M1 Maths

G1-2 Angles

- the meaning of 'turn' and 'angle'
- estimating, measuring and constructing angles

Summary Learn Solve Revise Answers

Summary

A turn is a change in direction. An angle is a difference in direction. Both can be measured in revolutions or in degrees. A revolution is one complete rotation back to the direction you started in. A revolution is divided into 360 degrees (360°), so a degree is $^1/_{360}$ of a revolution. An angle of 90° (¼ of a revolution) is called a right angle.

The word 'angle' is also used to refer to the shape made where two lines meet. The angle (number of degrees) is the difference in direction between the two lines.

To measure an angle with a protractor, put the centre of the protractor at the vertex (point where the two lines meet) and lay the base line along one of the lines (arms). Then, using the scale that starts from 0 on that arm, read around to the other arm.

To estimate angles, compare them to a revolution or a right angle.

To construct an angle, draw one line. Then place the centre of the protractor on one end of the line (the end that will be the vertex) with the base line along that line. Then, using the scale that starts from 0 on that line, read around the required number of degrees and make a mark at the edge of the protractor. Finally draw a second line from the vertex through that mark.

Angles are classified as acute, right, obtuse, straight or reflex, depending on their size (number of degrees).

Learn

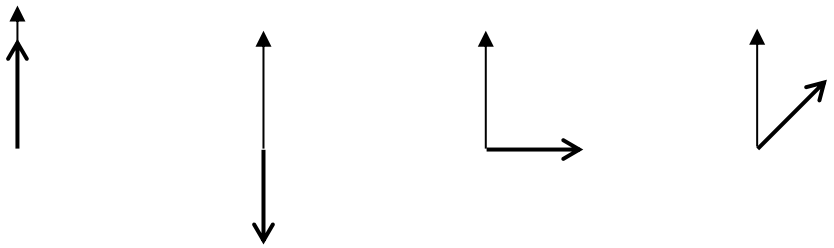
Turn and Angle

Stand facing the window. Then turn to face the door. You have performed a **turn**. A turn is a change in direction.

Turns can be measured in revolutions. Stand and face the window, then turn right around until you are facing the same window again. That turn is one **revolution**. Now turn right around three times (or maybe just imagine it so you don't get dizzy). That turn is 3 revolutions.

Face the window again, then turn and face the opposite direction. That was half a revolution. Face the window yet again, then turn to the right, then to the right again so you end up facing the opposite direction from the window. Each turn would be a quarter of a revolution.

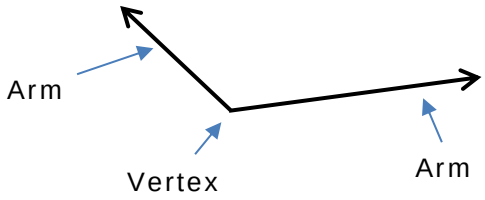
The diagram below shows some of these turns. The long thin lines with solid arrow heads show the starting directions. The short thick lines with hollow arrow heads show the finishing directions.



1 revolution ½ revolution ¼ revolution $\frac{1}{8}$ revolution

The word ***angle*** is used to describe the shape made where two lines that have different directions meet. So the diagrams above also show angles.

The lines making an angle are called the **arms** and the point where they meet is called the **vertex**.



Both turns and angles are differences in direction. So, like turns, angles can be measured in revolutions. The angle above is about a third of a revolution.

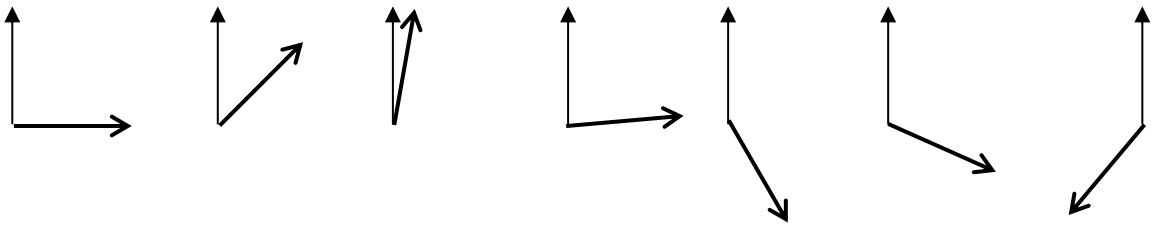
Degrees

Revolutions are a convenient unit for large turns and large angles, but they are not so convenient for small angles because the measures are always fractions. So, about 3000 years ago the Babylonians introduced a more convenient unit for angles called the **degree**. They divided a revolution into 360 equal turns and called each one a degree. So there are 360 degrees in a revolution. Degree is often written using the symbol °, e.g. 60°.

