M1 Maths

G1-3 Position

- describing position using distance and the eight compass points, distance and bearing, coordinates and latitude and longitude

Summary Learn Solve Revise Answers

Summary

The position of an object relative to a given position (origin) can be expressed by saying what direction it is in, chosen from one of the eight compass points, and how far in that direction.

It can be specified more accurately by giving the direction as a bearing (the number of degreed clockwise from north to that direction), along with the distance.

A position can also be expressed as a position on a grid – how many units to the right, followed by how many units up, from a given starting point (origin).

A position on the Earth can be given using its latitude and longitude – how many degrees up (north) or down (south) from the equator and how many degrees east or west of a given starting north-south meridian (usually taken as the one through Greenwich in London, UK).

Learn

The Eight Compass Points

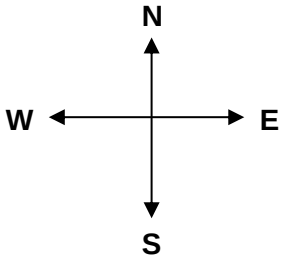
Directions on the Earth are usually described in terms of **north**, **south**, **east and west**.

North is the direction to the North Pole. It is the direction you would have to walk (and swim) without turning to arrive at the North Pole. The North Pole is the point on the Earth where the Earth's axis of rotation passes through the surface. It's in the Arctic Ocean north of Greenland – in case you are ever looking for it. It's cold there.

South is the direction opposite from north. If you face north and turn 180° in either direction, you will be facing south. It is also the direction you would have to walk (and swim) to reach the South Pole. The South Pole is in Antarctica, just beside an American research base. It's cold there too.

East is the direction you would face if you faced north, then turned 90° to the right (clockwise). West is opposite east. It is the direction you would face if you faced north, then turned 90° left.

Maps are usually drawn with north upwards (because people in the northern hemisphere invented maps). So the four directions on a map would look like this. Note that north is often written as an upper case N, east as E, south as S and west as W.



North, south, east and west are the ***four* compass points**. To remember which is which, some people remember the saying *Never Eat Soggy Weet-bix* (NESW). This is the order of the compass points starting from north and going clockwise.

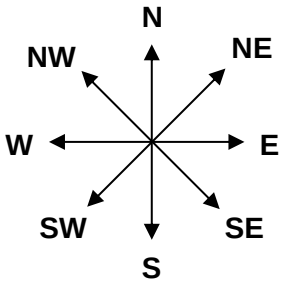
Sometimes we want to refer to the direction between north and east.

We call this northeast (NE).

NE is 45° clockwise from N

Likewise the direction between south and east is called southeast (SE). Southwest (SW) is between south and west. Northwest (NW) is between north and west.

Along with N, S, E and W, these make the ***eight* compass points**.



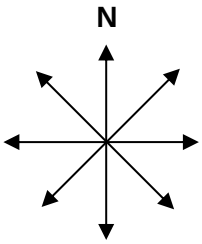
Note that in NE, NW, SE and SW, the N or S always comes first, the E or W second.

Distance and the Eight Compass Points

One way to specify the location of something is to say what distance and direction it is from a given starting point or origin. For instance, we might say the treasure is buried 80 m NE of the big tree or 30 m S of the cemetery gate.

Practice

Q1 Copy the diagram to the right and write on the other seven compass points.



Q2 (a) If you faced north then turned 90° left, what direction would you then be facing?

(b) If you faced SE then turned 90° to the right, what direction would you then be facing?

(c) What direction is opposite NW?

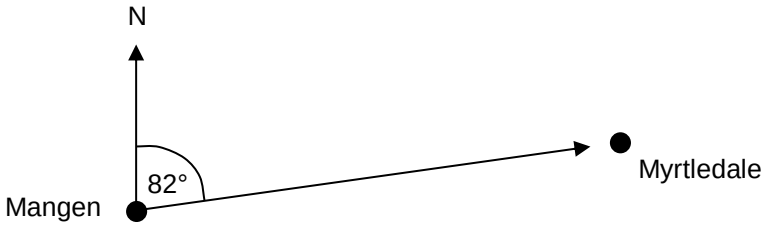
Q3 On a sheet of paper, mark a point (origin) and assume that north is directly up the page. Then mark the following locations relative to the origin. Assume also that 1 cm on the map represents 1 km. So, for instance for (a), mark the point 3 cm directly below the origin.

(a) 3 km S (b) 5 km NE (c) 4.5 km W (d) 2 km E

Distance and bearing

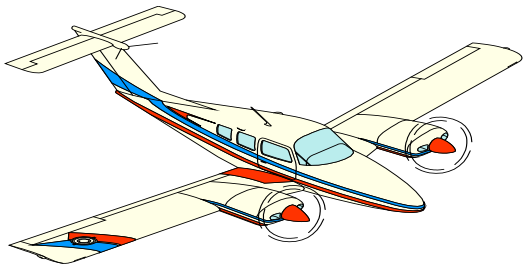
The distance and compass points method can be used in navigation and activities that require navigation like orienteering. However, in most cases, though, we need to specify the direction more precisely than just E or NE. For example, if someone was flying from Mangen to Myrtledale, you wouldn't want to tell them to fly E or NE, but somewhere in between.

There is a way of being more precise with our directions and that is to use **bearings** rather than the 8 compass points. To work out the bearing from Mangen to Myrtledale, imagine standing at Mangen facing North. Then turn clockwise (to the right) until you are facing Myrtledale. The number of degrees you turned is the bearing.



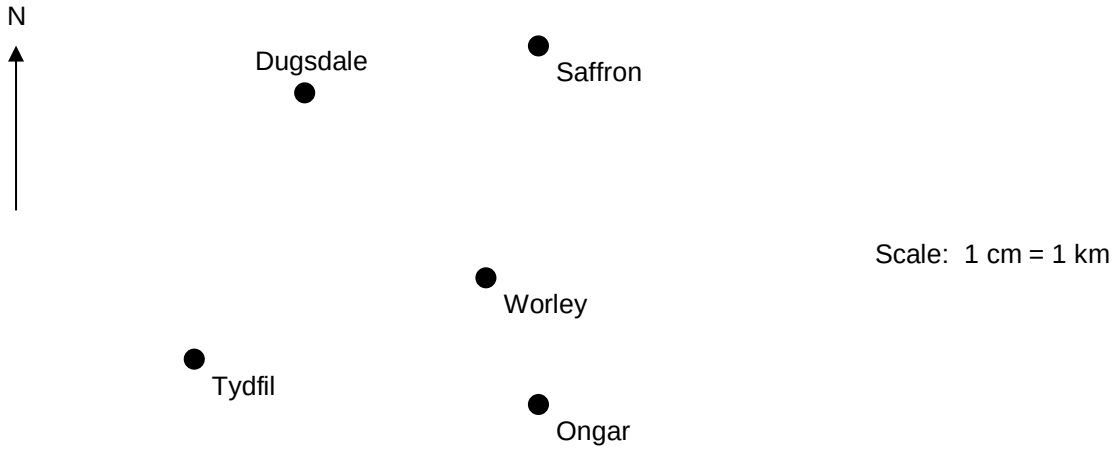
The diagram above shows that the bearing of Myrtledale from Mangen is 82°. Telling the pilot to fly on a bearing of 82° would be a lot better than telling her to fly E or NE.

