M1 Maths

A6-1 Combinations and the Binomial Expansion

- multiplication principle
- permutations and combinations
- binomial expansion
- Pascal's Triangle

Summary Learn Solve Revise Answers

Summary

By the multiplication principle, if you have to make a number of choices and there are numbers of options for each choice, then the total number of ways of doing the whole thing is found by multiplying the numbers of options for each choice.

${}^nP_r$, the number of permutations (placings) of r things from n, is given by $\frac{n!}{(n-r)!}$.

${}^nC_r$, the number of combinations (choices) of r things from n, is given by $\frac{n!}{(n-r)!r!}$.

${}^nC_r$ can be written $\binom{n}{r}$.

The binomial expansion is a short-cut for expanding $(x + y)^n$. It says:

$$(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y^1 + \binom{n}{2}x^{n-2}y^2 + \binom{n}{3}x^{n-3}y^3 + \ldots + \binom{n}{n-1}x^1y^{n-1} + \binom{n}{n}y^n$$

Learn

Multiplication Principle

A café serves 4 different appetizers, 5 different main courses and 3 different desserts. How many different meals are possible if you have one of each?

Well, you can pick your appetizer in 4 ways. For each of those 4 ways, you can pick your main course in 5 ways. So, there are 20 ways of choosing appetizer and main course. Then, for each of those 20 choices, you can pick dessert in 3 ways, making 60 choices for the meal.

The total number of possible meals is $4 \times 5 \times 3 = 60$.

If you have to make a number of choices and there are numbers of options for each choice, then the total number of ways of doing the whole thing is found by multiplying the numbers of options for each stage.

How many sequences of 3 letters are possible if no letter is allowed to be used more than once?

J P A

Well, there are 26 choice for the first letter, then just 25 for the second, then 24 for the third, making 26 × 25 × 24 = 15 600.

Practice

Q1 You can choose from 2 appetizers, 5 main courses and 4 desserts. How many different meals are possible if you have one of each?

Q2 You have 2 different pairs of shoes, 2 different pairs of socks, 3 different pairs of pants and 7 different tops. In how many ways can you dress if you wear one of each?

Q3 The vowels are A, E, I, O and U. How many sequences of 2 vowels are possible? (the two vowels can be the same, e.g. UU is ok.)

Q4 Number plates in a small country consist of just 2 letters followed by 2 digits. How many different number plates are possible?

Q5 In another country, the number plates consist of 3 characters, which can be letters or numbers. How many plates are possible?

Q6 How many different sequences of three digits are possible if none of the digits can be the same? (e.g. 774 is not allowed.)

Q7 Albert, Bruce and Carp run a race. All finish with no ties. In how many different orders can they finish?

Q8 Ally, Bree, Claudia, Dani and Ella run a race. All finish with no ties. In how many different orders can they finish?

Q9 10 people run a race. How many possible results are possible for the first three places?

Q10 How many different ways can 6 bad people be put into a line of 6 cells, one per cell?

Q11 How many different ways can 6 bad people be put into 6 cells if there are no restrictions on how many can go in one cell?

Factorials

If n is a counting number, $n!$ (pronounced *n factorial*), is n multiplied by all the counting numbers smaller than n. So,

$5! = 5 \times 4 \times 3 \times 2 \times 1$

$8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$2! = 2 \times 1$

$1! = 1$

0 isn't a counting number, but for convenience, we define 0! as 1. This is different from the definition of factorials of counting numbers, but it makes things easier if we define it that way.

Permutations – ^nP_r

There are 7 rowers in a rowing squad, Ann, Bella, Celine, Diedre, Edith, Fiona and Gerty. The coach has to man (woman) a 3-seater boat.

She could put Ann in seat 1, Edith in seat 2 and Celine in seat 3.

Or she could put Celine in seat 1, Edith in seat 2 and Ann in seat 3.

Or she could put Diedre in seat 1, Edith in seat 2 and Gerty in seat 3.

And there are many other ways she could do it. The question is 'How many different ways could she place 3 rowers in the boat, chosen from the 7 rowers available?'

Well, let's fill the seats in turn. In seat 1, she can place any of the 7 rowers. Then, for each choice for seat 1, she can place any of the remaining 6 in seat 2, giving 42 ways to fill seats 1 and 2. Then for each of these 42 ways, she can place any of the remaining 5 rowers in seat 3, giving 210 ways in total.

Thus, the number of ways to place 3 rowers chosen from 7 is $7 \times 6 \times 5$.

