M1 Maths

G2-2 Geometric Figures

- conventions for labelling geometric figures
- geometric theorems

Summary Learn Solve Revise Answers

Summary

Points on a geometric diagram are labelled with upper-case letters. The line joining points P and Q is called $\overline{PQ}$ or $\overline{QP}$. The angle between $\overline{PQ}$ and $\overline{PR}$ is called ∠QPR or ∠RPQ. A polygon is named using the names of the vertices in order as you go round.

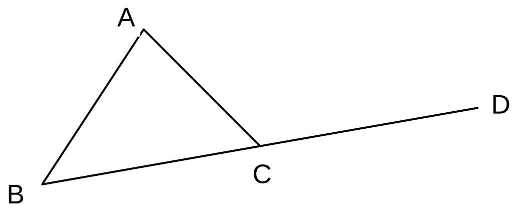
There are theorems for deducing angles on a geometric diagram. We will look at six: T, Y, X, D O, and H angles.

Learn

Conventions for naming and labelling geometric features

Points

Points on a geometric diagram are conventionally labelled with upper-case letters like this:

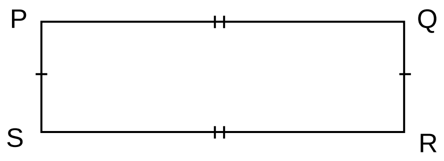


Lines

Line segments are then referred to by the names of the points at the two ends.

For example, in the diagram above, the line segment from B to C (the bottom of the triangle) is called $\overline{BC}$ or $\overline{CB}$. The line over the top shows that it is a line segment.

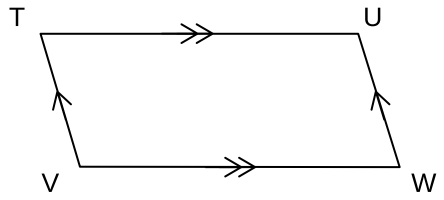
If two or more line segments are of equal length, this can be shown on the diagram by marking the line segments with the same number of cross-ticks, like this:



The cross ticks show that $\overline{PQ}$ and $\overline{RS}$ are the same length (because they both have two cross-ticks) and that $\overline{PS}$ and $\overline{QR}$ are the same length (because they both have one cross-tick).

We write this as $\overline{PQ} = \overline{RS}$ and $\overline{PS} = \overline{QR}$

In the same way, we can show that lines are parallel by putting the same number of arrow heads on them as in the following diagram.

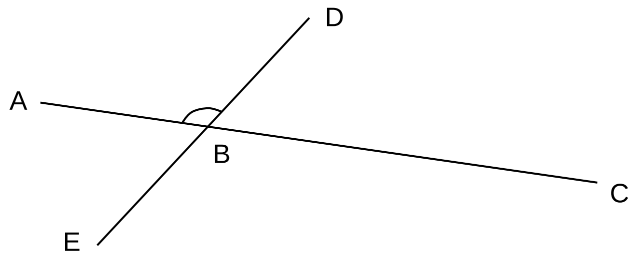


We write this as $\overline{TU} \parallel \overline{VW}$ and $\overline{TV} \parallel \overline{UW}$. ($\parallel$ means 'is parallel to'.)

[Parallel lines point in the same direction and stay the same distance apart along their lengths.]

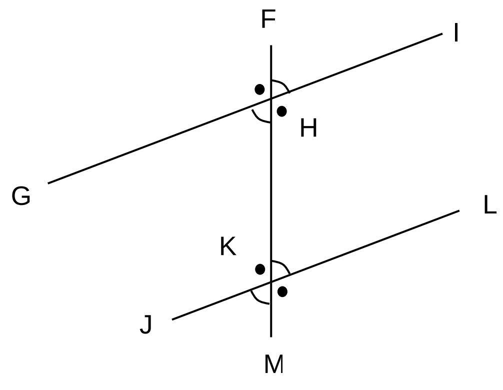
Angles

Angles on a diagram can be referred to using the names of three points, the second one being at the angle and the first and third showing which line segments are involved. For example in the diagram below, the angle marked is called ∠ABD or ∠DBA. The ∠ symbol is shorthand for 'angle'.