Bearings are used by pilots and sailors to navigate. They are also used by bush walkers and people doing orienteering. All these people use magnetic compasses to tell which way is north. Most compasses are designed so that it is easy to set the course on a particular bearing.



In the diagram below, the distances and bearings of the places shown from Worley are:

Saffron 3.2 km, 12°; Ongar 1.8 km, 158°;
Tydfil 4.0 km, 255°; Dugsdale 3.4 km, 316°.



Note that to work out the distances, we measure from the centre of one spot to the centre of the other spot. For instance the centre of the Worley spot to the centre of the Saffron spot is 3.2 cm. Then we use the scale on the map to change to real distance 1 cm = 5 km, so 3.2 cm = 3.2 ×5 km = 16 km.

To measure the bearings, we draw a line going north from Worley (parallel to the North line given at the side of the map, then we draw lines between Worley and the other towns, then we use a protractor to measure the bearing angles.

Practice

Q4 Copy and complete the following table showing the distance and bearing of each town from each of the others.

To

From	Dugsdale	Saffron	Tydfyl	Worley	Ongar
Dugsdale	–				
Saffron		–			
Tydfyl			–		
Worley	3.4 km, 316°	3.2 km, 12°	4.0 km, 255°	–	1.8 km, 158°
Ongar					–

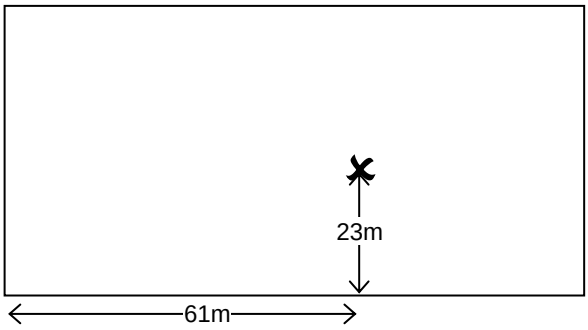
Positive Coordinates

Short John Silver was making a collection of animal skeletons. He picked up dead animals, then buried them for a couple of months until the worms had eaten everything but the bones, then he dug them up again. He buried the animals in different places on the school football field. Trouble was, when it came time to dig them up, the grass had all grown back and he couldn't find where he had buried them.

But Short John devised a scheme. Each time he buried an animal he recorded where it was. He did this by starting at the corner of the field near the library. Then he walked so many metres along the side of the field. Then he turned 90° and walked so many metres across the field.

He wrote down the number of metres he walked along the side and the number of metres he walked across. Then, when it came time to dig up the skeleton, he just walked the same numbers of metres along and across, then dug. And hey presto, there were the bones.

Short John was using **coordinates** to record the locations of his animals. If the rectangle below is the football field, then the measurements shown are the coordinates.



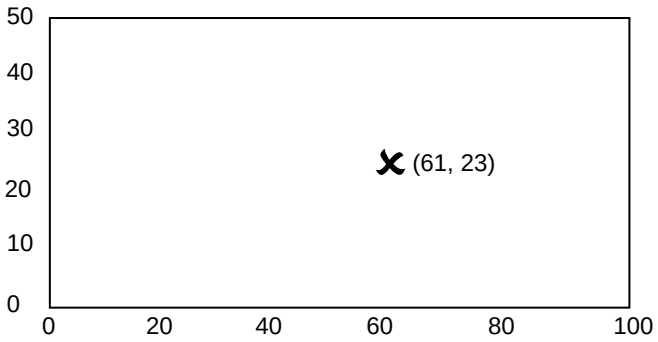
On paper we generally use the bottom left hand corner as the starting point for coordinates (this is called the **origin**). Then we give the distance we travel to the right as the first coordinate. Then we give the distance we travel upwards as the second coordinate. So the × would be at 61 m right, 23 m up.

In fact, we always give the distance to the right first and the distance up second and, because of this, we can abbreviate the coordinates to (61, 23). We know that the first number (61) is the distance to the right and that the second number (23) is the distance up.

Remember always: **RIGHT** then **UP**

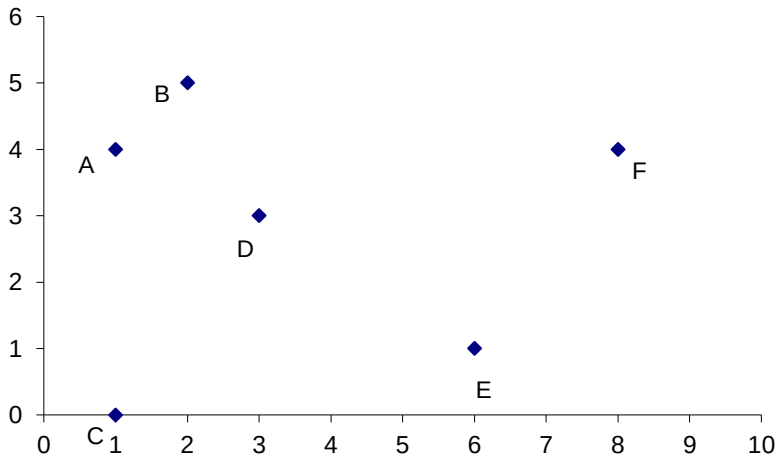
You can remember this because R comes before U in the alphabet, so we go RIGHT then UP.

If we put scales along the bottom edge of the field and up the left side of the field to make it a graph, we would notice that the point (61, 23) is in line with the 61 on the bottom (horizontal) axis and in line with the 23 on the side (vertical) axis.



Practice

Q5 Give the coordinates of points A to F on the diagram below.



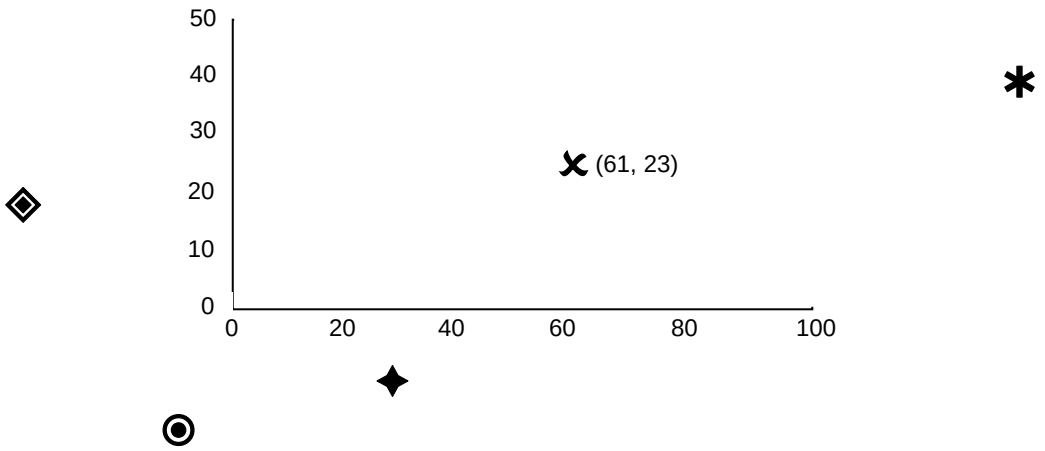
Q6 Copy the axes above and plot the following points:
(a) G (3, 1) (b) H (4, 6) (c) I (0, 2) (d) J (9, 2) (e) K (0, 0)

