M1 Maths

C6-6 Chain, Product and Quotient Rules

- differentiating compound functions using the chain, product and quotient rules

Summary Learn Solve Revise Answers

Summary

The **chain rule** is used to differentiate a function of a function. The outside function is differentiated with respect to the inside function as if it were x, but then the result is multiplied by the derivative of the inside function. If $y = y(u(x))$, then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = y'(u(x)) \times u'(x)$.

The **product rule** is used where a function can be written as one function multiplied by another function. If $y = uv$, then $y' = vu' + uv'$.

The **quotient rule** is used where a function consists of one function divided by another function. If $y = \frac{u}{v}$, then $y' = \frac{vu' - uv'}{v^2}$

Learn

You should know how to differentiate x^n and sums and multiples of such terms. The chain, product and quotient rules allow you to differentiate more complex functions like

$$\sqrt{x^3 + 5x}, \quad (x^6 + 4x^5 - 3)(2x + \sqrt{x}), \quad \frac{x^2 - 5x}{x + 3}$$

The Chain Rule – Function of a Function

We use the chain rule to differentiate a function of a function.

$x^2 + 7x - 3$ is a function of x. Think of a function as something we do to x. In this case we squared it, added 7 times x, then subtracted 3.

We could then make a function of $x^2 + 7x - 3$ by doing something to it, e.g. raising it to the power of 10. This would give us $(x^2 + 7x - 3)^{10}$. But, as $x^2 + 7x - 3$ is a function of x, $(x^2 + 7x - 3)^{10}$ is a function of a function of x.

Now we could of course expand that, then differentiate, but that would be messy and time-consuming. The chain rule offers us a much easier alternative.

To differentiate $y = (x^2 + 7x - 3)^{10}$, look at the large-scale structure of the function: it is *something* to the power of 10. Another way of looking at this is that raising to the power of 10 was the last thing done in constructing the function. Anyway, think of it as *something* to the power of 10.

The *something* is $x^2 + 7x - 3$. Call this u.

So $y = u^{10}$ $\quad u = x^2 + 7x - 3$.

Then differentiate y and u.

$\frac{dy}{du} = 10u^9 \quad \frac{du}{dx} = 2x + 7$

Now $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ (because you can cancel the dus)

So $\frac{dy}{dx} = 10u^9 \times (2x + 7)$

$= 10(x^2 + 7x - 3)^9 \times (2x + 7)$

A good way to lay this out is like this:

$$y = (x^2 + 7x - 3)^{10}$$

$$y = u^{10} \qquad u = x^2 + 7x - 3$$

$$\frac{dy}{du} = 10u^9 \qquad \frac{du}{dx} = 2x + 7$$

$$\frac{dy}{dx} = 10u^9 \times (2x + 7)$$

$$= 10(x^2 + 7x - 3)^9 \times (2x + 7)$$

Here is another example.

$$y = \frac{1}{\sqrt{1-x^2}} = (1 - x^2)^{-\frac{1}{2}}$$

$$y = u^{-\frac{1}{2}} \qquad u = 1 - x^2$$

$$\frac{dy}{du} = -\tfrac{1}{2}u^{-3/2} \qquad \frac{du}{dx} = -2x$$

$$\frac{dy}{dx} = -\tfrac{1}{2}u^{-3/2} \times -2x$$

$$= \frac{x}{u\sqrt{u}}$$

$$= \frac{x}{(1-x^2)\sqrt{1-x^2}}$$

Practice

Q1 Use the chain rule with the layout above to differentiate the following. Don't forget to write the functions in differentiable form before differentiating by expressing fractions as negative powers and roots as fractional powers.

