specific sense – a change in the position, orientation or size of something on a two-dimensional plane.

We will consider transformations of points and 2D shapes. And we will consider 5 types of transformation – translations, reflections, rotations, dilations from a line and dilations from a point.

Translation

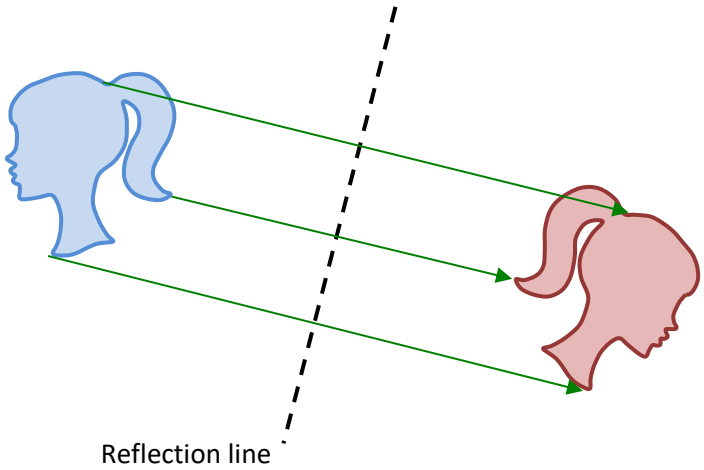
A **translation** is a movement by specific distance in a specific direction. The diagram below shows a translation of 5 cm to the right.



Throughout this module, the original figure, called the object, will be drawn in blue; the transformed figure, called the image, will be in red.

Reflection

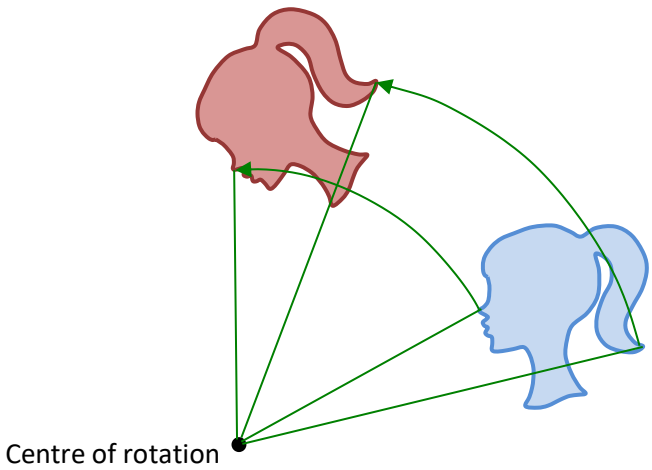
A point or 2D shape can be reflected in a **reflection line**, just like a 3D object can be reflected in a mirror. In a **reflection**, every point on the object moves perpendicularly towards the reflection line, then continues the same distance past it.



Rotation

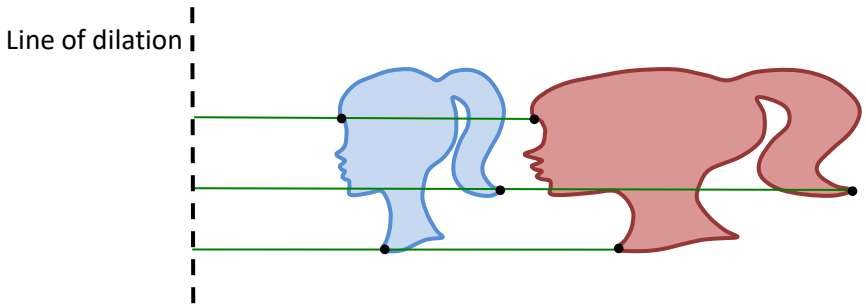
A point or 2D shape can be rotated a given angle around a given point, called the **centre of rotation**. Every point on the object moves around the point by the same angle, keeping the same distance from the centre of rotation.

The picture to the right shows a **rotation** of 60° anticlockwise.

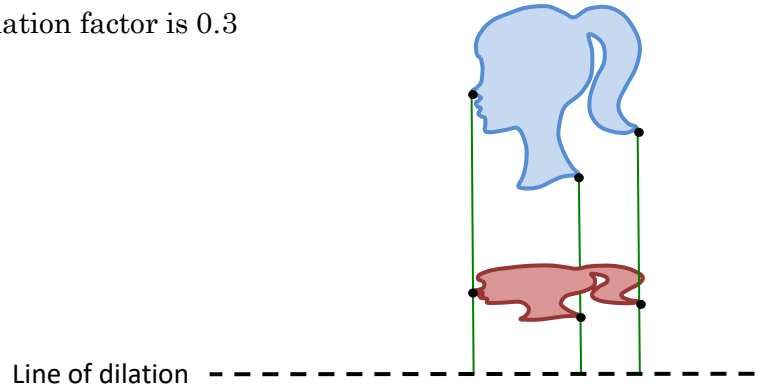


Dilation from a line

In a **dilation from a line**, for all points in the object, the distance from a given **line of dilation** is multiplied by a certain factor called the **dilation factor**. In the diagram below, the dilation factor is 2. Dilations from a line tend to produce a distorted image.