Positive and Negative Coordinates

You have just learnt how to define position using positive coordinates. We have two axes at right angles meeting at a starting point called the origin. We move so many units to the right along one axis and then so many units upwards parallel to the other axis. To get to the point marked × below from the origin, we move 61 units to the right, then 23 units upwards. The coordinates of × are (61, 23). The brackets and the comma show that they are coordinates and are always included when writing coordinates.

The distance right is always written first. Because we know this, we don't have to write (61 right, 23 up).

Remember always: **RIGHT** then **UP** (alphabetical order)



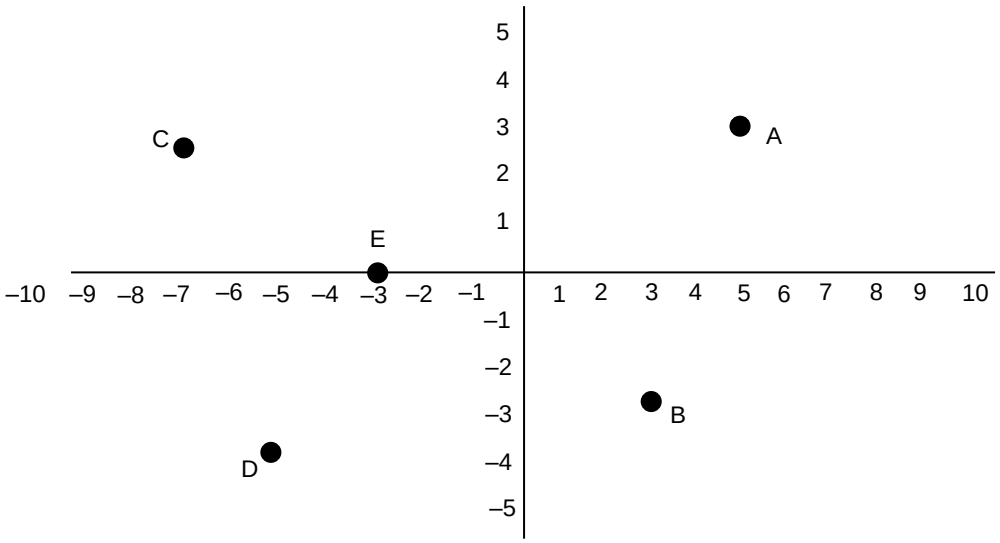
The horizontal axis is often called the ***x*-axis** and the vertical axis is often called the ***y*-axis**. Likewise, the first coordinate is called the *x*-coordinate and the second coordinate is called the ***y*-coordinate**. The *x*-coordinate of × is 61 and the *y*-coordinate is 23.

This method allows us to specify the location of any point as long as it is above the *x*-axis and to the right of the *y*-axis. We could go off the area shown: the point ✱ has an *x*-coordinate greater than 100. In fact its coordinates are (134, 41). The point ◈ is left of the *y*-axis, so its *x*-coordinate is less than 0. In fact it is –37. The coordinates of ◈ are (–37, 18).

Likewise, we could define a point below the *x*-axis, but its *y*-coordinate would be negative. The point ✦ has coordinates (29, –12). The point ⊙ is both to the left of the

y-axis and below the x-axis, so its x-coordinate and its y-coordinate are both negative. Its coordinates are (–6, –20).

To allow for negative coordinates, we often draw the axes going in all four directions from the origin like this:

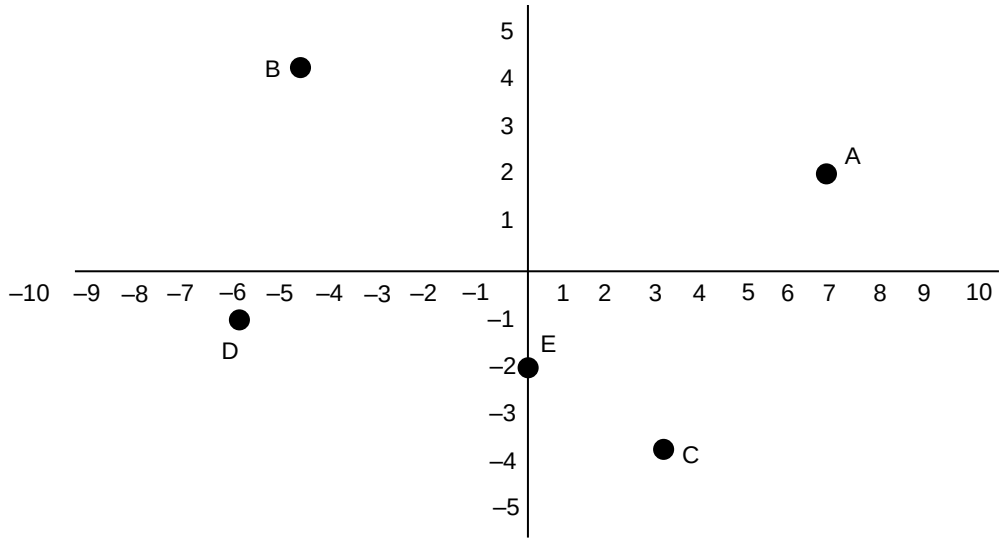


The coordinates of the points shown are as follows:
A (5, 3); B (3, –3); C (–7, 2.5); D (–5.3, –3.8); E (–3, 0).

If we use x and y instead of right and up, then, instead of **RIGHT** then **UP**, we remember $\boldsymbol{x}$ then $\boldsymbol{y}$ – still in alphabetical order. Easy.

Practice

Q7 Give the coordinates of points A to E on the diagram below.