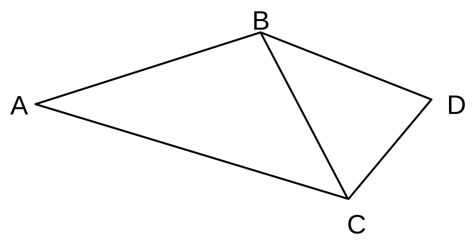
We can show that angles are equal by putting the same symbol in the angles, for instance like this:

We write this as ∠GHF = ∠KHI = ∠JKH = etc.

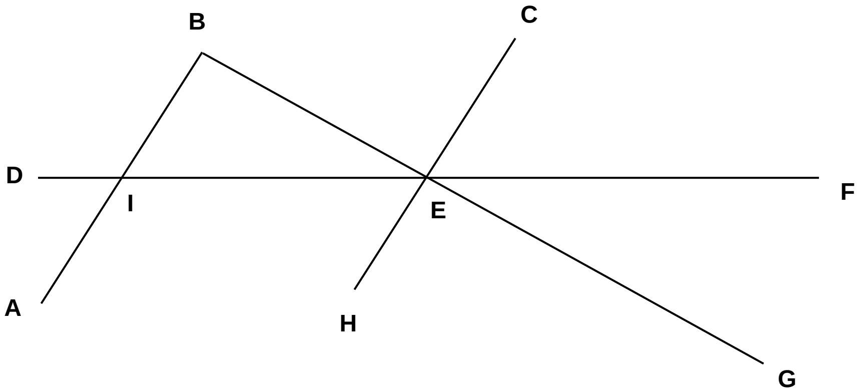


Polygons

Polygons can be referred to by listing the points at their vertices in order as you go around the polygon. The diagram below contains triangle ABC, triangle BCD and quadrilateral ABDC.



Practice



Q1 For the diagram above, write:

(a) the two possible names for the line segment from A to B

(b) the two possible names for the line segment from F to D

(c) the two possible names for the angle made by the line segment from E to B and the line segment from E to F

(d) the name for the triangle in the picture

Q2 Copy the diagram above and put symbols on it to show that:

(a) the line segment from F to E is the same length as that from E to G

(b) the line segment from A to I is the same length as that from I to B

(c) the line segment from I to B is parallel to that from E to C

(d) $\angle$CEF is equal to $\angle$BIE

(e) $\angle$DIB is equal to $\angle$AIE

Q3 Write the statements in Q2 in short form. (The first one is $\overline{FE} = \overline{EG}$.)

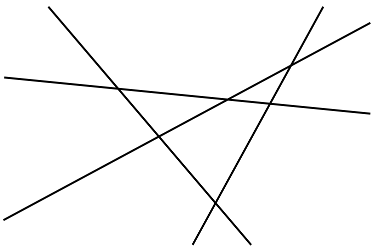
Geometric Theorems

There are ways of working out the size of angles on a geometric diagram without having to measure them all. These methods are called **geometric theorems**. We will look at some of those theorems here. However, doing Q4 will lead you to discover some of them yourself and that is the best way to learn them.

Practice

Q4 Take a sheet of A4 paper, a pencil, a ruler and a protractor. Draw four lines on the paper so that each line crosses all the others, none at 90°, and so that the intersections are all at least 2 cm apart.

This should produce 6 intersections and 24 angles. Measure all the angles.

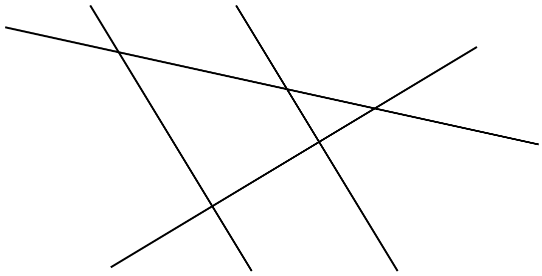


Then, on another sheet, draw another four lines at different angles. This time try to find all 24 angles without measuring them all.

Repeat again. Try to find all 24 angles with as few measurements as possible. How many angles do you actually need to measure?

List the short-cuts you used to avoid measuring angles.

Repeat the activity with two of the lines parallel.

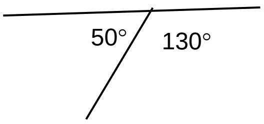


TYXDOH

We will look at six theorems that can be used to work out angles produced by intersecting lines.