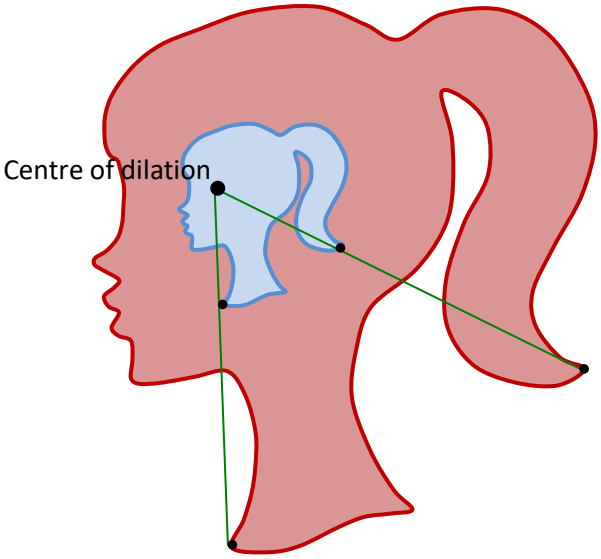


In this next diagram, the dilation factor is 0.3

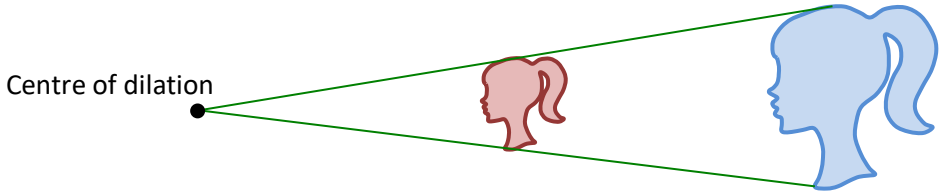


Dilation from a point

A **dilation from a point** produces a change is size without distortion. In a dilation from a point, for all points in the object, the distance from a given point, the **centre of dilation**, is multiplied by a certain factor, the dilation factor. In the diagram below, the dilation factor is 3 and the centre of dilation is inside the object.



In this next diagram, the dilation factor is 0.5 and the centre of dilation is outside the object.



Note that when performing a transformation on a shape, the best way is to pick a few distinctive points on the shape, transform them, then connect the images of the points to construct the image of the shape. If the shape is a polygon, use the vertices of the polygon.

Practice

Q1 Copy the drawing below, leaving adequate space around it, then, using a pencil, ruler and protractor:

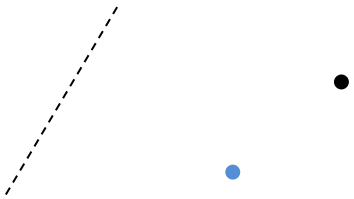
(a) Translate the blue dot 3 cm down the page

(b) Reflect the blue dot in the dotted line (use the original shape)

(c) Rotate the blue dot 120° anti-clockwise about the black dot

(d) Dilate the blue dot by a factor of 0.75 using the dashed line as the line of dilation

(e) Dilate the blue dot by a factor of 1.5 using the black dot as the centre of dilation



Q2 Copy the drawing below and, using a pencil, ruler and protractor:

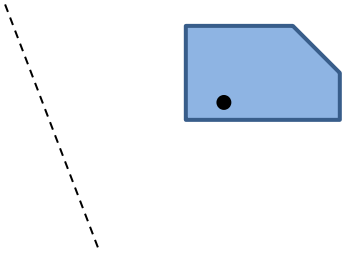
(a) Translate the shape 6 cm to the right

(b) Reflect the shape in the dotted line (use the original shape)

(c) Rotate the shape 90° clockwise about the black dot

(d) Dilate the shape by a factor of 2 using the dashed line as the line of dilation

(e) Dilate the shape by a factor of 1.5 using the black spot as the centre of dilation