Q8 Copy the axes from Question 1 and plot the following points on them:

(a) F (3, 1) (b) G (4, –5) (c) H (–2, 0)

(d) I (–5, –3) (e) J (0, 0)

Latitude and Longitude

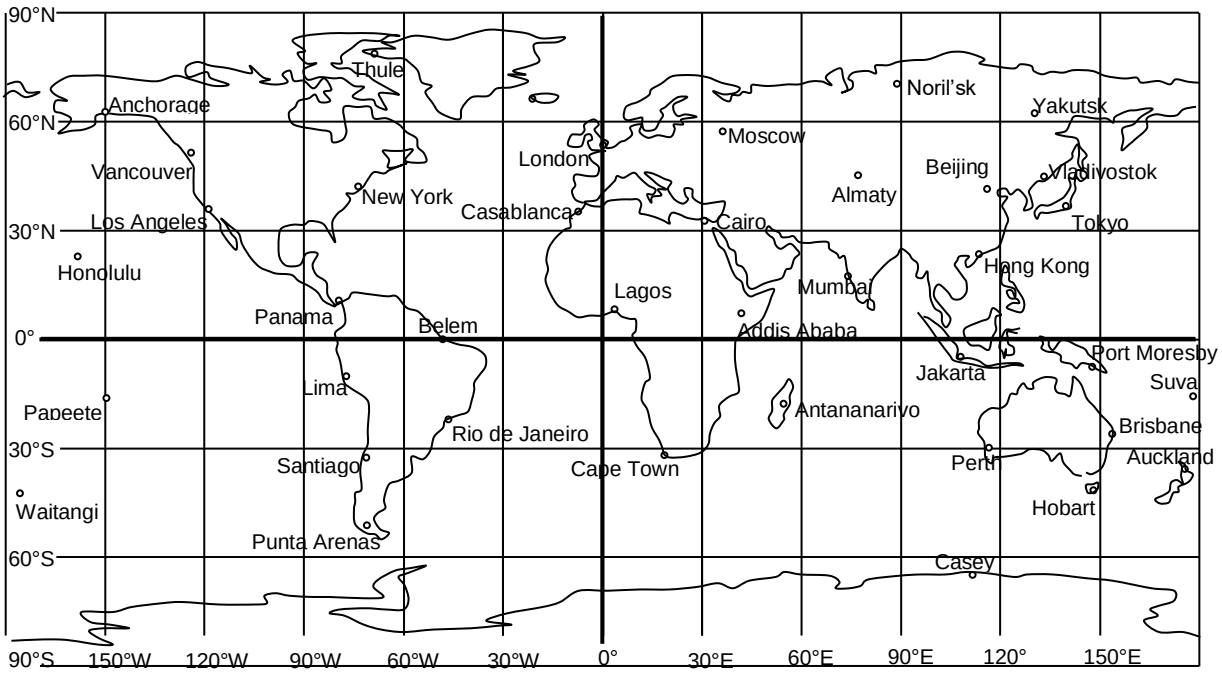
Latitude and longitude are a way of describing position on the surface of the Earth. The latitude-longitude method is very similar to coordinate method.

Latitude is how far north or south of the **equator** we are. The equator is the thick horizontal line on the map below. Latitude varies from 0° at the equator to 90° south at the south pole and 90° north at the north pole. Brisbane is about 27° S and London is about 51° N.

Longitude is how far east or west we are. With latitude the equator is an obvious starting point. But there is no obvious starting point for longitude. So we use London (Greenwich to be precise) as 0° and measure east and west from there. The thick vertical line on the map is 0° longitude.

We use London because the Brits invented the system of latitude and longitude so they got to choose. It also happens to be convenient because then 180° longitude is mostly in the Pacific Ocean.

Brisbane has a longitude of about 153° East. New York has a longitude of about 75° W.



Reading latitude and longitude

Using a map like the one above, we read latitude and longitude pretty much the same way that we read coordinates. The main difference is that we give latitude before longitude. With coordinates we give across, then up. With latitude and longitude, it is up/down, then across.

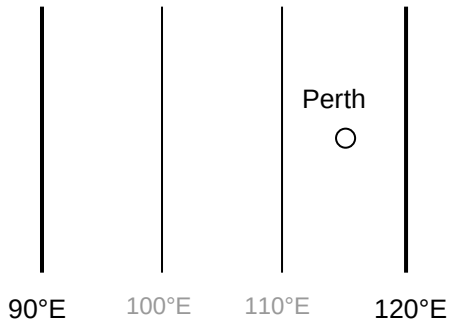
A good way to remember this is like this. We read across before up because across comes before up in the dictionary. We use alphabetical order. It is the same with latitude and longitude. Latitude comes before longitude in the dictionary. So we give latitude first, then longitude.

So using the map, we would give the locations of cities like this:

Perth	30°S 115°E
Moscow	56°N 37°E
Santiago	34°S 72°W

Reading between the lines

Note that the latitude of Perth is easy to read because it is on the line – 30°S. But the longitude is a bit harder because it is between the lines. It is between 90°E and 120°E. In this case we have to estimate it. We can see that it is quite a bit closer to 120°E. We estimate 115°E. It can help if we use a pencil to lightly mark in lines between 90° and 120° - say at 100° and 110°.



Then we can see that Perth is about half way between 110° and 120° – about 115°

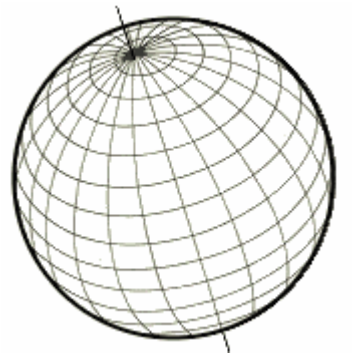
Lines of latitude and longitude

On the map of the world, the vertical line through Greenland and the Atlantic Ocean, marked 30°W, goes through all places with a longitude of 30°W. This line is called a **line of longitude** – the 30°W line of longitude. All the vertical lines are called lines of longitude. In the same way the horizontal lines are called **lines of latitude**.

It can be a little confusing that latitude is how far N or S you are, but the lines of latitude run E-W. Make sure that this makes sense to you.

The 0° line of latitude has a special name – the Equator. In the same way, the 0° line of longitude has a special name – the **Greenwich Meridian or the Prime Meridian**. Meridian is another name for line of longitude.

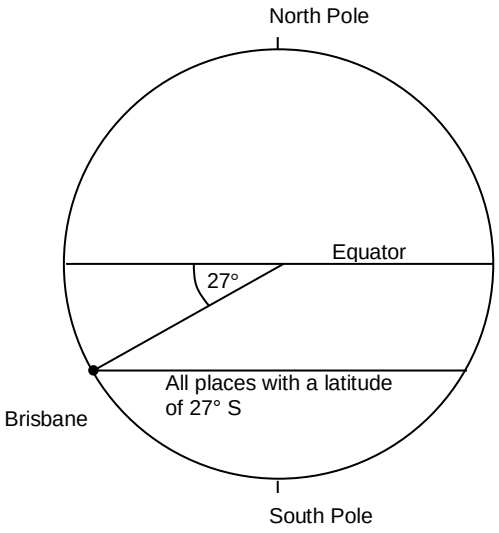
Lines of latitude and longitude are shown as making a square grid on the map. If we look at a globe, though, they make a bit of a different pattern as shown below.