1 revolution is 360°, ½ revolution is 180°, ¼ revolution is 90°, $\frac{1}{8}$ revolution is 45° and so on. Angles of ¼ revolution or 90° are very common and important and are given a special name – right angles. The corners of buildings, boxes, sheets of paper etc. are usually right angles.

Estimating angles

You need to be able to estimate angles in degrees. Try these ones.



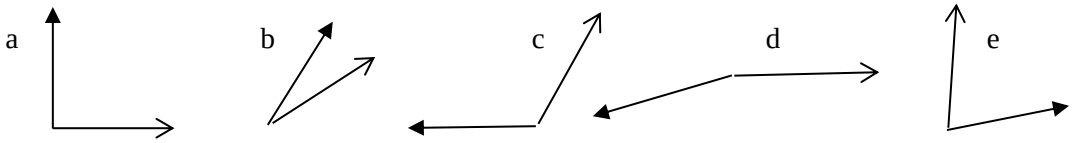
The angles are, from left to right, 90°, 45°, 10°, 85°, 150°, 114°, 220°.

Note that the last angle was taken to be a turn of 220° clockwise. It could also have been 140° anticlockwise. In fact all angles can be thought of in either direction. So the first one could be 270° anticlockwise. What would the others be if they were thought of as anticlockwise turns?

In the absence of a reason to the contrary, we normally take the smaller of the two possibilities, so we would call the last one 140°. 220° isn't wrong though.

Practice

Q1 Estimate the size of the following angles in degrees. Assume that you are estimating the smaller of the two possibilities. You should be able to get each to within 15°.

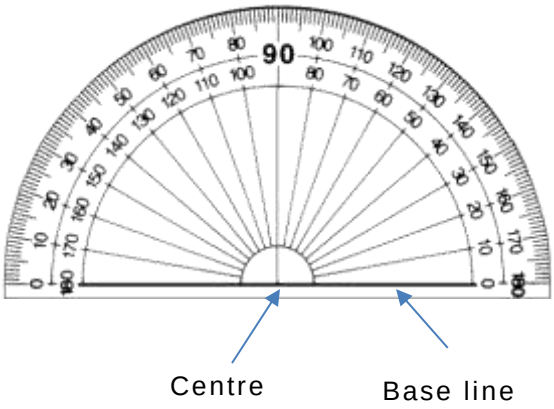


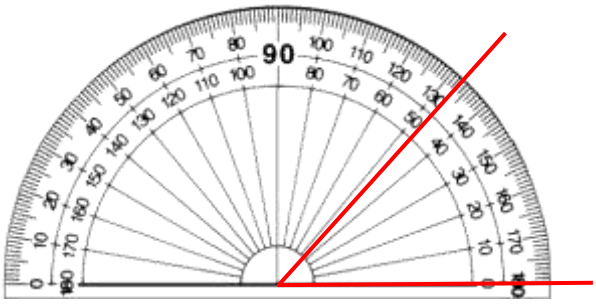
Measuring angles

If we want to know the size of an angle more accurately, we can measure it. To measure an angle we use a **protractor**.

A protractor is a transparent circular or semicircular device marked in degrees. There is a picture of a semicircular protractor below on the left. You place it over the angle (drawn in red on the diagram below on the right) with the centre on the vertex and one half of the base line along one of the arms, as shown.

You will notice that the protractor has two scales going in opposite directions. Choose the scale (inside or outside) which starts at 0 on the arm which runs along the protractor base line. Then read off the position of the other arm using that scale. With the angle in the right-hand diagram, the inside scale has zero on the angle arm, so we use that one. The angle is then 47°.