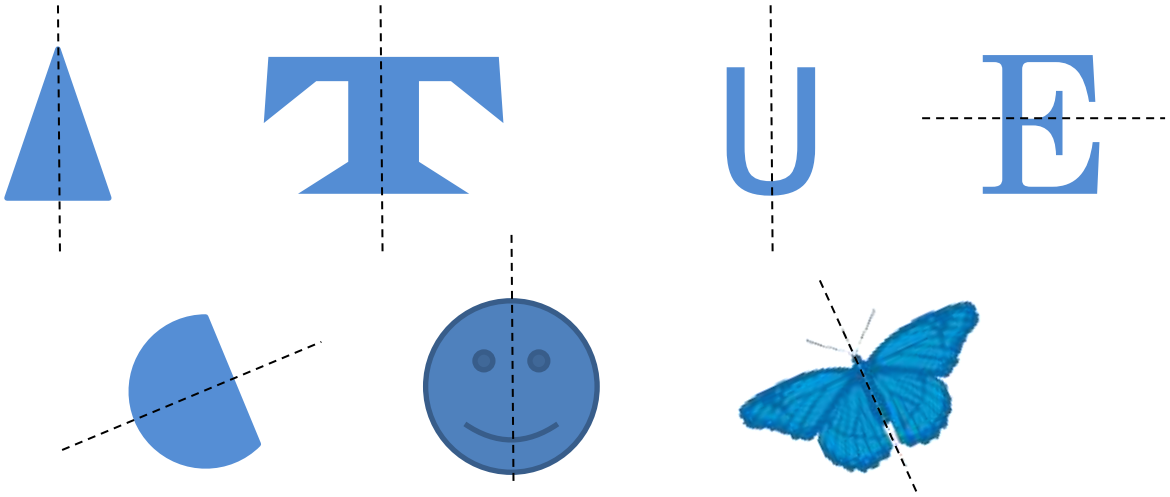
Symmetry

An object, shape or pattern has **symmetry** if it still looks the same after a transformation. We will consider symmetry of 2D shapes.

Reflectional Symmetry

The most familiar type of symmetry is **reflectional symmetry** (sometimes called mirror symmetry or bilateral symmetry). Something has reflectional symmetry if it looks the same after being reflected in a line, the **line of symmetry** or **line of reflectional symmetry**.

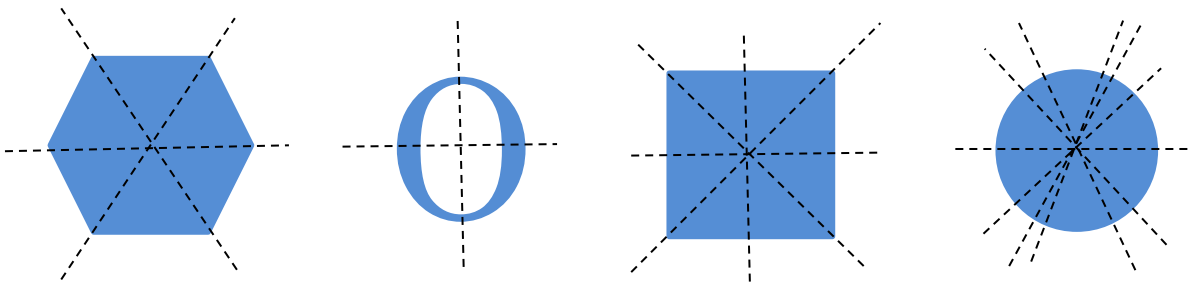
Some shapes are shown below. The dashed lines are the lines of symmetry.



Butterfly from Bea.miau, Wikipedia

The following shapes do not have reflectional symmetry.

Some shapes have more than one line of symmetry. Here are some examples.



Note that a circle has an infinite number of lines of symmetry. In fact, any line passing through its centre is a line of symmetry.

Practice

Q3 Copy the drawings below and mark on them all lines of reflectional symmetry.

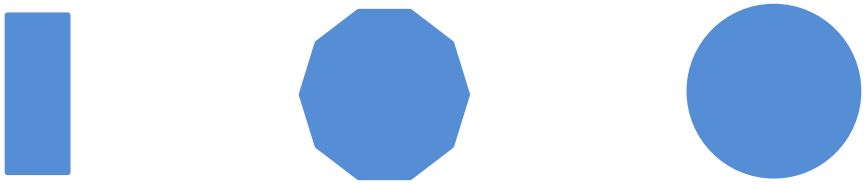
Rotational Symmetry

Something has **rotational symmetry** if it looks the same after being rotated about a point. For example, if a square is rotated 90° about its centre, it will look exactly the same.

