M1 Maths

G2-3 Properties of Polygons

- polygon names
- internal angles
- types of triangles and their properties
- types of quadrilateral and their properties

Summary **Learn** **Solve** **Revise** **Answers**

Summary

Polygons are named according to their number of sides: triangle, quadrilateral, pentagon . . . They can also be called regular if all their sides and internal angles are equal or irregular otherwise.

The sum of the internal angles of any polygon with n sides is $180(n-2)°$. If the polygon is regular, each internal angle is $180(n-2)° \div n$.

Triangles can be named scalene, isosceles or equilateral if 0, 2 or 3 of their sides are equal. They can also be named acute-angled, right-angled or obtuse-angled according to their largest angle. There are names for quadrilaterals with particular properties.

Particular types of triangles and quadrilaterals have other consequent properties.

Learn

Polygon Names

A polygon is a 2D shape with all its sides straight. Polygons are named primarily according to the number of sides. The names are listed below.

Sides	Name
3	**Triangle**
4	**Quadrilateral**
5	**Pentagon**
6	**Hexagon**
7	Heptagon
8	**Octagon**
9	Nonagon

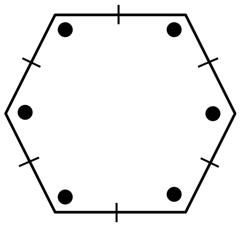
10	Decagon
11	Undecagon
12	**Dodecagon**
13	13-gon
14	14-gon

It is worth knowing the names in bold red. For polygons with more than 12 sides, the n-gon names are generally used: 13-gon, 14-gon, 15-gon, 16-gon and so on. There are long names for polygons with more than 12 sides. For example, a 451-sided polygon is called a tetrahectapentacontahenagon. But these names aren't commonly used and they're not worth knowing.

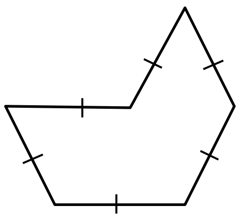
Regular and Irregular Polygons

A polygon is called a **regular** polygon if all its sides are of equal length ***and*** all its internal angles are equal. Otherwise it is **irregular**.

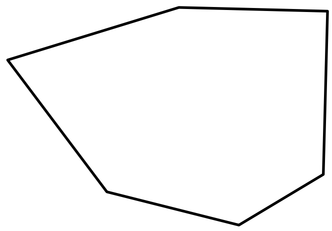
Regular hexagon



Irregular hexagon



Irregular hexagon

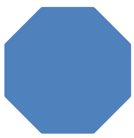


Note that the second figure has all its sides equal, but it is irregular because its internal angles are not all equal.

Practice

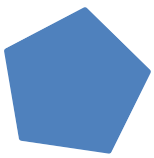
Q1 Name each of the following polygons according to the number of sides and whether they are regular or irregular. For example, (b) is an irregular quadrilateral. If they look regular, assume that they are.

(a)



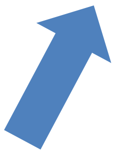
(b)

(c)



(d)

(e)



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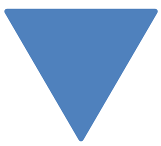
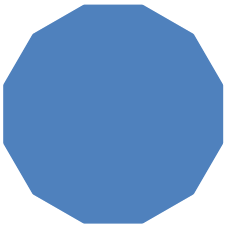


(g)



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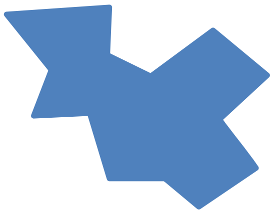
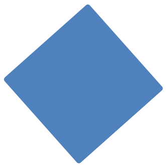
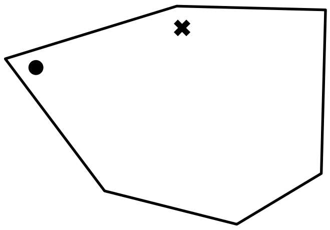


(i)



Sums of the Internal Angles



The **internal angles** of a polygon are the angles inside the shape.