(a) $y = (3x^2 + 2x)^8$ (b) $y = (x^4 - 5x)^4$ (c) $y = (x^3 + 4x^2 + 1)^7$

(d) $y = \frac{1}{(x^3+2x)^4}$ (e) $y = \frac{4}{(5x^4 - x - 7)^2}$ (f) $y = \frac{-2}{3x - x^3}$

(g) $y = (x^2 + {}^1/_x)^6$ (h) $y = (2x^2 + \sqrt{x})^5$ (i) $y = \left(x^2 + \frac{1}{\sqrt{x}}\right)^4$

(j) $h = \sqrt{1 - x^2}$ (k) $r = \sqrt[3]{4x + x^2}$ (l) $s = (2t + 1)\sqrt{2t + 1}$

(m) $h = \frac{1}{\sqrt{a^3+5a}}$ (n) $t = \sqrt{(4x^3 + 1)^5}$ (o) $y = (\sqrt{x} + 2x^2)^5$

Doing it without the dus

Once you have had a bit of practice with this, it is possible (and in fact preferable) to do the differentiations without writing down the working steps with the dus etc. To do this, we think about it just slightly differently – like this:

Suppose we need to differentiate $y = (x^2 + 7x)^5$.

Instead of thinking of $x^2 + 7x$ as u and y as u^5, we thinking of $x^2 + 7x$ as *something* and y as *something* to the power of 5.

We differentiate y with respect to the something to get 5 × *something*^{4}, i.e. $5(x^2 + 7x)^4$.

But then, because the something wasn't just x, we have to multiply by the derivative of the *something*, which is $2x + 7$. This gives us $5(x^2 + 7x)^4\,(2x + 7)$.

Of course, the first part of this is the $\frac{dy}{du}$ and the second part is the $\frac{du}{dx}$. So we get the same result.

As another example, let's differentiate $y = \sqrt{x^3 + 3x^2}$.

First we have to put it in index form: $y = (x^3 + 3x^2)^{½}$

This is *something* raised to the power of ½. We differentiate to get ½ × *something* to the power of –½, i.e. $½(x^3 + 3x^2)^{-½}$. Then we multiply by the derivative of the *something* to get $½(x^3 + 3x^2)^{-½} \times (3x^2 + 6x)$.

Finally we convert it back to the original form with a root: $y = \frac{3x^2 + 6x}{2\sqrt{x^3+3x^2}}$

Practice

Q2 Use the chain rule to differentiate the following without writing the dus etc. You will still need to write down the original function, re-write it in index form (if necessary), write down the derivative, then re-write it again back in the original form. Don't worry about simplifying your answer too much. The answers at the end are given with little simplification. This makes it easier to see how they are arrived at.

(a) $y = (4x^2 + x)^3$ (b) $y = (2x^4 - 5x + 5)^{10}$ (c) $h = (t^3 - 4t^2)^6$

(d) $s = \frac{1}{(t^4+2t)^3}$ (e) $y = \frac{5}{x^4 - 3}$ (f) $p = \frac{-8}{(2x-3x^3)^7}$

(g) $y = (3x^2 + {}^4/_x)^4$ (h) $y = (x^2 + 2\sqrt[3]{x})^5$ (i) $y = \left(x^5 + x + {}^1/_{\sqrt{x}}\right)^4$

(j) $h = \sqrt{4 + 4r^2}$ (k) $s = 5\sqrt[3]{4t^3 - t^2}$ (l) $s = (4h + 3) \div \sqrt{4h + 3}$

(m) $h = \frac{4}{\sqrt{a^2 - a}}$ (n) $t = \sqrt[3]{(4x^3 + x)^4}$ (o) $y = (x + {}^3/_{x^2})^3$

You need to become fluent with the chain rule and that comes with practice. So here are a few more.

Practice

Q3 Differentiate the following without writing any working, but simplifying your answer where possible.

(a) $y = (\tfrac{1}{2}x^2 + x)^5$ (b) $y = (5x^3 - 11x^2 + 2x)^2$ (c) $P = (t - t^2)^{21}$

(d) $a = \frac{1}{v^5 - 4v}$ (e) $y = \frac{5}{\left(\frac{1}{2}x^4 - 3\right)^2}$ (f) $p = \frac{-1}{(5x - x^3)^4}$

(g) $y = (3x^2 + {}^4/_x)^4$ (h) $y = ({}^5/_{x^2} + 7\sqrt[3]{x})^3$ (i) $y = \left(x^5 + \frac{1}{2\sqrt{x}}\right)^5$

(j) $h = 1 \div \sqrt{2x + 3x^2}$ (k) $s = 2\sqrt[3]{5t^4 - t^3}$ (l) $s = (2t + 3) \times \sqrt{2t + 3}$

(m) $w = \frac{-3}{\sqrt{2x^2 - 5}}$ (n) $t = 6 \div \sqrt[4]{(x^3 + 2x)^5}$ (o) $y = (2\sqrt{x} + 3x)^{-4}$

The Product Rule

The product rule can be used for any function which can be written as one expression multiplied by another expression.