Before After

In fact, in rotating through each revolution, the square will look the same 4 times. Because of this we say that it has 4-fold rotational symmetry.

The rectangle on the left below has 2-fold rotational symmetry and the regular decagon in the centre has 10-fold rotational symmetry. The circle on the right has ∞-fold rotational symmetry. However, rather than say that, we say that it has circular symmetry.



Practice

Q4 In each of the following shapes, mark any points of rotational symmetry and indicate whether it is 2-fold, 3-fold etc.

Last shape: Washiuko, Wikipedia

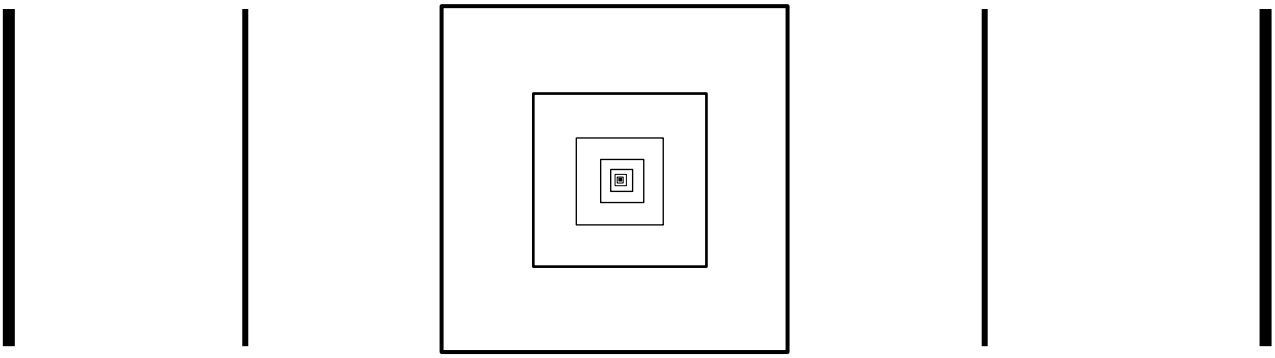
Translational and Dilational Symmetry

You may not need to know about these, but they are included for completeness.

Obviously, if you translate a shape, it won't look the same. Unless it is replaced by another one the same. So **translational symmetry** applies only to endlessly repeating patterns. Such patterns can be found on material or wallpaper.

commons.wikipedia.org

In the same way, if you dilate a shape, it won't look the same. Unless there is another one there to take its place. Imagine a square with a square half its size in the centre and a square half the size again in the centre of that and so on for ever. If the biggest square is infinitely big, then the resulting pattern will have **dilational symmetry**. It will always look the same after a dilation with factor 2 from the centre of the squares. Part of the pattern might look like this.

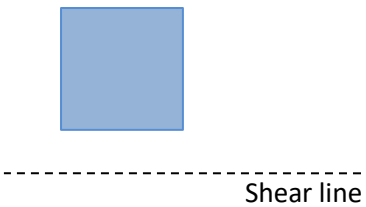


Solve

Q51 Another type of transformation is the shear. In a particular shear every point is moved to the right (as viewed from the shear line) parallel to the shear line by a distance equal to the distance of that point from the shear line.

(a) What shape would the square in the diagram be after such a shear?

(b) What would be the ratio of the area of the image to the area of the object?



Revise

Revision Set 1

Q61 Copy the drawing below and, using a pencil, ruler and protractor:

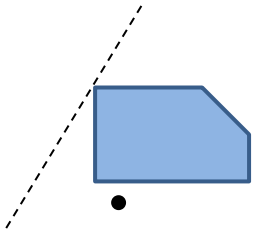
(a) Translate the blue shape 5 cm to the right

(b) Reflect the shape in the dotted line (use the original shape)

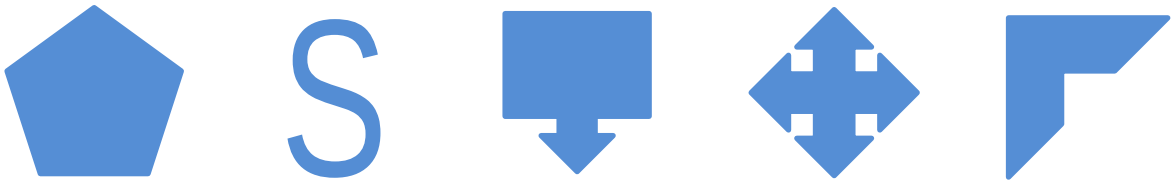
(c) Rotate the shape 180° clockwise about the black spot

(d) Dilate the shape by a factor of 2 using the dashed line as the line of dilation