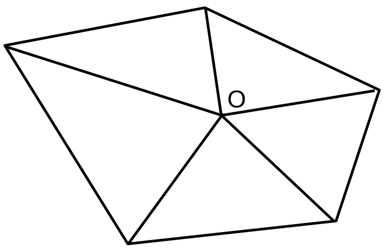
Two internal angles are marked in the polygon to the right. The one marked with a dot is about 70°; the one marked with a cross is about 160°.

For a given number of sides, the sum of the internal angles is always the same, irrespective of the shape of the polygon. The table below shows the sums for various numbers of sides.

Sides	Name	Sum of internal angles
3	Triangle	180°
4	Quadrilateral	360°
5	Pentagon	540°
6	Hexagon	720°
7	Heptagon	900°
8	Octagon	1080°

This pattern continues. The formula is $s = 180\ (n - 2)$, where n is the number of sides and s is the sum of the internal angles.

This can be shown to be the case as follows, though you don't need to be able to show it.



Let O be any point inside an n-sided polygon.
Connect O to each vertex of the polygon to make n triangles.
The sum of the angles in each triangle is 180.
$\therefore$ The total in the n triangles is $180n$.
360 of these are at O.
The rest form the internal angles of the polygon.
$\therefore$ The sum of the internal angles of the polygon is $180n - 360$.
$= 180n - 180 \times 2$
$= 180\ (n - 2)$

Individual Internal Angles

The individual internal angles of a regular polygon can be calculated by dividing the sum by the number of sides:

$$a = 180° \times (n - 2) \div n$$

For a regular pentagon, therefore, the internal angles are each $180° \ (5 - 2) \div 5 = 108°$

Obviously, the individual internal angles of an irregular polygon can't be calculated.

Practice

Q2 For each of the following polygons, give the sum of the internal angles and the value of each internal angle if the polygon is regular.

(a) triangle (b) quadrilateral (c) octagon

(d) 15-gon (e) 100-gon (f) 47-gon

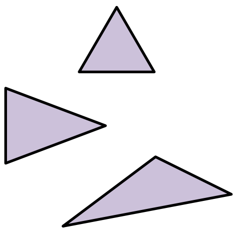
Names of Triangles

Triangles can be named according to the number of equal sides:

3 sides equal: **equilateral**

2 sides equal: **isosceles**

No sides equal: **scalene**

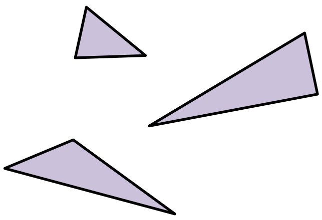


They can also be named according to the size of the largest angle:

acute-angled

right-angled

obtuse-angled

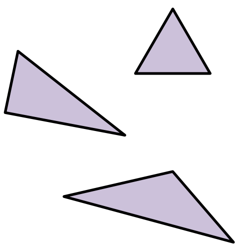


Either or both names can be attached to a triangle, e.g.

equilateral triangle

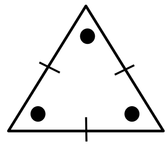
right-angled triangle

obtuse-angled isosceles triangle

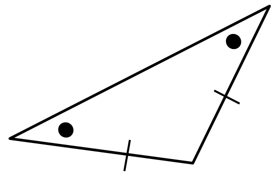


Of course, you wouldn't say 'acute-angled equilateral triangle' because all equilateral triangles are acute-angled.

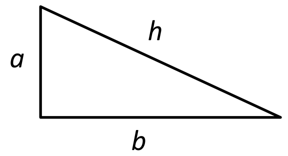
Properties of Triangles



In an equilateral triangle, all angles are equal and all are 60°. It works the other way round too: if all the angles in a triangle are 60°, then the triangle is equilateral.

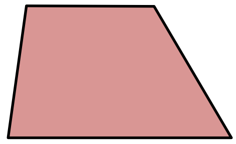


In an isosceles triangle, the angles at the opposite ends of the equal sides are equal. It works the other way too: if two angles in a triangle are equal, then two sides will also be equal; the two properties always go hand-in-hand.



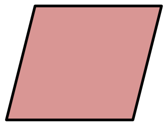
In a right-angled triangle, the longest side (the side opposite the right angle) is called the **hypotenuse**. The other two sides are called the **legs**.

Names of Quadrilaterals



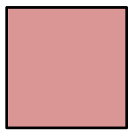
A quadrilateral with exactly one pair of parallel sides is called a **trapezium**.