Consider $y = (4x + 2)(x^2 + 3x - 7)$

This is not a function of a function; it is two functions multiplied together, i.e. a product of two functions. The way to tell if a function is a product and therefore if we can use the product rule is to see if it can be split with a vertical line into two parts which are multiplied together.

$(4x + 2)(x^2 + 3x - 7)$ can be split thus: $(4x + 2) \mid (x^2 + 3x - 7)$ and written as $(4x + 2) \times (x^2 + 3x - 7)$

$(4x^2 + 2x)^3$ cannot be. Neither can $\sqrt{x^2 + 3x}$. They are both functions of functions rather than products of functions and so require the chain rule.

The product rule is easy. We call the two factors of the product u and v. So in the case of $(4x + 2)(x^2 + 3x - 7)$, $u = 4x + 2$ and $v = x^2 + 3x - 7$.

[We use the dash notation for derivatives. So the derivatives of y, u and v are y', u' and v' (pronounced *y-dash* etc.). $y' = \frac{dy}{dx}$ $u' = \frac{du}{dx}$ and $v' = \frac{dv}{dx}$. The dash notation makes it quicker to write out as well as easier to remember.]

The product rule says that, if $y = uv$, then $y' = vu' + uv'$. In other words it is $u \times v$ with one of the factors differentiated plus $u \times v$ with the other factor differentiated. The order doesn't matter: $u'v + v'u$ is just as good, and so is $uv' + vu'$.

In the case of $y = (4x + 2)(x^2 + 3x - 7)$

$y = (4x + 2)(x^2 + 3x - 7)$

$u = 4x + 2 \qquad v = x^2 + 3x - 7$

$u' = 4 \qquad v' = 2x + 3$

$y' = vu' + uv'$

$\quad = 4(x^2 + 3x - 7) + (4x + 2)(2x + 3)$

This could then be simplified to $y' = 12x^2 + 32x - 22$, but don't worry about simplifying in the practice questions; the answers at the end are mostly unsimplified.

It is recommended that this layout be used as it provides cues as to what to do next.

You might have noticed that we could avoid using the product rule by expanding the original expression first. Later you will be using the product rule for expressions that can't be expanded like $2^x \sin x$, so it is good to practice using it anyway.

Practice

Q4 Use the product rule to differentiate the following, laying out the working as shown above. Don't worry about simplifying your answers too much. The answers at the end are mostly unsimplified.

(a) $y = x^3(3x^2 + 2)$ (b) $y = (4x^3 - 3x)(x^2 + 5x)$ (c) $y = \sqrt{x}(x^2 + 5x)$

(d) $y = (x + 3) \times {}^1/_{x^2}$ (e) $y = (-x^2 + 3x)\sqrt[3]{x}$ (f) $y = (x^3 - x^2 + 3x)(2x + 5)$

(g) $y = 4x^2 ({}^1/_{x^3} + 3x)$ (h) $y = (3x^2 - 5x)(4 - x)$ (i) $y = 6x(x^2 - 2)$

(j) $k = x^{-2}(2x^3 - 4x^2)$ (k) $t = -4x(\sqrt{x} + {}^2/_x)$ (l) $A = 3x \times (2x + x^3)$

As with the chain rule, once you've had a bit of practice laying out the working, it is good to learn to do it without writing the *u*s and *v*s. It can be helpful to think of it as writing out the product twice, once with the first half differentiated, once with the second half differentiated.

Practice

Q5 Use the product rule to differentiate the following without writing the *u*s and *v*s. Again, don't worry about simplifying.