(e) Dilate the shape by a factor of 0.5 using the black spot as the centre of dilation

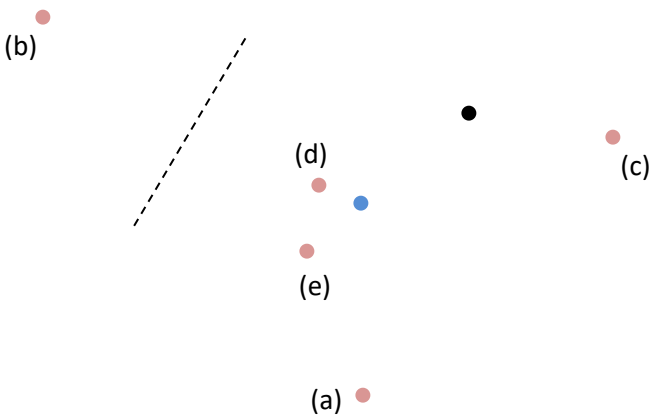


Q62 In each of the following shapes, mark any lines of reflectional symmetry and any points of rotational symmetry. Also indicate whether the rotational symmetry is 2-fold, 3-fold etc.

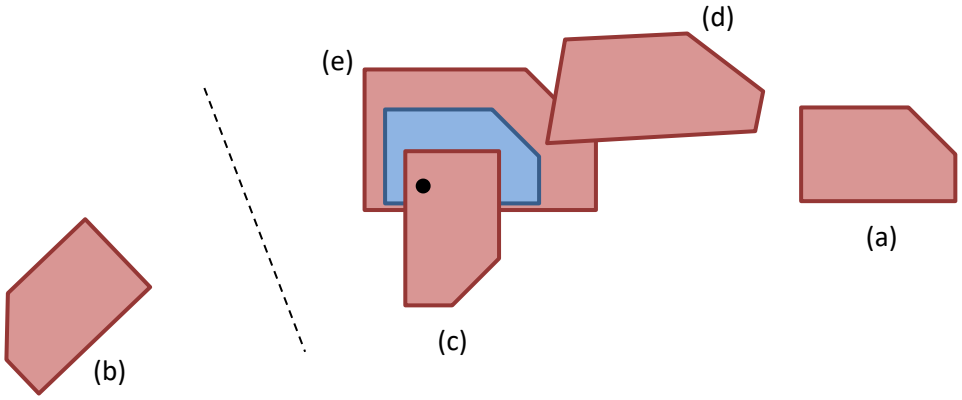


Answers

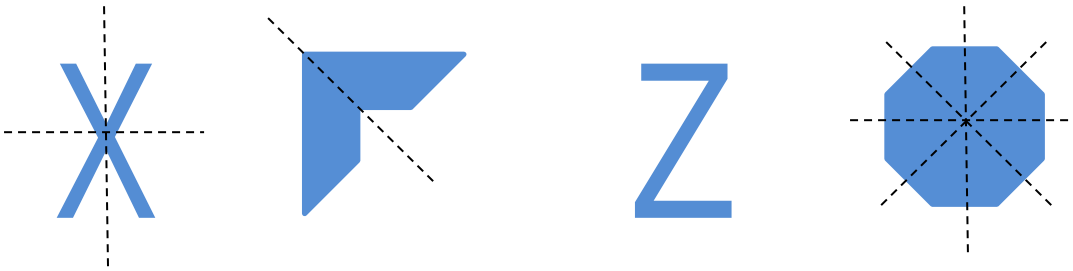
Q1



Q2



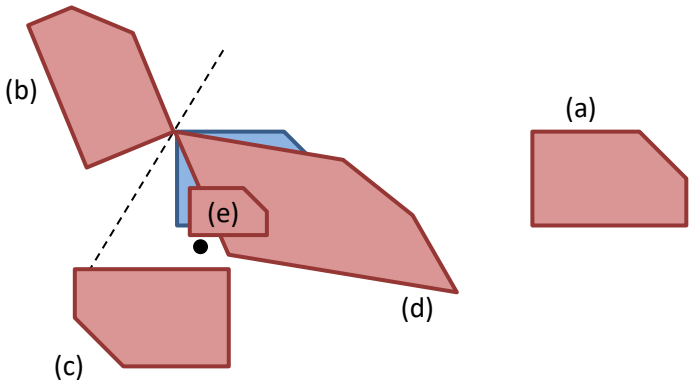
Q3



Q4

Q51 (a) Parallelogram (not a rhombus) (b) 1:1

Q61



Q62