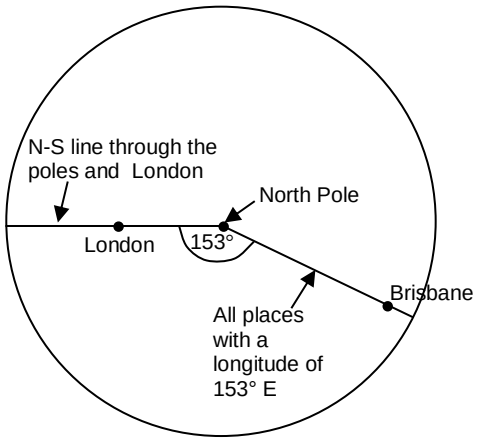


The lines of longitude all run from the North Pole to the South Pole and the lines of latitude run around the poles. The difference is because you cannot produce a flat map of a spherical Earth without distorting it.

Why latitude and longitude are measured in degrees rather than kilometres

Imagine you were at the centre of the Earth and the Earth was transparent. You could look at the equator north of Brisbane. Then you would have to turn your eyes 27° down to look at Brisbane. So we say the latitude of Brisbane is 27° S. Make sense of the diagram on the left below to make this more obvious.





Likewise, imagine yourself at the centre of the Earth looking at the Greenwich Meridian where it crosses the equator. You would then have to turn your eyes 153° to the left (east) to look at where the 153° E line of longitude crosses the equator. Brisbane is on this line of longitude, so we say the longitude of Brisbane is 153° E. Make sense of the diagram on the right to make this more obvious.

Pronouncing longitude

Note also that longitude is spelt LONGITUDE, not LONGTITUDE and is pronounced LONGITUDE without a TIT. Many people wrongly pronounce it with a TIT. Try not to do that.

Practice

Use the map of the world to do the following.

Q9 Give the latitude and longitude of the following cities:

(a) Hobart (Australia) (b) Hong Kong (China)
(c) New York (USA) (d) Rio de Janeiro (Brazil)
(e) London (England) (f) Cape Town (South Africa)

Q10 Name the city at each of the following locations:

(a) 18° S 178° E (b) 56° N 38° E (c) 43° N 132° E
(d) 49° N 123° W (e) 34° S 18° E (f) 1° S 49° W

Solve

Q51 What would be your approximate distance and bearing from the grain silo if you started at the silo, then walked 400 m east, then 400 m south?

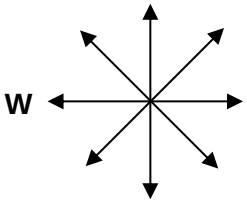
Q52 Frappleton is at 34°N 92°W. Marville is at 37°N 89°W. Roughly what direction is Marville from Frappleton? Answer as a compass point and a bearing.

Q53 Anchorage in Alaska is 62°N 150°W. Saint Petersburg in Russia is 60°N 30°E. If you set off from Anchorage to fly the shortest route to Saint Petersburg, in roughly which direction would you fly?

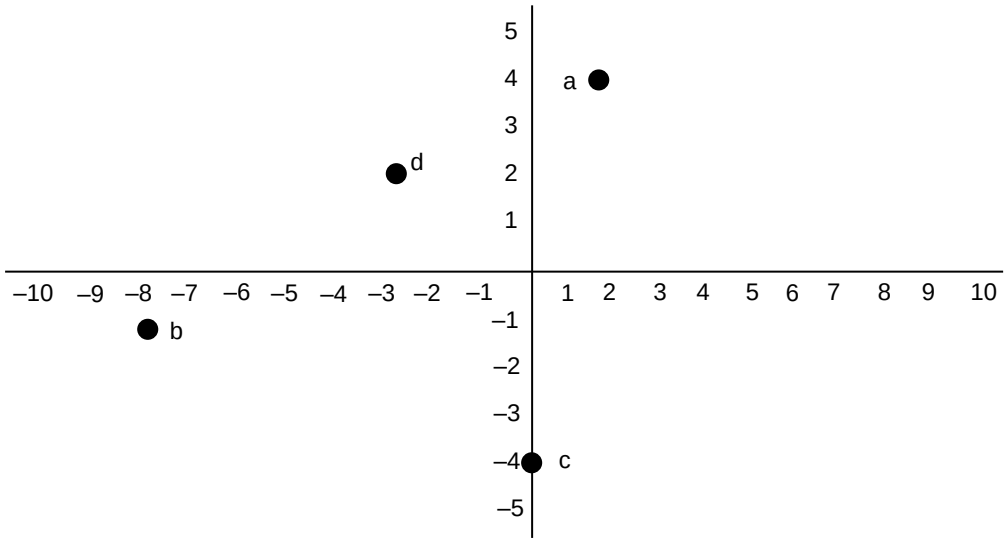
Q54 If the square with vertices (0, 0), (1, 0), (1, 1) and (0, 1) has an area of 1 cm^2, What is the area of the rectangle with vertices at (−2, −1), (6, −1), (6, 3) and (−2, 3)?

Revision Set 1

Q61 Copy the diagram to the right and write on the other seven compass points.



Q62 Give the coordinates of points (a) to (d) on the diagram below.

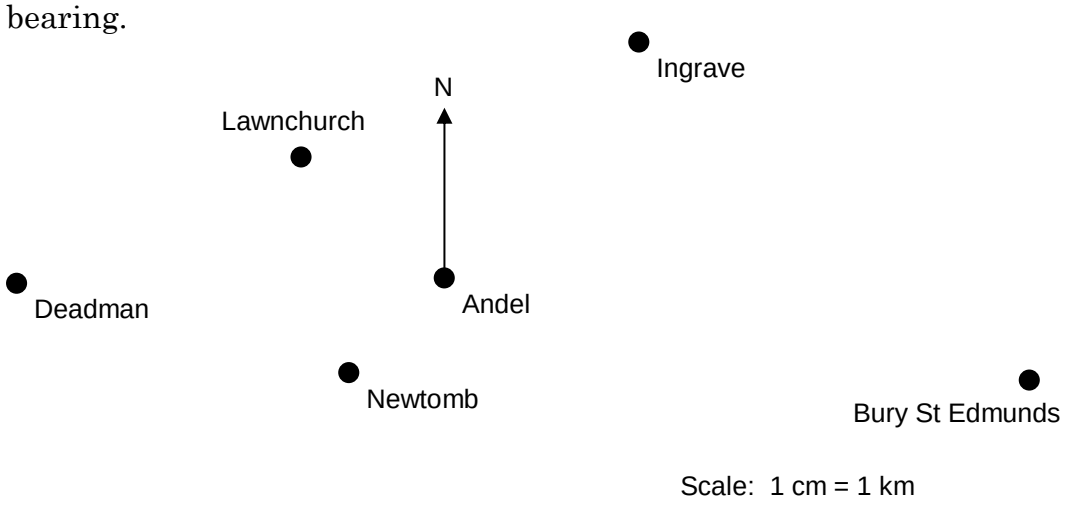


Q63 Copy the axes above and plot the following points on them:
(a) (–6, 0) (b) (2, –4) (c) (–1, 4)

Q64 Use the map of the world to give the latitude and longitude of:
(a) Perth (Australia) (b) Moscow (Russia)
(c) Lima (S. America) (d) London (England)

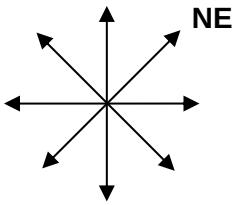
Q65 Use the map of the world to find the towns at the following locations:
(a) 11° N 78° W (b) 62° N 130° E (c) 52° S 71° W

Q66 Give the positions of the towns below relative to Andel as a distance and a bearing.