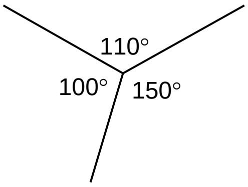
T-Angles

Where two (or more) angles together make a straight angle, the sum of the angles is 180°. So each is 180° minus the other(s).



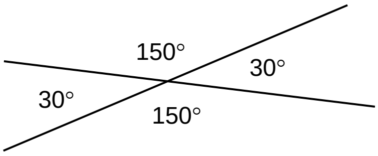
Y-Angles

The angles surrounding a point add up to 360°



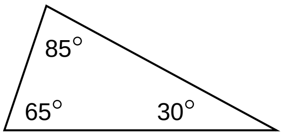
X-Angles

Where two lines cross, the angles opposite each other are equal.



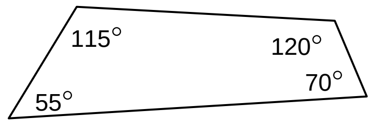
D-Angles

The three internal angles in a triangle always add up to 180°. They are called D-Angles because the letter D is a bit like a triangle. Mm…



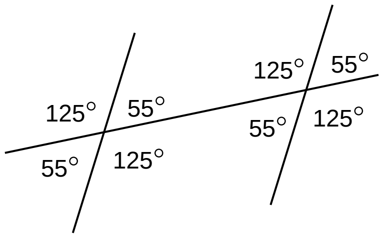
O-Angles

The four angles in a quadrilateral always add up to 360°. They are called O-Angles because the letter O is a bit like a quadrilateral.



H-Angles

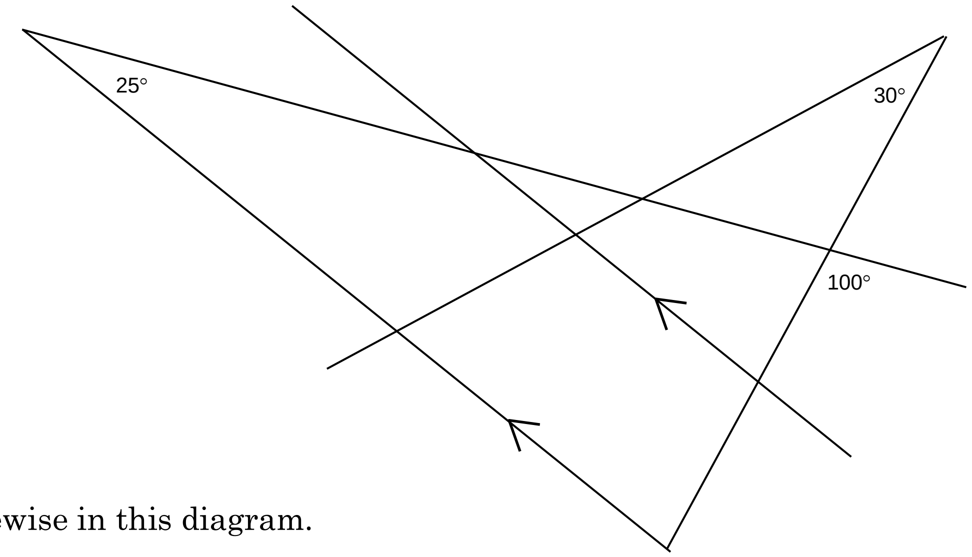
When a line crosses two parallel lines, the four angles at one intersection are the same as the four angles in the same positions at the other intersection.