(a) $y = x^2(x^3 - 4x)$ (b) $y = (5x^3 + 1)(4x - 3)$ (c) $y = (3x^2 + 2x)(\sqrt{x} + \sqrt[3]{x})$

(d) $y = \sqrt{x}(x^2 - x^3)$ (e) $h = (x^2 - 5x)(4 - 8x)$ (f) $r = {}^1/_{x^2}(2x + 3)$

(g) $y = x\sqrt{x}(x^2 + 4x)$ (h) $P = (7x^2 - 2x)(4x - x^3)$ (i) $s = 4(x^3 - x)(2x^2 + 1)$

(j) $f = 2a^2(a + \sqrt{a})$ (k) $t = 4p^7(3 - p)$ (l) $A = x^2 x^3$

The Quotient Rule

The quotient rule can be used for any function which can be written as one expression divided by another expression.

Consider $y = \frac{x^2+3}{2x-5}$

We let the top expression (the numerator) be u and the bottom expression (the denominator) be v. Again, we let their derivatives be u' and v'.

The quotient rule says that $y' = \frac{vu' - uv'}{v^2}$

We write:

$y = \frac{x^2+3}{2x-5}$

$u = x^2 + 3$ $\quad v = 2x - 5$

$u' = 2x$ $\quad v' = 2$

$y' = \frac{vu' - uv'}{v^2}$

$= \frac{2x(2x-5)-2(x^2+3)}{(2x-5)^2}$

$= \frac{2x^2-10x-6}{(2x-5)^2}$

(Note that the top of $y' = \frac{vu' - uv'}{v^2}$ is the same as for the product rule except for the minus.) With the quotient rule, however, it does matter which way round it's done.

Practice

Q6 Use the quotient rule to differentiate the following, laying out the working as shown above. Don't worry about simplifying your answers too much.

(a) $y = \frac{x^2}{x+3}$ (b) $y = \frac{x^2 - x}{x^2 + x}$ (c) $y = \frac{2x-5}{x+1}$

(d) $y = \frac{\sqrt{x}}{1-x^2}$ (e) $y = \frac{1+\sqrt[3]{x}}{2x}$ (f) $y = \frac{1}{x+1}$

(g) $y = \frac{4-x}{x^2+7x+1}$ (h) $y = \frac{4}{x^3 - 3}$ (i) $y = \frac{x^2-3}{2\sqrt{x}}$

(j) $y = \frac{4x}{1+x^4}$ (k) $y = \frac{x+2}{x^2+4x-5}$ (l) $y = \frac{6}{x}$

Because it matters which way round you get the us and vs, many people prefer always to write the rule when using the quotient rule, then to sub in the expressions for u, v etc. You can please yourself in the next exercise.

Practice

Q7 Use the quotient rule to differentiate the following.

(a) $y = \frac{x^3}{x+4}$ (b) $y = \frac{3x^2 - 2x}{x^2 - 4}$ (c) $y = \frac{x-5}{x}$

(d) $y = \frac{2x}{x^2+5}$ (e) $y = \frac{x^2}{5x+1}$ (f) $h = \frac{2c^4}{1+c^3}$

Mixed Practice

The questions in the next exercise require the chain rule, product rule or quotient rule. They will give you practice in choosing as well as more practice in using the rules.

Practice

Q8 Use the chain rule, product rule or quotient rule to differentiate the following. Again, don't worry about simplifying.

(a) $y = (7 + 3x)^4$ (b) $y = x^2 (2x^2 + 5x - 3)$ (c) $y = \frac{x+3}{2x^2-7}$

(d) $y = \frac{5x+2\sqrt{x}}{x^3}$ (e) $h = \sqrt{x^2 + 2x - 7)}$ (f) $r = {}^1/_x \times (\sqrt[4]{x} + 1)$

(g) $y = -4x\,(2x^2 + {}^1/_x)$ (h) $P = x^9 + (7x^2 - 2x)^8$ (i) $y = \frac{1}{t+t^2}$

(j) $s = \frac{4x+2}{x^2-6}$ (k) $t = (4x + x^2)^5$ (l) $y = \frac{x}{x^2+2}$

And a few word problems

Practice

Q9 (a) The height of a rocket at time t is given by $h = \frac{200+20t}{1+t^2}$. Find its velocity when $t = 4$.

(b) The formula for the area of a hexagon with side length s is $A = \frac{3\sqrt{3}}{2}s^2$. A hexagonal prism has side length s and height $s + 5$. What is the rate of increase of its volume with respect to s, $(\frac{dV}{ds})$ when $s = 8$.