This can be written $\frac{7\times6\times5\times4\times3\times2\times1}{4\times3\times2\times1}$, which is $\frac{7!}{(7-3)!}$.

We call this the number of placings of 3 things from 7 or, more conventionally, the number of permutations of 3 things from 7. It is generally abbreviated to $^7P_3$.

In general, ${}^{n}P_{r}$, the number of ways of placing r things from n, is given by $\frac{n!}{(n-r)!}$.

Your calculator will be able to give you factorials and permutations. On a Casio, you will find them in OPTN then PROB. To get 6! press 6 then x! To get ${}^{6}P_{3}$ press 6 nPr 3.

Practice

Q12 Find (a) 4! (b) 8! (c) 12! (d) 26! (e) 1! (f) 0!

Q13 Find (a) ${}^{3}P_{3}$ (b) ${}^{8}P_{8}$ (c) ${}^{8}P_{3}$ (d) ${}^{12}P_{4}$ (e) ${}^{20}P_{10}$ (f) ${}^{2}P_{1}$

Q14 A coach needs to place 4 of his 9 rowers into the 4 seats in a quad boat. In how many ways can he do it?

Q15 In a police lock-up, there are 6 cells. Assuming you put only one prisoner in a cell, in how many ways can you place the following numbers of prisoners:
(a) 1 (b) 2 (c) 3 (d) 4 (e) 5 (f) 6

Q16 5 people run a race. Assuming there are no ties, in how many orders can they finish?

Q17 12 people run a race. How many ways can the first 3 places be filled?

Q18 In how many orders can you place the letters ADFK?

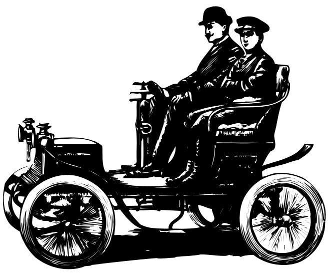
Q19 In how many orders can you place the letters ABCDEFGHIJ?

Q20 How many 3-letter sequences can be made using the letters ABCDEFGHIJ if no letter is used more than once?

Q21 How many 3-letter sequences can be made using the letters ABCDEFGHIJ if letters can be used more than once?

Q22 Number plates in Koryan Province consist of a sequence of 5 letters chosen from the first half of the alphabet. The same letter cannot appear twice. How many different number plates are possible?

Q23 How many number plates would be possible in the previous question if the letters could be used more than once?



Combinations – ${}^nC_r$

Let's say the rowing coach has to choose 3 of the 7 rowers for the boat, but doesn't have to worry about who sits where in the boat – the girls will decide that. How many different choices can she make?

The best way to work this out is to think about how many placings of 3 rowers from 7 that there are. This is ${}^7P_3$, which is $\frac{7!}{(7-3)!}$.

Then consider how many times each choice of 3 is repeated. This is 6 times: ABC, ACB, BAC, BCA, CAB, CBA. We can work this out by considering how many ways we can place the 3 rowers. This is ${}^3P_3$, which is $\frac{3!}{(3-3)!}$, which is 3!

So, the number of ways of choosing the 3 rowers from the 7 is ${}^7P_3 \div 3!$ i.e. $\frac{7!}{(7-3)!3!}$.

We call this the number of choices of 3 things from 7 or, more conventionally, the number of combinations of 3 things from 7. The abbreviation is ${}^7C_3$.

In general, ${}^nC_r$, the number of choices of r things from n, is given by $\frac{n!}{(n-r)!r!}$.

Your calculator can give you combinations. On a Casio, you will find them in OPTN PROB. For 6C3 press 6 nCr 3.

Alternative Notation for Combinations

${}^nC_r$ is often written $\binom{n}{r}$. So ${}^7C_3$ would be written $\binom{7}{3}$. There is no equivalent notation for permutations.

Summary

${}^nP_r$ is the number of placements or permutations of r things from n. It is $\frac{n!}{(n-r)!}$.

${}^nC_r$ or $\binom{n}{r}$ is the number of choices or combinations of r things from n. It is $\frac{n!}{(n-r)!r!}$.

The essential difference is that in working out the number of permutations, the order matters – ABC is a different permutation from ACB, whereas in working out the number of combinations, the order doesn't matter – ABC and ACB are not different combinations.

'Placements' and 'choices' are the more self-explanatory words, but 'permutations' and 'combinations' are the ones traditionally used and the ones generally used by mathematicians. Fortunately, the notation ${}^{n}P_r$ and ${}^{n}C_r$ can be read either way.