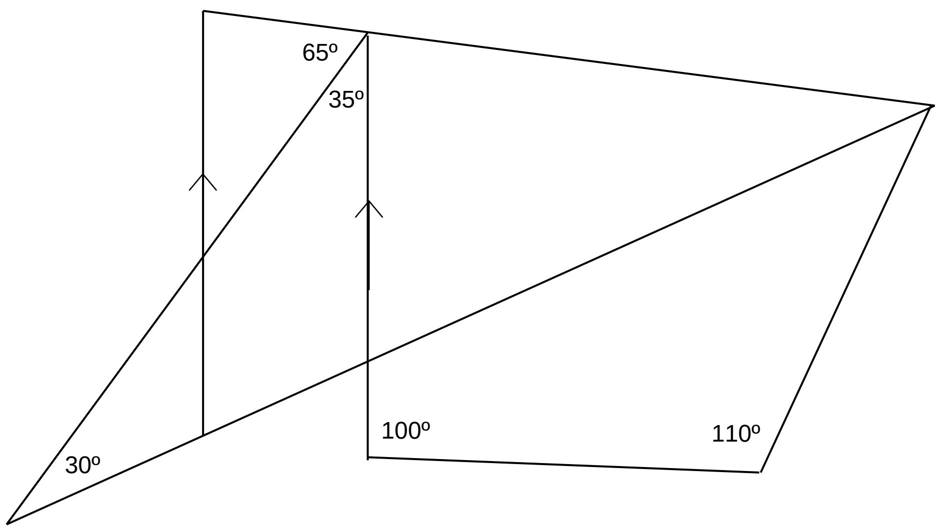
TYXDOH is a word that can help you to remember the six theorems.

Practice

Q5 Using these theorems, it is possible to work out all the angles in this diagram. Copy the diagram and write in all the angles.



Q6 Likewise in this diagram.

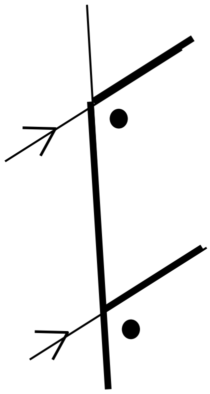


Formal Names

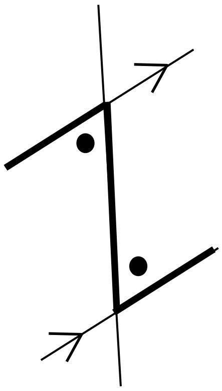
Before finishing this module, it might be worth being aware of the more formal names for X-angles, T-angles etc. You may need to know these. However, be aware that the main purpose of the module is to help you learn to use logical deduction to find angles rather than to know the proper names for the theorems. Anyway, here they are.

T-angles	**supplementary angles**
Y-angles	**angles at a point**
X-angles	**vertically opposite angles**
D-angles	**angles in a triangle**
O-angles	**angles in a quadrilateral**

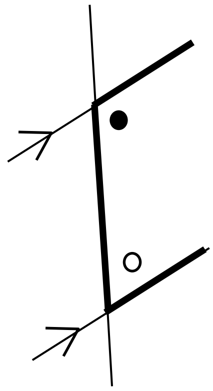
H angles are divided into F, Z and U angles, depending on which part of the H is used.



F: **Corresponding angles**
These are equal



Z: **Alternate angles**
These are equal



U: **Co-interior angles**
These add to 180°

Solve

Q51 Use the T-angle theorem to prove the X-angle theorem.

Note that you need to show that the theorem works for any angles, so you have to use variables for the angles rather than specific values. If you're not sure about this, have a look at how it's done in the Answers section, then try it by yourself.

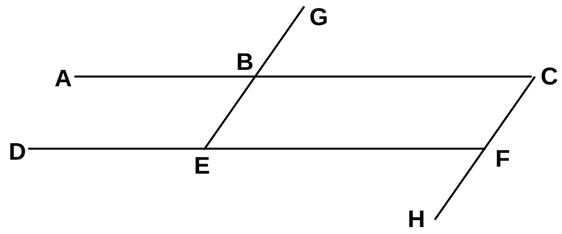
Q52 Using just the Z-angle theorem and the Y-angle theorem, prove the D-angle theorem.

Q53 Using the D-angle theorem, show that the sum of the internal angles in any polygon with n sides is $180\,(n-2)°$.