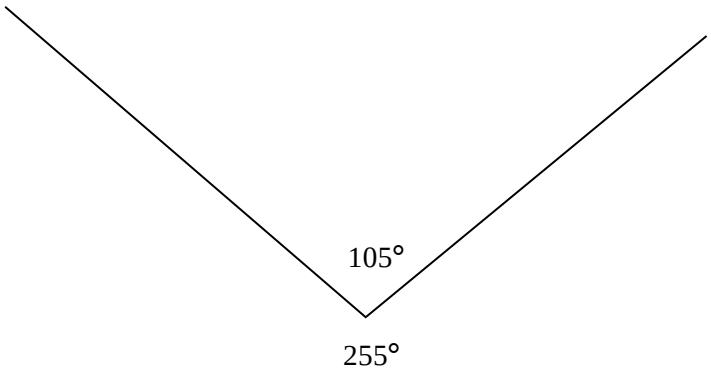




Be careful always to use the scale that runs from 0 at the first arm. Using the wrong scale is the most common error made in using a protractor. It will help if you look at the angle before you measure it and decide whether it is less than 90° or more than 90°. Then, after you have measured, make sure the measurement agrees with what you decided.

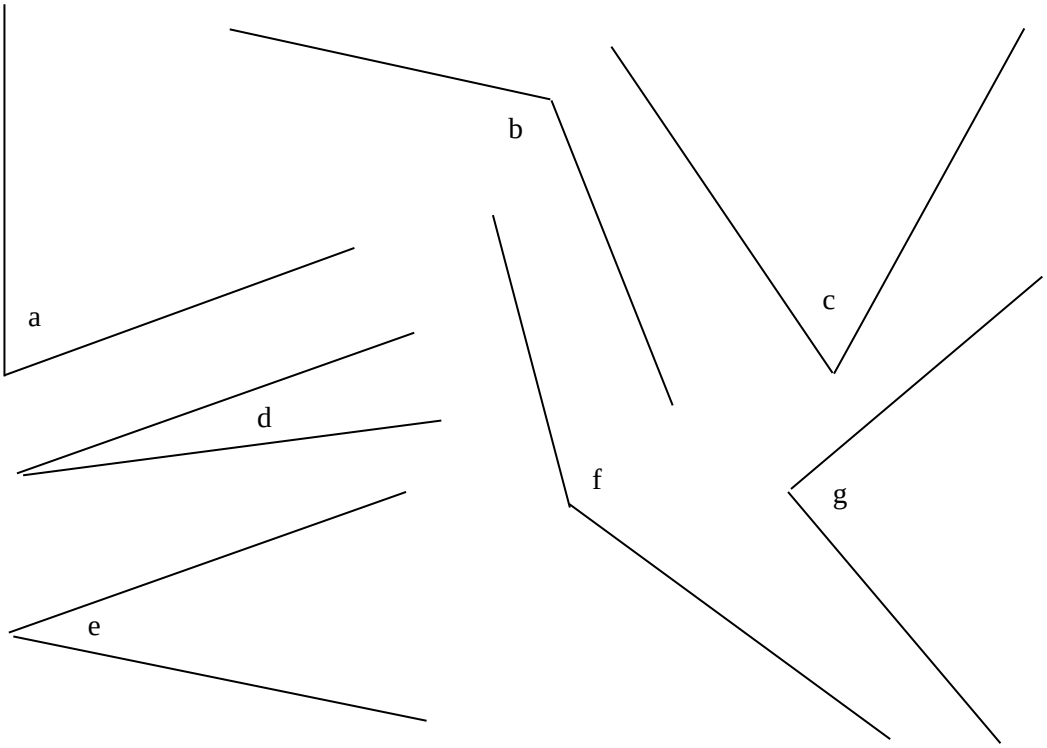
You need to get quite a bit of practice with this, so that you can do it reliably and accurately. You should be able to measure an angle to the nearest degree. You certainly shouldn't be more than 2° out.

If you have to measure an angle greater than 180°, the easiest way is to measure the smaller angle and subtract the result from 360°. So, to measure the larger angle below, we can measure the smaller angle and find it is 105°, then subtract that from 360° to find that the larger angle is 255°.



Practice

Q2 Use a protractor to measure the following angles. Take the smaller angle in diagrams a to e and the larger angle in diagrams f and g. You need to be within 2° with each. It should be possible to do this on a computer screen. Or you could print the page.