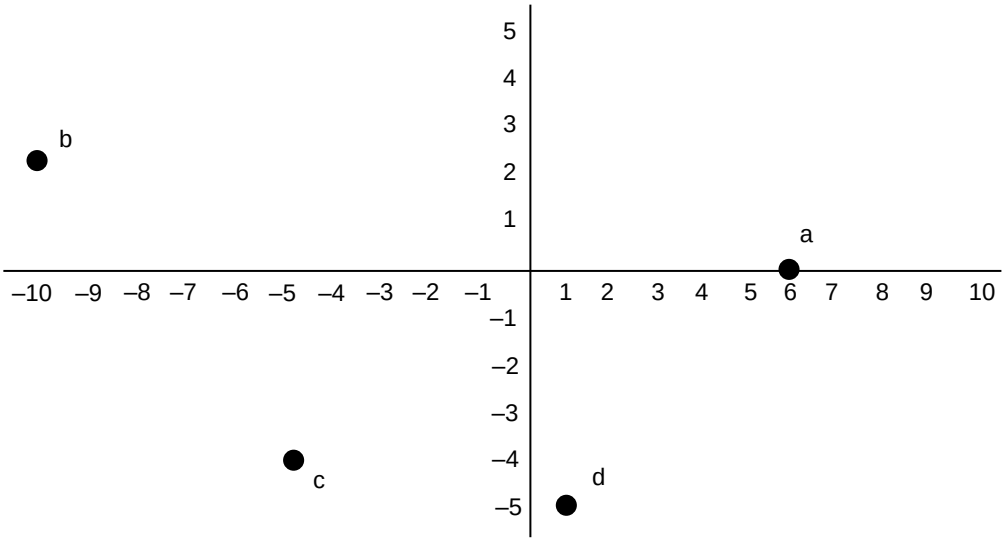


Revision Set 2

Q71 Copy the diagram to the right and write on the other seven compass points.



Q72 Give the coordinates of points (a) to (d) on the diagram below.

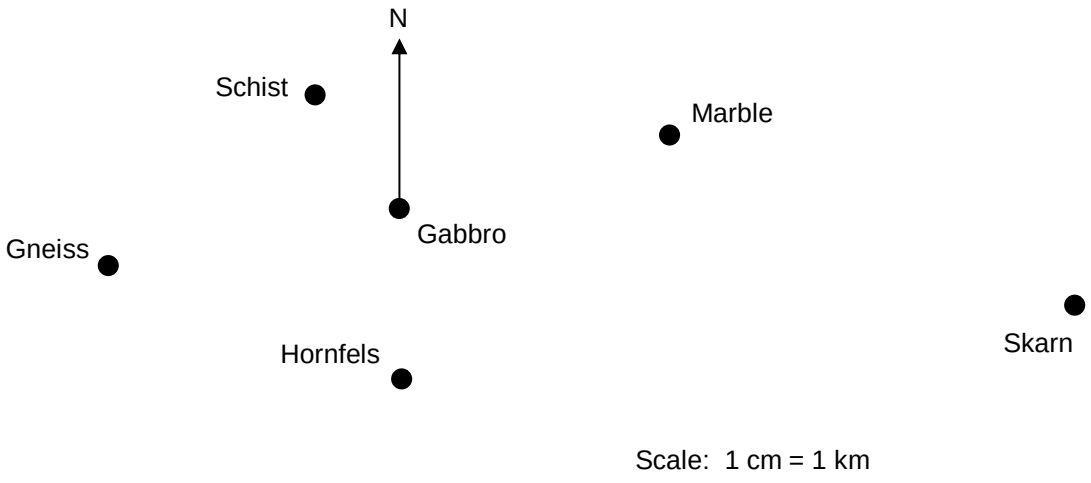


Q73 Copy the axes above and plot the following points on them:
(a) (2, 5) (b) (–6, –4) (c) (–1, 3)

Q74 Use the map of the world to give the latitude and longitude of:
(a) Thule (Greenland) (b) Cape Town (South Africa)
(c) Santiago (S. America) (d) Tokyo (Japan)

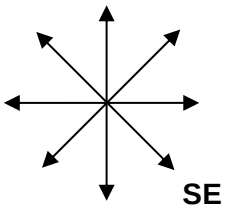
Q75 Use the map of the world to find the towns at the following locations:
(a) 8° N 42° E (b) 64° N 150° W (c) 36° N 9° W

Q76 Give the positions of the towns below relative to Gabbro as a distance and a bearing.

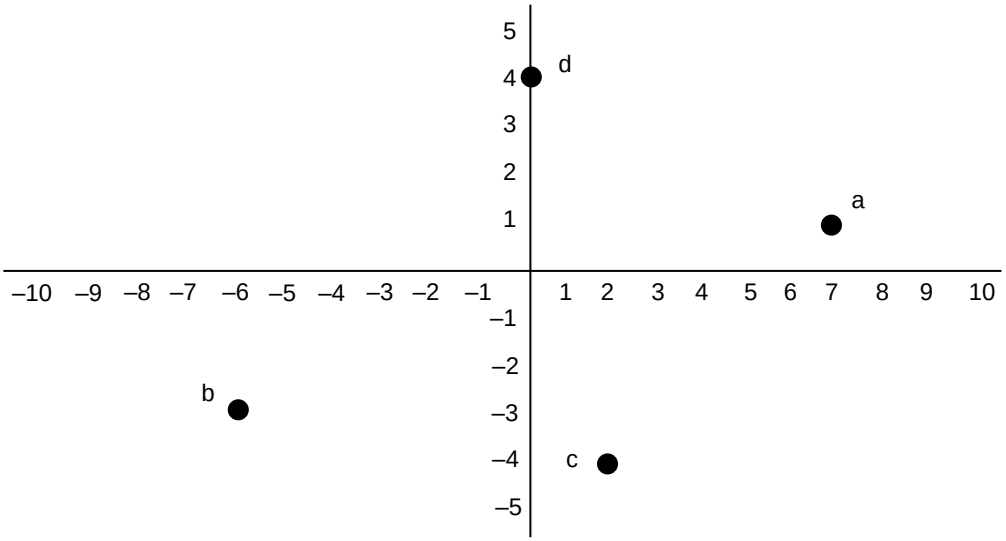


Revision Set 3

Q81 Copy the diagram to the right and write on the other seven compass points.



Q82 Give the coordinates of points (a) to (d) on the diagram below.