Revise

Revision Set 1

Q61 Look at the diagram to the right and write:



(a) the two possible names for the line segment from B to G

(b) the two possible names for the angle made by the line segment from F to E and the line segment from F to C

(c) the name for the parallelogram in the picture

Q62 Copy the diagram and put symbols on it to show:

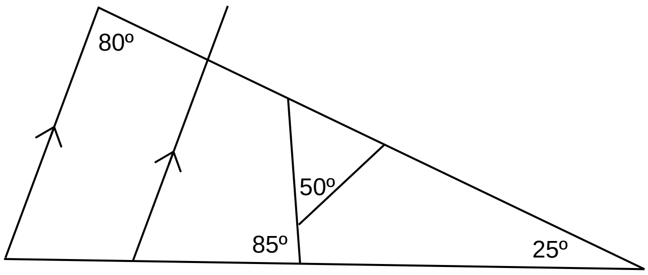
(a) that the line segment from B to E is the same length as that from C to F

(b) that the line segment from A to B is parallel to that from D to E

(c) that the angle at C is equal to one of those at F

(d-f) Rewrite the statements in (a-c) in short form (e.g. ∠ABC = ∠XYZ).

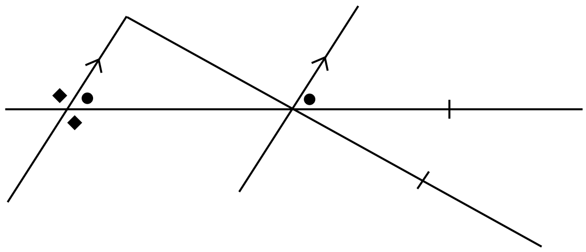
Q63 Copy this diagram and mark in the sizes of all the angles.



Answers

Q1 (a) $\overline{AB}$, $\overline{BA}$ (b) $\overline{FD}$, $\overline{DF}$ (c) ∠BEF, ∠FEB
(d) triangle BIE (or those letters in different orders)