(c) A rectangle is 4 m wide. At was rate is the length of its diagonal increasing with respect to its length when it is 5 m long?

(d) Cindy wants a rectangular pool with area 40 m^2. The cost of building it is given by $c = (l + w) \times \$1500$, where c is the cost, l is the length of the pool and w is the width, both in metres. Express c in terms of just l, then find the value of l which gives the minimum value for c, and hence find the length that will make the pool the cheapest. How much will it cost?

(e) Find the coordinates of the stationary points of $y = (2x^2 - 7x)^5$.

Solve

Q51 If $y = uvw$, then $y' = uvw' + uv'w + u'vw$.
Use this to differentiate (without expanding) $y = x^2(3x^2 - 4)(x^3 + 2x)$.

Revise

Revision Set 1

Q61 Use the chain, product and quotient rules to differentiate the following. Don't worry about simplifying your answers too much.

(a) $y = (x^2 + 5x)^6$ (b) $y = (x^2 + 3x)(\sqrt{x} + 1)$ (c) $y = \frac{2x}{x^2+5}$

(d) $p = r^2(4r - 1)$ (e) $m = \frac{5}{\sqrt{x^2+3x}}$ (f) $y = \sqrt{4x^2 - 3x}$

Answers

Q1 (a) $y' = 8(3x^2 + 2x)^7(6x + 2)$ (b) $y' = 4(x^4 - 5x)^3(4x^3 - 5)$ (c) $y' = 7(x^3 + 4x^2 + 1)^6(3x^2 + 8x)$

(d) $y' = \frac{-12x^2-8}{(x^3+2x)^5}$ (e) $y' = \frac{8-160x^3}{(5x^4-x-7)^3}$ (f) $y' = \frac{6-6x^2}{(3x-x^3)^2}$

(g) $y' = 6(x^2 + {}^1/_x)^5(2x - {}^1/_{x^2})$ (h) $y' = 5(2x^2 + \sqrt{x})^4(4x + \frac{1}{2\sqrt{x}})$ (i) $y' = 4\left(x^2 + \frac{1}{\sqrt{x}}\right)^3\left(2x - \frac{1}{2\sqrt{x}}\right)$

(j) $h' = {}^{-1}/_{\sqrt{1-x^2}}$ (k) $r' = \frac{4-2x}{3\sqrt[3]{(4x+x^2)^2}}$ (l) $s' = 3\sqrt{2t+1}$

(m) $h' = \frac{-3a^2-5a}{\sqrt{(a^3+5a)^3}}$ (n) $t' = 30x\sqrt{(4x^3+1)^3}$ (o) $y' = 5(\sqrt{x} + 2x^2)^4(\frac{1}{2\sqrt{x}} + 4x)$

Q2 (a) $y' = 3(4x^2 + x)^2(8x + 1)$ (b) $y' = 10(2x^4 - 5x + 5)^9(8x^3 - 5)$ (c) $h' = 6(t^3 - 4t^2)^5(3t^2 - 8t)$

(d) $s' = \frac{-12t^3-6}{(t^4+2t)^4}$ (e) $y' = \frac{-20x^3}{(x^4-3)^2}$ (f) $p' = \frac{56(2-9x^2)}{(2x-3x^3)^8}$

(g) $y' = 4(3x^2 + {}^4/_x)^3(6x - {}^4/_{x^2})$ (h) $y' = 5(x^2 + 2\sqrt[3]{x})^4(2x + \frac{2}{3\sqrt[3]{x^2}})$

(i) $y' = 4\left(x^5 + x + \frac{1}{\sqrt{x}}\right)^3\left(5x^4 + 1 - \frac{1}{2\sqrt{x^3}}\right)$

(j) $h' = {}^{4r}/\sqrt{4+4r^2}$ (k) $s' = \frac{60t^2-10t}{3\sqrt[3]{(4t^3-t^2)^2}}$ (l) $s' = {}^2/\sqrt{4h+3}$

(m) $h' = \frac{2-4a}{\sqrt{(a^2-a)^3}}$ (n) $t' = {}^4/_3\sqrt[3]{4x^3+x}(12x^2 + 1)$ (o) $y' = 3(x + {}^3/_{x^2})^2(1 - {}^6/_{x^3})$