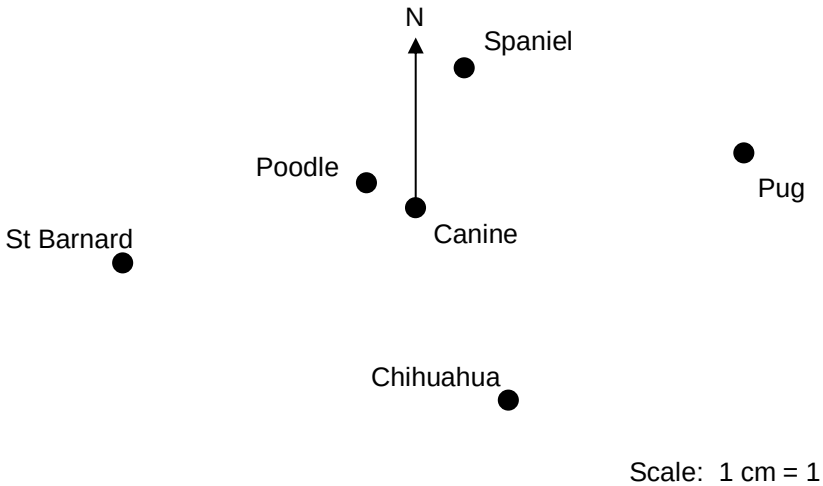


Q83 Copy the axes above and plot the following points on them:
(a) (2, 0) (b) (–2, –5) (c) (–1, 3)

Q84 Use the map of the world to give the latitude and longitude of:
(a) Casey (Antarctica) (b) Panama (Central America)
(c) Mumbai (India) (d) Beijing (China)

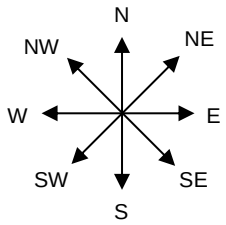
Q85 Use the map of the world to find the towns at the following locations:
(a) 1° S 48° W (b) 19° S 52° E (c) 70° N 89° E

Q86 Give the positions of the towns below relative to Canine as a distance and a bearing.



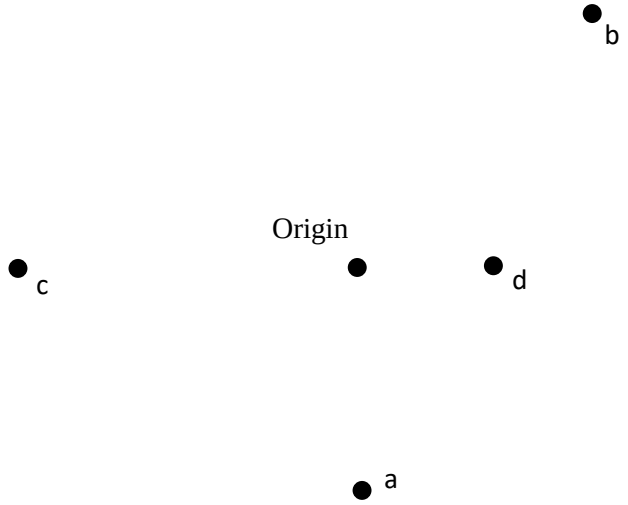
Answers

Q1



Q2 (a) W (b) SW (c) SE

Q3

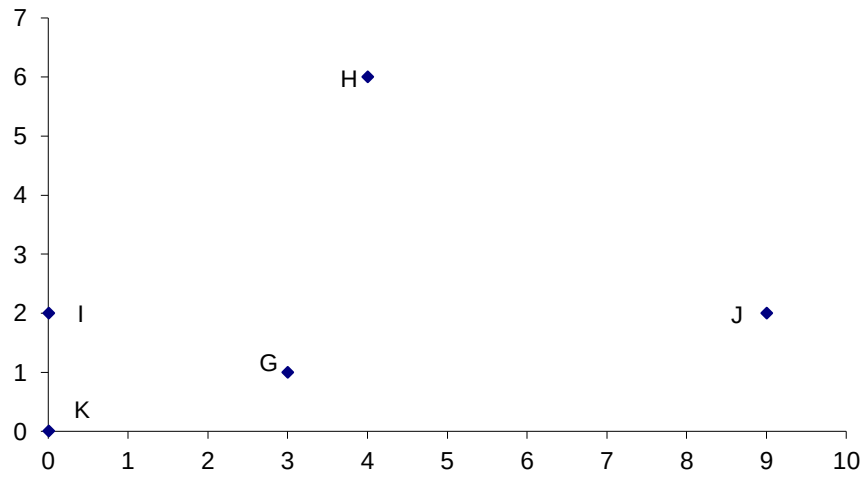


Q4

From \ To	Dugsdale	Saffron	Tydfyl	Worley	Ongar
Dugsdale	–	3.2 km, 78°	3.8 km, 202°	3.4 km, 136°	5.2 km, 142°
Saffron	3.2 km, 258°	–	5.2 km, 227°	3.2 km, 192°	4.6 km, 179°
Tydfyl	3.8 km, 22°	5.2 km, 227°	–	4.0 km, 75°	3.6 km, 98°
Worley	3.4 km, 316°	3.2 km, 12°	4.0 km, 255°	–	1.8 km, 158°
Ongar	5.2 km, 322°	4.6 km, 179°	3.6 km, 98°	1.8 km, 338°	–

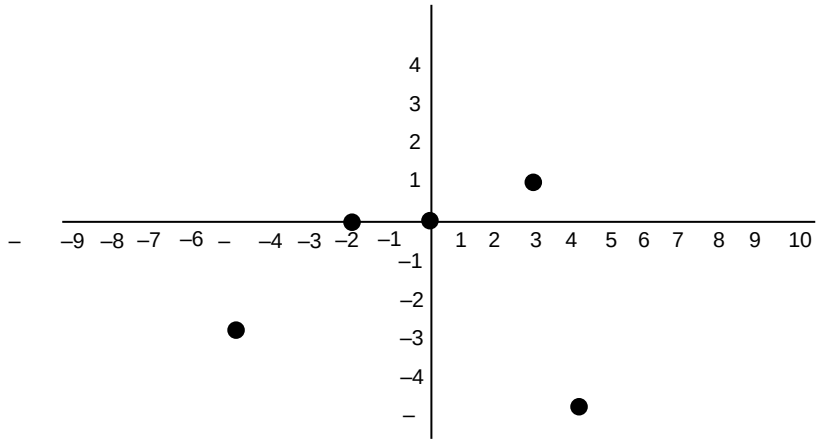
Q5 A (1, 4) B (2, 5) C (1, 0) D (3, 3) E (6, 1) F (8, 4)

Q6.



Q7 A (7, 2) B (–5, 4) C (3, –4) D (–6, –1) E (0, –2)

Q8



Q9 (a) 43° S 147° E (b) 22° N 114° E (c) 41° N 74° W
(d) 23° S 43° W (e) 51° N 0° (f) 34° S 18° E

Q10 (a) Suva (b) Moscow (c) Vladivostok
(d) Vancouver (e) Capetown (f) Belem

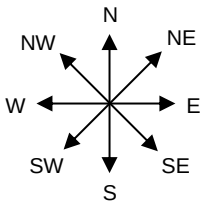
Q51 560 m 135°

Q52 NE, 045°

Q53 North. If this doesn't make sense, get a globe of the world and connect a bit of string tightly between Anchorage and Saint Petersburg.