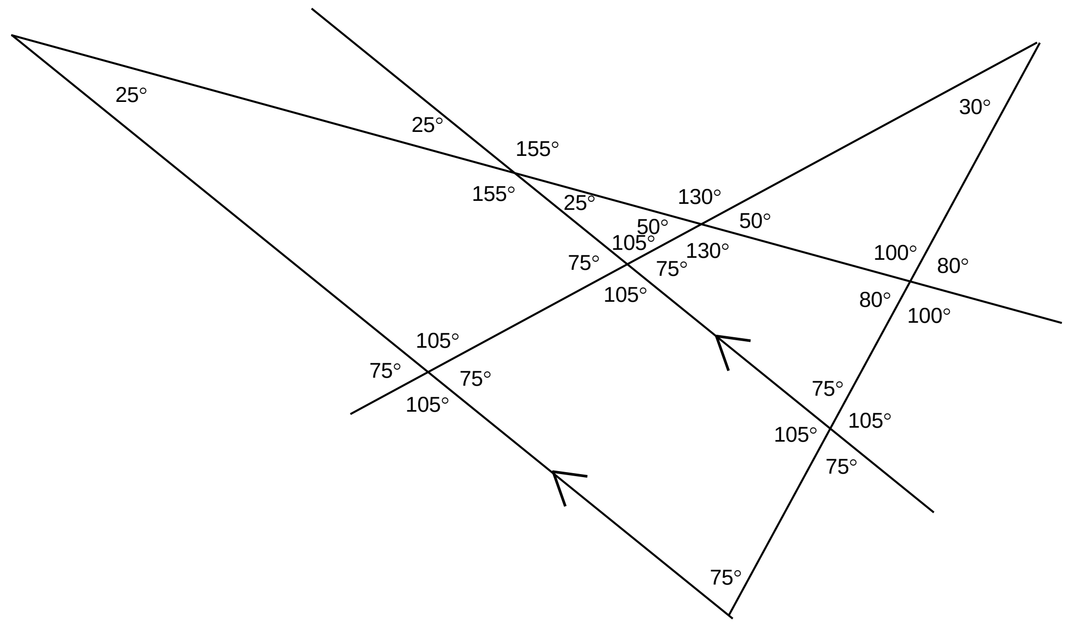
Q2



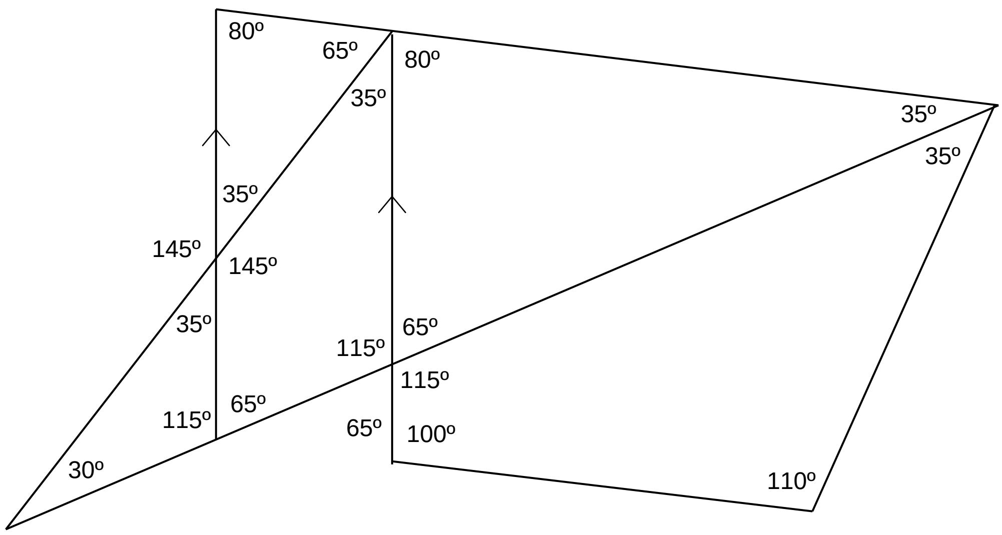
Q3 (a) $\overline{FE} = \overline{EG}$ (b) $\overline{AI} = \overline{IB}$ (c) $\overline{IB} \parallel \overline{EC}$
(d) ∠CEF = ∠BIE (e) ∠DIB = ∠AIE

Q4 With none of the lines parallel, you should be able to find all the angles, measuring only 3 of them. With two of the lines parallel, you should only have to measure 2 angles.

Q5



Q6



Q51 Let ∠AEB be θ

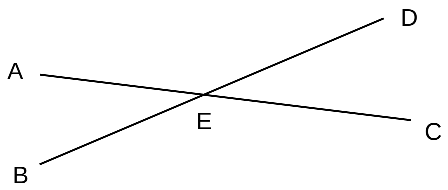
Then ∠BEC = 180° – θ (T-angles)

Then ∠CED = 180° – (180° – θ)

= 180° – 180° + θ

= θ

∠AEB always equals ∠CED



Q52 Construct a line parallel to the base through the apex as shown.

Let ∠BAD be a

Let ∠ABD be b

∠ADC = a (Z-angles)

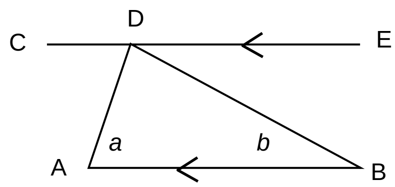
∠BDE = b (Z-angles)

∠CDE = 180°

∠ADB = 360° – 180 – a – b (Y-angles)

= 180° – a – b

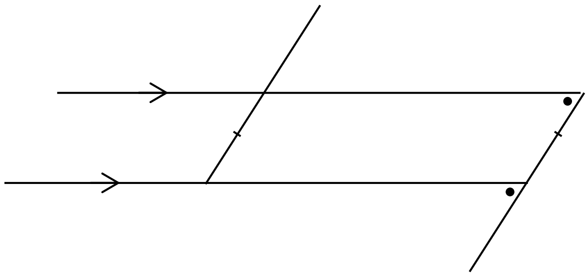
The three angles of the triangle add to 180° – a – b + a + b = 180°



Q53 Let O be any point inside an n-sided polygon.
Connect O to each vertex of the polygon to make n triangles.
The sum of the angles in each triangle is 180.
$\therefore$ The total in the n triangles is $180n$.
360 of these are at O.
The rest form the internal angles of the polygon.
$\therefore$ The sum of the internal angles of the polygon is $180n - 360$.
$= 180n - 180 \times 2$
$= 180(n - 2)$

Q61 (a) $\overline{BG}, \overline{GB}$ (b) $\angle$EFC, $\angle$CFE
(f) BCFE, CFEB, FEBC, EBCF, BEFC, EFCB, FCBE or CBEF

Q62



(d-f) $\overline{BE} = \overline{CF}$; $\overline{AB} \parallel \overline{DE}$; e.g. $\angle$BCF = $\angle$EFH

Q63