A quadrilateral with two pairs of parallel sides is called a **parallelogram**.

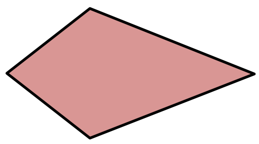


A parallelogram with all sides equal is called a **rhombus**.

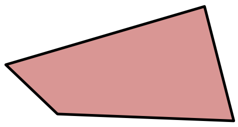
A parallelogram with right angles is called a **rectangle**.



A quadrilateral that is a rhombus and a rectangle is called a **square**.

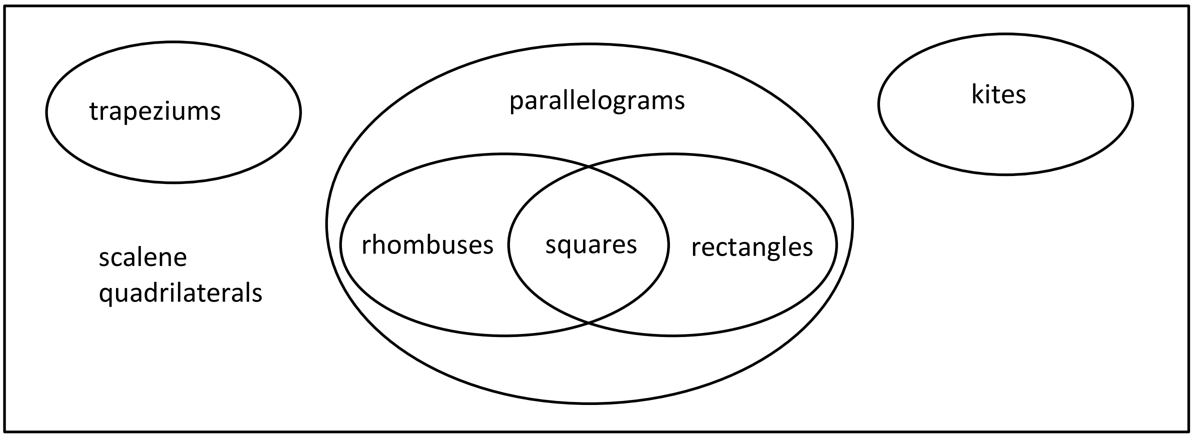


A quadrilateral with two pairs of equal adjacent sides is called a **kite**.



A **scalene quadrilateral** is a quadrilateral that is not a parallelogram, trapezium or kite.

The relation between the various types of quadrilateral is shown in the diagram below.



Note that this means that a square is a type of rectangle. Rhombuses and rectangles are parallelograms, so squares are parallelograms too.

Strictly speaking, rhombuses and squares are kites, though, in practice, we never refer to them as such. Doing so would make our diagram messy.

Don't read this – it's confusing!

Trapezium is the word used in the UK, Australia and New Zealand for a quadrilateral with one pair of parallel sides. It is called a *trapezoid* in the USA and Canada. To complicate things further, a quadrilateral with no sides parallel is called a *trapezoid* in the UK, Australia and New Zealand, and a *trapezium* in the USA and Canada. In practice, we rarely need to give a quadrilateral with no parallel sides a specific name – we can usually just call it a scalene quadrilateral or kite. In this module, the word *trapezium* will be used in the UK/Australian sense and the word *trapezoid* will not be used.

To add even further confusion, a trapezium is sometimes defined as a quadrilateral with at least one pair of parallel sides. We will stick to the definition of having exactly on pair of parallel sides. Mathematicians ought to get their act together!

Properties of Quadrilaterals

Different types of quadrilaterals have various properties. For example, the diagonals of any parallelogram bisect each other (divide each other into two equal halves) and the diagonals of a rhombus bisect each other at right angles.