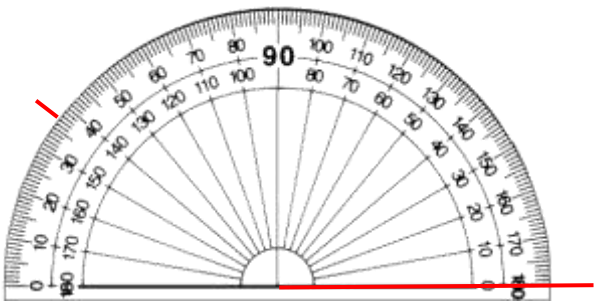


Drawing angles

Protractors can also be used to draw angles of a particular size.

Suppose we want to make an angle of 143°. We draw one of the arms. We then place the protractor over it so that the centre of the protractor is on the vertex end of the arm and so that the base line is along the arm (as in the diagram on the left below).

Then, using the scale that starts from 0 at our first arm (the inside scale in the picture below), we make a mark on the paper at 143°. We then take the protractor off the paper and draw a line from the vertex to the mark we have made (and further if we wish) (as in the diagram on the right below).



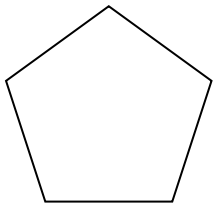
As with measuring, if we have to construct an angle of more than 180°, say 310°, the best way is to subtract the 310° from 360° to get 50° and construct that.

Practice

Q3 Use a protractor to construct angles of the following sizes. You need to be within 2°.

(a) 50° (b) 37° (c) 124° (d) 229°

Q4 To test your accuracy, construct a regular pentagon as follows.



Draw a straight 5 cm line. Then make a 108° angle at one end. Make the new line of the angle 5 cm long also. Then make a 108° angle at the end of that and so on until you get back to your starting point. If you are accurate, the fifth line will get you back to exactly where you started and the pentagon will look regular and beautiful. If you are not accurate, your last line will not join up with your first line. You can then fudge it by just connecting to the first line, but your pentagon will look distorted and ugly.

Q5 Do other regular polygons. Use 5 cm for side lengths for the first three, 4 cm for the last three. Use the following angles:

square	90°
hexagon	120°
octagon	135°
nonagon (9 sides)	140°
decagon (10 sides)	144°
dodecagon (12 sides)	150°

Classification of Angles

It is sometimes useful to classify angles according to their size (number of degrees). These are the words used:

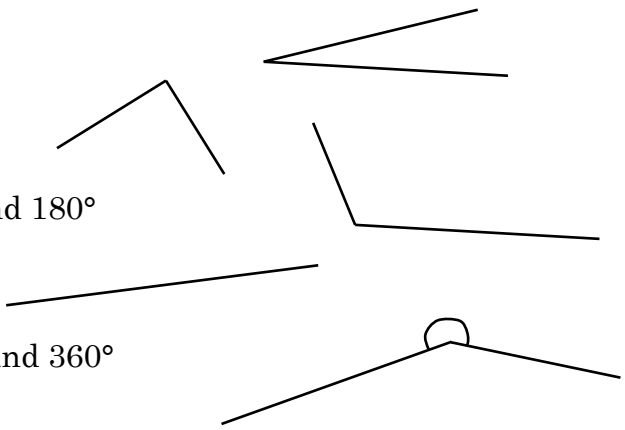
Acute: Less than 90°

Right: 90°

Obtuse: Between 90° and 180°

Straight: 180°

Reflex: Between 180° and 360°



Practice

Q6 For each of the following angles, say whether it is acute, right, obtuse, straight or reflex.

(a) 45°	(b) 200°	(c) 90°	(d) 179°	(e) 2°	(f) 335°
(g) 180°	(h) 94°	(i) 88°	(j) 140°	(k) 270°	(l) 60°

Protractor Golf

The Protractor Golf game in the Fun and Games section of Black Star Maths will provide further practice at estimating, measuring and constructing angles.

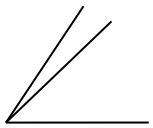
Solve

Q51 Draw a triangle, then measure the three angles (use the inside angles) and add them up. Do the same with other triangles with different shapes and sizes. What do you notice?