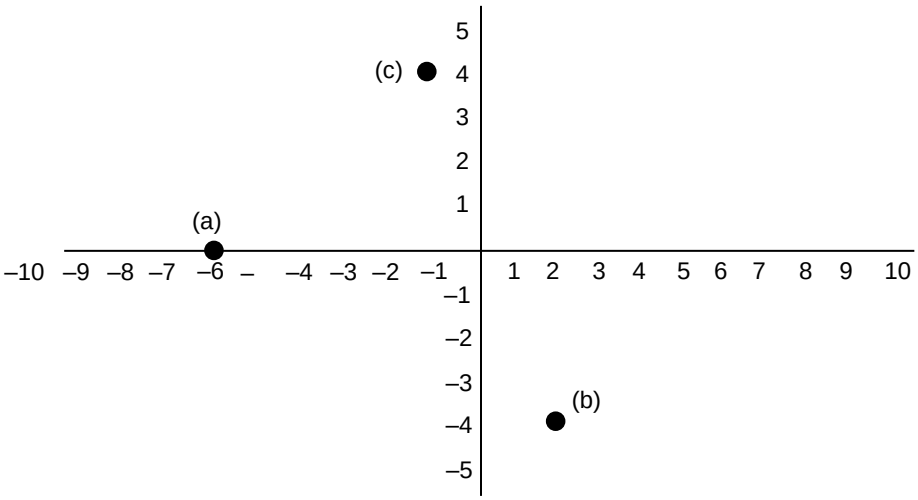
Q54 32 cm²

Q61



Q62 (a) (1, 4) (b) (–8, –1) (c) (0, –4) (d) (–3, 2)

Q63



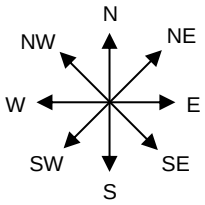
Q64 (a) 30°S 114°E (b) 57°N 38°E (c) 12°S 77°W (d) 52°N 0°

Q65 (a) Panama (b) Yakutsk (c) Punta Arenas

Q66 For these answers the distances are what you should get if you print the question on paper. If viewed on the screen, they may be different – unless you make the paper width 21 cm.
Deadman 5.7 km 270°, Lawnchurch 2.5 km 310°, Ingrave 4.1 km 39°, Bury St Edmunds

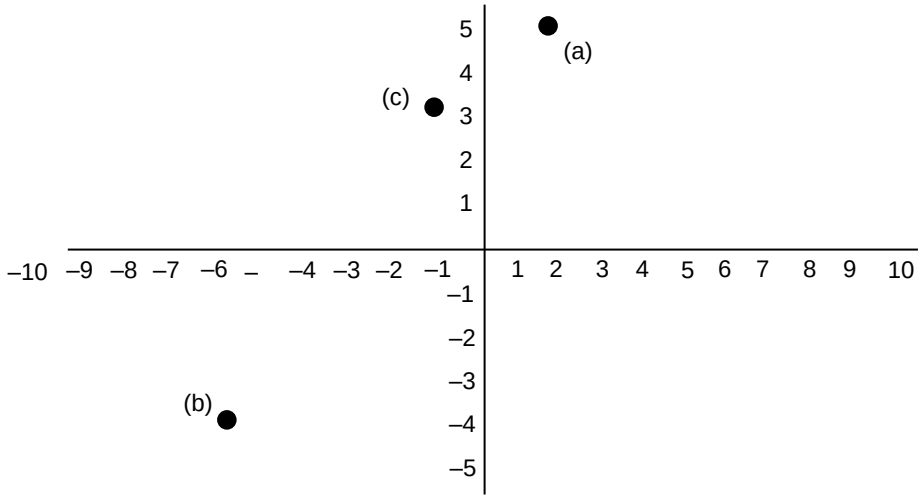
7.8 km 100°, Newtomb 1.8 km 226°.

Q71



Q72 (a) (6, 0) (b) (–10, 2) (c) (–5, –4) (d) (1, –5)

Q73

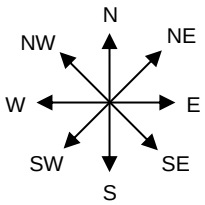


Q74 (a) 77°N 69°W (b) 32°S 18°E (c) 34°S 72°W (d) 38°N 138°E

Q75 (a) Addis Ababa (b) Anchorage (c) Casablanca

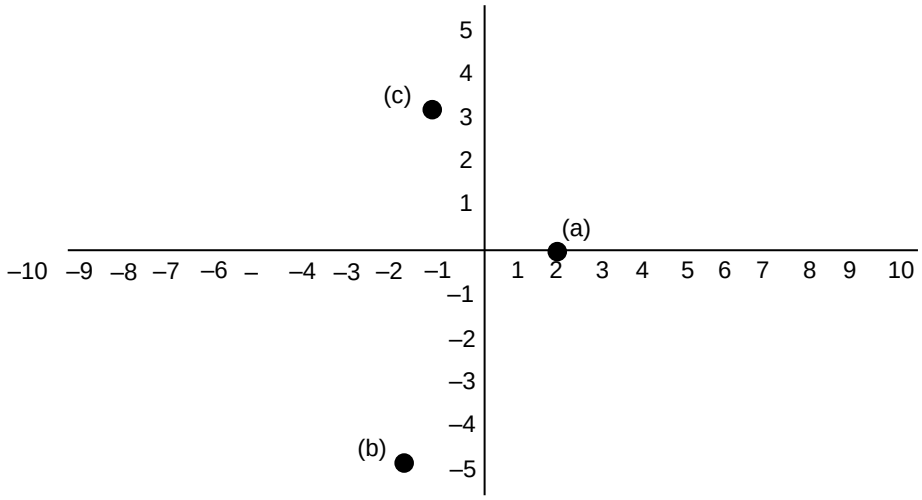
Q76 For these answers the distances are what you should get if you print the question on paper. If viewed on the screen, they may be different – unless you make the paper width 21 cm.
Gneiss 3.9 km 255°, Schist 1.8 km 330°, Hornfels 2.3 km 180°, Marble 3.7 km 70°, Skarn 9.0 km 100°.

Q81



Q82 (a) (7, 1) (b) (–6, –3) (c) (1, –4) (d) (0, 4)

Q83



Q84 (a) 66°S 112°E (b) 10°N 78°W (c) 17°N 74°E (d) 42°N 114°E

Q85 (a) Belem (b) Antananarivo (c) Noril'sk

Q86 For these answers the distances are what you should get if you print the question on paper. If viewed on the screen, they may be different – unless you make the paper width 21 cm.
St Bernard 3.9 km 260°, Poodle 0.8 km 300°, Chihuahua 2.8 km 160°, Spaniel 1.9 km 10°, Pug 4.4 km 75°.