However, most of these are fairly esoteric (of interest only to a few people). The ones worth remembering are the following:

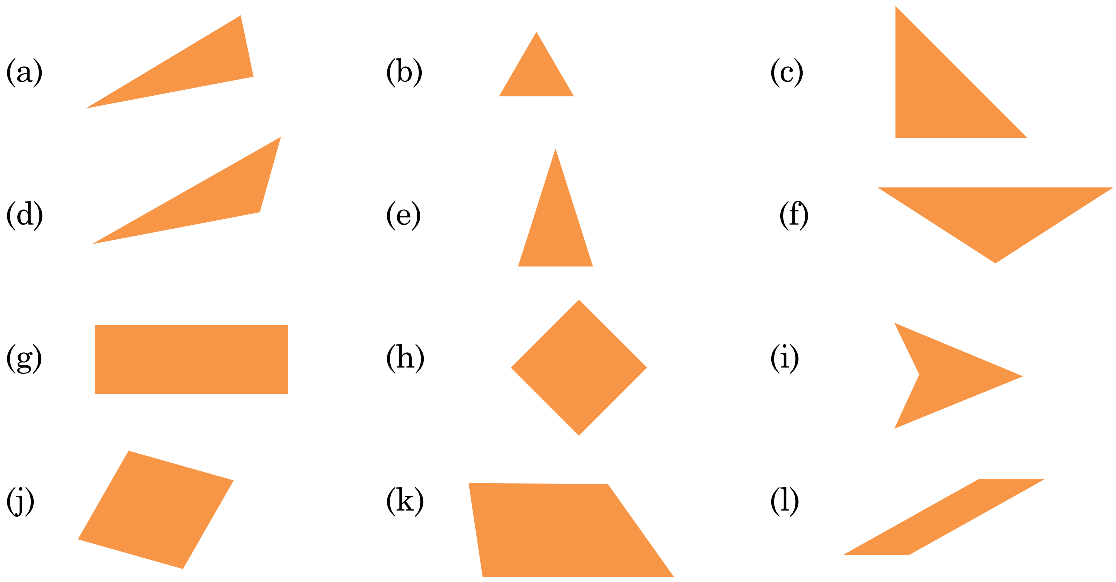
- Opposite sides of a parallelogram are equal.
- Opposite angles of a parallelogram are equal
- Diagonals of a parallelogram bisect each other

(Of course these properties apply to rhombuses, rectangles and squares too because they are parallelograms.)

The first two of these you probably already knew. The third one might be new.

Practice

Q3 Give the most specific names for each of the following shapes (e.g. 'obtuse-angled scalene triangle' rather than just 'triangle' or 'polygon' or 'shape'). If sides and angles look equal or like they are right angles, assume that they are.



Q4 True or false

(a) all squares are rectangles

(b) all rectangles are squares

(c) all rhombuses are polygons

(d) all trapeziums are quadrilaterals

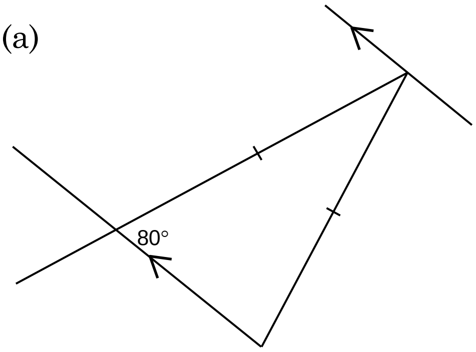
(e) all squares are parallelograms

The properties of triangles and quadrilaterals can be used to deduce angles on geometric diagrams (like in Module G2-2).

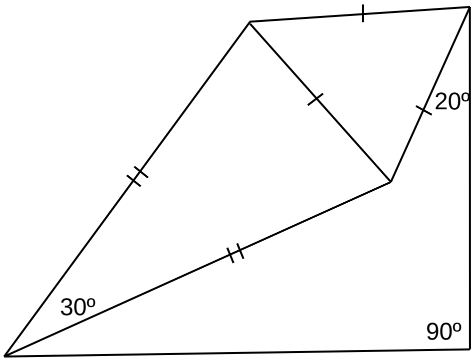
Practice

Q5 Copy these diagram and write in all the angles.

(a)

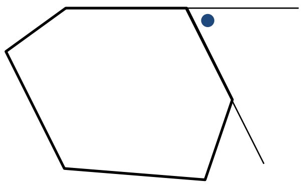


(b)



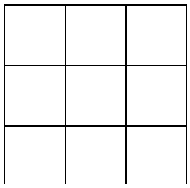
Solve

Q51 An alternative way to find the sum of the internal angles of an n-gon is to consider the external angles at each vertex (blue spot in the diagram).
Use this approach to show that the sum of the internal angles is $180\,(n-2)$.