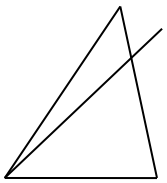
Q52 Do the same as Q51, but with quadrilaterals (4-sided shapes).

Q53 In Q5 you saw the internal angles of regular polygons with various numbers of sides. Try to find the pattern, then work out the angle sizes for regular polygons with 20 sides and 30 sides.

Q54 How many angles in this diagram? (Don't forget the angles which are more than 180°, but don't include any which are 0° or less or 360° or more.)



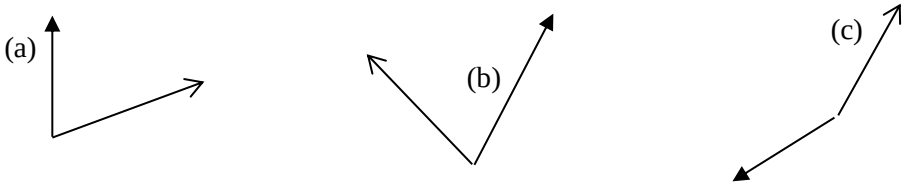
Q55 How many angles in this diagram? (Again, don't forget the angles which are more than 180°, but don't include any which are 0° or less or 360° or more.)



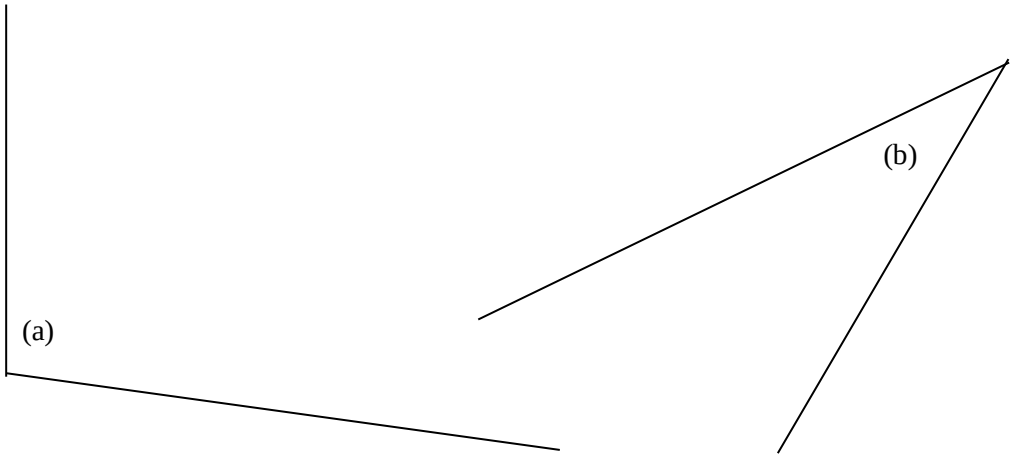
Revise

Revision Set 1

Q61 Estimate the size of the following angles in degrees. Assume that the turn is clockwise from the solid arrow to the hollow arrow. You need to be within 15° with each.



Q62 Use a protractor to measure the following angles. Find the smaller angle in (a) and the larger one in (b). You need to be within 2° with each.



Q63 Use a protractor to construct angles of the following sizes.
(a) 73° (b) 237°

Q64 Give the size range for each of the following classes of angle:
(a) acute (b) right (c) obtuse (d) straight (e) reflex

Answers

Q1 (a) 90° (b) 25° (c) 120° (d) 195° (e) 285°
Q2 (a) 70° (b) 124° (c) 297° (d) 12° (e) 31°
(f) 157° (g) 90°
Q3 If possible, get someone to check your angles with a protractor.
Q6 (a) acute (b) reflex (c) right (d) obtuse (e) acute (f) reflex
(g) straight (h) obtuse (i) acute (j) obtuse (k) reflex (l) acute

Q51 They always add up to 180°
Q52 They always add up to 360°
Q53 162°, 168°
Q54 6
Q55 22

Q61 (a) 70° (b) 228° (c) 151°
Q62 (a) 98° (b) 326°
Q63 (a, b) If possible, get someone to check your angles with a protractor.
Q64 (a) Less than 90° (b) 90° (c) Between 90° and 180°
(d) 180° (e) Between 180° and 360°