Q3 (a) $y' = 5(\frac{1}{2}x^2 + x)^4(x + 1)$ (b) $y' = 2(5x^3 - 11x^2 + 2x)(15x^2 - 22x + 2)$ (c) $P' = (t - t^2)^{20}(21 - 42t)$

(d) $a' = \frac{4-5v^4}{(v^5-4v)^2}$ (e) $y' = \frac{-20x^3}{\left(\frac{1}{2}x^4-3\right)^3}$ (f) $p' = \frac{20-12x^2}{(5x-x^3)^5}$

(g) $y' = 4(3x^2 + {}^4/_x)^3(6x - {}^4/_{x^2})$ (h) $y' = 3(\frac{5}{x^2} + 7\sqrt[3]{x})^2(\frac{7}{3\sqrt[3]{x^2}} - \frac{10}{x^3})$ (i) $y' = 5\left(x^5 + \frac{1}{2\sqrt{x}}\right)^4\left(5x^4 - \frac{1}{4\sqrt{x^3}}\right)$

(j) $h' = (-1 - 3x) \div \sqrt{(2x+3x^2)^3}$ (k) $s' = \frac{40t^3-6t^2}{3\sqrt[3]{(5t^4-t^3)^2}}$ (l) $s' = 3\sqrt{2t+3}$

(m) $w' = \frac{-12x}{2\sqrt{(2x^2-5)^3}}$ (n) $t' = \frac{-45x-30}{2\sqrt[4]{(x^3+2x)^9}}$ (o) $y' = -4(2\sqrt{x} + 3x)^{-5}(\frac{1}{\sqrt{x}} + 3)$

Q4 (a) $y' = 6x^4 + 3x^2(3x^2 + 2)$ (b) $y' = (12x^2 - 3)(x^2 + 5x) + (4x^3 - 3x)(2x + 5)$

(c) $y' = \sqrt{x}(2x + 5) + \frac{1}{2\sqrt{x}}(x^2 + 5x)$

(d) $y' = \frac{1}{x^2} - \frac{2(x+3)}{x^3}$ (e) $y' = (-2x + 3)\sqrt[3]{x} + (-x^2 + 3x)\,\frac{1}{3\sqrt[3]{x^2}}$

(f) $y' = 2(x^3 - x^2 + 3x) + (3x^2 - 2x + 3)(2x + 5)$

(g) $y' = 8x({}^1/_{x^3} + 3x) + 4x^2({}^{-3}/_{x^4} + 3)$ (h) $y' = -(3x^2 - 5x) + (6x - 5)(4 - x)$

(i) $y' = 6(x^2 - 2) + 12x^2$

(j) $k' = x^{-2}(6x^2 - 8x) - 2x^{-3}(2x^3 - 4x^2)$ (k) $t' = -4(\sqrt{x} + {}^2/_x) - 4x(\frac{1}{2\sqrt{x}} - {}^2/_{x^2})$

(l) $A' = 3(2x + x^3) + 3x(2 + 3x^2)$

Q5 (a) $y' = 2x(x^3 - 4x) + x^2(3x^2 - 4)$ (b) $y' = 4(5x^3 + 1) + 15x^2(4x - 3)$

(c) $y' = (6x + 2)(\sqrt{x} + \sqrt[3]{x}) + (3x^2 + 2x)(\frac{1}{2\sqrt{x}} + \frac{1}{3\sqrt[3]{x^2}})$

(d) $y' = \sqrt{x}\,(2x - 3x^2) + \frac{1}{2\sqrt{x}}(x^2 - x^3)$ (e) $h' = (2x - 5)(4 - 8x) - 8(x^2 - 5x)$

(f) $r' = {}^2/_{x^2} - {}^2/_{x^3}\,(2x + 3)$

(g) $y' = x\sqrt{x}\,(2x + 4) + {}^3/_2\sqrt{x}\,(x^2 + 4x)$ (h) $P' = (7x^2 - 2x)(4 - 3x^2) + (14x - 2)(4x - x^3)$

(i) $s' = 4(3x^2 - 1)(2x^2 + 1) + 16x(x^3 - x)$

(j) $f' = 4a\,(a + \sqrt{a}) + 2a^2\,(1 + \frac{1}{2\sqrt{a}})$ (k) $t' = 28p^6(3 - p) - 4p^7$ (l) $A' = 2x\,x^3 + 3x^2\,x^2$