Q52 5 equal-sized regular hexagons are joined together so that they are touching along at least part of a side. What are the largest and smallest possible numbers of sides on the resulting polygon?

Q53 (a) How many squares in this shape?
(There are squares of different sizes.)
(b) How many rectangles?
(c) How many rhombuses?
(d) How many parallelograms?



Revise

Revision Set 1

Q61 Give the names for polygons with
(a) 4 sides (b) 5 sides (c) 8 sides (d) 12 sides

Q62 For each of the polygons in Q61, find
(i) the sum of the internal angles
(ii) the size of each internal angle if the polygon is regular

Q63 What is the more common name for
(a) a regular quadrilateral
(b) a regular triangle

Q64 Give two properties of an isosceles triangle which are not shared by a scalene triangle.

Q65 True or false
(a) all squares are parallelograms
(b) all rectangles are parallelograms
(c) all rhombuses are squares
(d) all trapeziums are 4-sided polygons
(e) all squares are parallelograms
(f) all isosceles triangles are scalene triangles
(g) some right-angled triangles are isosceles

Answers

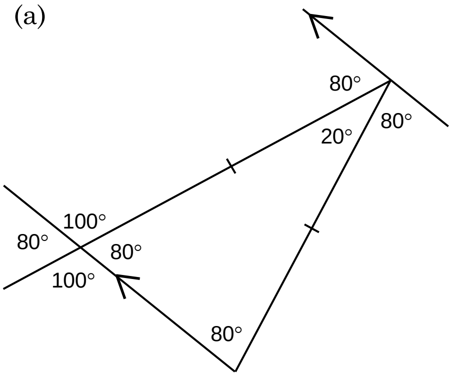
Q1
(a) regular octagon
(b) irregular quadrilateral
(c) regular pentagon
(d) irregular triangle
(e) irregular heptagon
(f) regular triangle
(g) regular dodecagon
(h) irregular 14-gon
(i) regular quadrilateral

Q2
(a) 180°, 60°
(b) 360°, 90°
(c) 540°, 108°
(d) 2340°, 156°
(e) 17 640°, 176.4°
(f) 8100°, 172.34°

Q3
(a) scalene right-angled triangle
(b) equilateral triangle
(c) right-angled isosceles triangle
(d) obtuse-angled scalene triangle
(e) acute-angled isosceles triangle
(f) obtuse-angled isosceles triangle
(g) rectangle
(h) square
(i) kite
(j) rhombus
(k) trapezium
(l) parallelogram

Q4 (a) True (b) False (c) True (d) True (e) True

Q5 (a)



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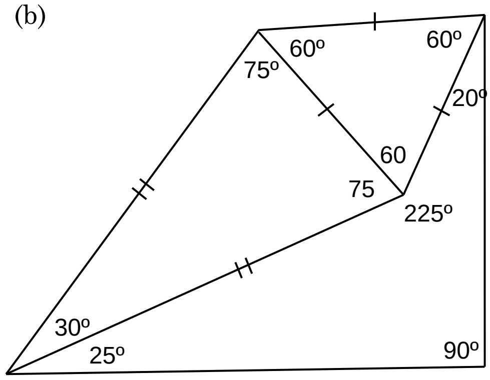

Q51 Consider walking around the polygon. The external angles are the amount you have to turn at each vertex.
To get back to the starting point, you need to turn 360°, so the sum of the external angles is 360°
Each internal angle is 180° – the external angle.
The sum of the n internal angles is, therefore, $n \times 180°$ minus the sum of the external angles.
This is $180\,n - 360$, which $180\,n - 180 \times 2$, which is $180\,(n - 2)$.

Q52 Largest 30; smallest 16

Q53 (a) 14 (b) 36 (c) 14 (d) 36

Q61 (a) quadrilateral (b) pentagon (c) octagon (d) dodecagon

Q62 (a) 360, 90 (b) 540, 108 (c) 1080, 135 (d) 1800, 150

Q63 (a) square (b) equilateral triangle

Q64 Two sides are equal, two angles are equal

Q65 (a) True (b) True (c) False (d) True
(e) True (f) False (g) True