M1 Maths

C6-15 Simpson's Rule

- Simpson's rule

Summary Learn Solve Revise Answers

Summary

Simpson's rule is a modification of the trapezoidal rule which models the ends of the strips with quadratic curves instead of straight lines. In most cases it gives a slightly more accurate result.

Learn

Using Simpson's rule, we divide the shape into strips just like when we use the trapezoidal rule. The difference is that Simpson's rule calculates the area of each strip assuming it to be a quadratic curve whose parameters are affected by the points in the far corners of the adjacent strips.

The mathematics behind it is tricky, but using the rule is simple and very much like using the trapezoidal rule.

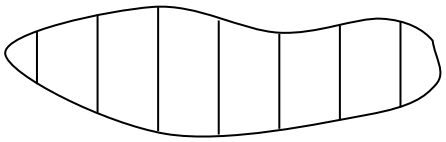
We need to have an even number of strips, i.e. an odd number of lines. We then measure each line as with the trapezoidal rule. Then we multiply the first line by 1, the second line by 4, the third line by 2, the fourth line by 4, the fifth by 2 and so on alternating . . . the second-last line by 4 and the last line by 1.

Diagrammatically, the multipliers look like this:

1 4 2 4 2 . . . . . 2 4 2 4 1

Then we add them. Then divide by 3. Then multiply by the line spacing.

Going back to the example at the start of this module, the working for Simpson's rule might look like this:



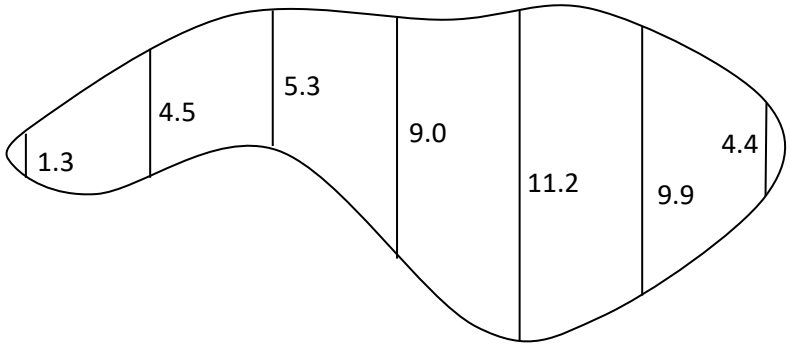
Length	1.5	2.7	3.4	3.2	2.6	2.6	2.4
Multiplier	1	4	2	4	2	4	1
Product	1.5	10.8	6.8	12.8	5.2	10.4	2.4

Total: 49.9 Line spacing = 2 cm. Area $49.9 \div 3 \times 2 = 33.3$ cm^2.

The difference with Simpson's rule is the multipliers (which average just below 3 rather than just below 2) and dividing by 3 instead of 2.

Practice

Q1 Use Simpson's rule to find the area of the shape in P1, reproduced below (line spacing = 5 cm).



Q2 Use Simpson's rule to find the following definite integrals. Use 8 strips for (a) and (b), 6 strips for (c).

(a) $\int_0^4 x \sin x \, dh$ (b) $\int_2^3 p^2 \times 2^p \, dp$ (c) $\int_2^5 x^x dx$

The trapezoidal rule will give an exact result for the integral of a linear function. This should be obvious if you think about it.

Simpson's rule will give an exact result for the integral of any linear, quadratic or cubic function.

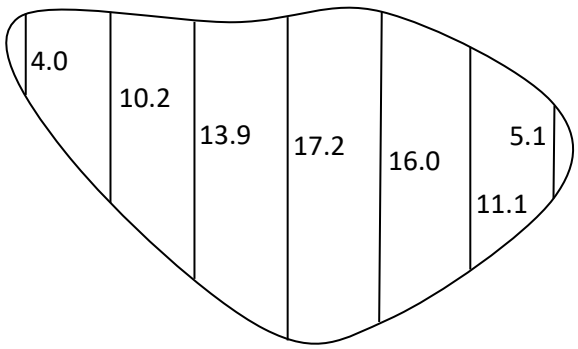
Solve

Q51 Find the integral of x^4 from $x = 1$ to $x = 2$ using calculus, using the trapezoidal rule and using Simpson's rule. (Use just 4 strips in each case.) Compare the error in the Simpson's rule result to the error in the trapezoidal rule result.

Revise

Revision Set 1

Q61 Use Simpson's rule to find the area of the shape to the right. The line lengths are in metres and the strip width is 5 m.



Q62 Use Simpson's rule to find $\int_0^4 x2^x dx$ using 8 strips.

Answers

Q1 221 m²
Q2 (a) 1.86 (b) 306 (c) 1256

Q51 Calculus: 6.1667 Trapezoidal: 6.3457 (error: 0.179) Simpsons: 6.201 (error: 0.034)

Q61 372 m²
Q62 61.1