Q6 (a) $y' = \frac{2x(x+3)-x^2}{(x+3)^2}$ (b) $y' = \frac{(x^2+x)(2x-1)-(x^2-x)(2x+1)}{(x^2+x)^2}$ (c) $y' = \frac{2(x+1)-(2x-5)}{(x+1)^2}$

(d) $y' = \frac{\frac{1}{2\sqrt{x}}(1-x^2)+2x\sqrt{x}}{(1-x^2)^2}$ (e) $y' = \frac{\frac{2x}{3\sqrt[3]{x^2}}+2(1+\sqrt[3]{x})}{4x^2}$ (f) $y' = \frac{-1}{(x+1)^2}$

(g) $y' = \frac{-(x^2+7x+1)-(4-x)(2x+7)}{(x^2+7x+1)^2}$ (h) $y' = \frac{-12x^2}{(x^3-3)^2}$ (i) $y' = \frac{4x\sqrt{x}-(x^2-3)/\sqrt{x}}{4x}$

(j) $y' = \frac{4(1+x^4)-16x^4}{(1+x^4)^2}$ (k) $y' = \frac{x^2+4x-5-(x+2)(2x+4)}{(x^2+4x-5)^2}$ (l) $y' = \frac{-6}{x^2}$

Q7 (a) $y' = \frac{3x^2(x+4)-x^3}{(x+4)^2}$ (b) $y' = \frac{(x^2-4)(6x-2)-2x(3x^2-2x)}{(x^2-4)^2}$ (c) $y' = \frac{x-(x-5)}{x^2}$

(d) $y' = \frac{2(x^2+5)-4x^2}{(x^2+5)^2}$ (e) $y' = \frac{2x(5x+1)-5x^2}{(5x+1)^2}$ (f) $h' = \frac{8c^3(1+c^3)-6c^6}{(1+c^3)^2}$

Q8 (a) $y' = 12(7 + 3x)^3$ (b) $y' = 2x(2x^2 + 5x - 3) + x^2(4x + 5)$ (c) $y' = \frac{2x^2-7-4x(x+3)}{(2x^2-7)^2}$

(d) $y' = \frac{x^6(5+1/\sqrt{x}-6x^5(5x+2\sqrt{x})}{x^6}$ (e) $h' = \frac{x+1}{\sqrt{x^2+2x-7)}}$ (f) $r' = -{}^1/_{x^2} \times (\sqrt[4]{x} + 1) + (\frac{1}{4\sqrt[4]{x^3}}) \div x$

(g) $y' = -4(2x^2 + {}^1/_x) - 4x\,(4x - {}^1/_{x^2})$ (h) $P' = 9x^8 + 8(7x^2 - 2x)^7(14x - 2)$ (i) $y' = \frac{1+2t}{(t+t^2)^2}$

(j) $s' = \frac{4(x^2-6)-2x(4x+2)}{(x^2-6)^2}$ (k) $t' = 5(4x + x^2)^4(4 + 2x)$ (l) $y' = \frac{x^2+2-2x^2}{(x^2+2)^2}$

Q9 (a) –7.8 m/s (b) 707 (c) 0.78 m/m

(d) 6.325 m, $18 974 (e) (0, 0), (1.75, –8620), (3.5, 0)

Q51 $y' = x^2 (3x^2 - 4)(3x^2 + 2) + 6x^3 (x^3 + 2x) + 2x (3x^2 - 4)(x^3 + 2x)$

Q61 (a) $y' = 6(x^2 + 5x)^5(2x + 5)$ (b) $y' = (2x + 3)(\sqrt{x} + 1) + (x^2 + 3x)(\frac{1}{2\sqrt{x}})$ (c) $y' = \frac{2(x^2+5)-4x^2}{(x^2+5)^2}$

(d) $p' = 4r^2 (4r - 1) + 2r (4r - 1)$ (e) $m' = \frac{-10x-15}{2\sqrt{(x^2+3x)^3}}$ (f) $y' = \frac{8x-3}{2\sqrt{4x^2-3x}}$