Practice

Q24 Find (a) ${}^{3}C_3$ (b) ${}^{8}C_8$ (c) ${}^{8}C_3$ (d) ${}^{12}C_4$ (e) ${}^{20}C_{10}$ (f) ${}^{2}C_1$
(g) ${}^{7}P_5$ (h) ${}^{7}C_5$ (i) ${}^{7}P_7$ (j) ${}^{7}C_7$ (k) ${}^{7}P_1$ (l) ${}^{7}C_1$

Q25 There are 7 rowers in the team and 4 of them need to be chosen to row in the quad. In how many ways can this be done?

Q26 There are 12 applicants for 5 jobs. How many different sets of 5 people is it possible to choose?

Q27 On a lock there are 10 knobs. 5 of these have to be pushed in and 5 left out to open the door. How many different combinations of 5 can be pushed in?

Q28 What is the probability that the lock in the last question will open if 5 knobs are pushed in at random?

Q29 You are dealt a hand of 5 cards from a standard pack of 52. How many different hands are possible? [The order of your 5 doesn't matter because you can rearrange them in your hand.]

Q30 In a lottery, you choose 6 numbers from the numbers 1 to 46. Order doesn't matter. How many different choices are possible?

Q31 In the draw for the lottery in the previous question, 6 numbers are picked at random. If they are the same as yours, you win. What is the probability of winning?

Q32 If you toss a coin 5 times, what is the probability of getting exactly 2 heads? [Hint: find the probability of getting HHTTT, then find how many ways there are of selecting 2 out of 5 tosses to be heads.]

Q33 Each worker in an office of 8 workers has a 10% chance of being chosen for jury service this year. What is the probability that exactly 2 will be chosen.

Q34 If you roll a die 10 times, find the probability of getting exactly 3 sixes.

Binomial Expansion

When expanding a pair of binomial factors, we multiply every term in one factor by every term in the other to get 4 new terms.

$$(x + y)^2 = (x + y)(x + y) = x^2 + xy + yx + y^2 = x^2 + 2xy + y^2$$

The number of xy terms is the number of ways of choosing 1 of the two factors to contribute the y, i.e. ${}^{2}C_1$ or $\binom{2}{1}$ or 2

When expanding three binomial factors, we multiply every term in the first by every term in the second and then by every term in the third to get 8 new terms.

$$\begin{aligned}(x+y)^3 &= (x+y)(x+y)(x+y)\\ &= x^3 + xxy + xyx + yxx + xyy + yxy + yyx + y^3\\ &= x^3 + 3x^2y + 3xy^2 + y^3\end{aligned}$$

The number of xxy terms is the number of ways of choosing 1 of the three factors to contribute the y, i.e. ${}^3C_1$ or $\binom{3}{1}$ or 3.

The number of xyy terms is the number of ways of choosing 2 of the three factors to contribute the ys, i.e. ${}^3C_2$ or $\binom{3}{2}$ or 3.

When expanding four binomial factors, we multiply every term in the first by every term in the second and then by every term in the third and then by every term in the fourth to get 16 new terms.

$$\begin{aligned}(x+y)^4 &= (x+y)(x+y)(x+y)(x+y)\\ &= x^4 \;+\; xxxy + xxyx + xyxx + yxxx \;+\; xxyy + xyxy + yxxy + xyyx + yxyx + yyxx\\ &\quad + \; xyyy + yxyy + yyxy + yyyx \;+\; y^4\\ &= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4\end{aligned}$$

The number of $xxxx$ terms is the number of ways of choosing none of the four factors to contribute a y, i.e. ${}^4C_0$ or $\binom{4}{0}$ or 1.

The number of $xxxy$ terms is the number of ways of choosing 1 of the four factors to contribute the y, i.e. ${}^4C_1$ or $\binom{4}{1}$ or 4.

The number of $xxyy$ terms is the number of ways of choosing 2 of the four factors to contribute the ys, i.e. ${}^4C_2$ or $\binom{4}{2}$ or 6.

The number of $xyyy$ terms is the number of ways of choosing 3 of the four factors to contribute the ys, i.e. ${}^4C_3$ or $\binom{4}{3}$ or 4.

The number of $yyyy$ terms is the number of ways of choosing 4 of the four factors to contribute the ys, i.e. ${}^4C_4$ or $\binom{4}{4}$ or 1.

So we could write $(x+y)^4 = \binom{4}{0}x^4 + \binom{4}{1}x^3y + \binom{4}{2}x^2y^2 + \binom{4}{3}xy^3 + \binom{4}{4}y^4$

Make sure the above makes sense. It is probably actually easier to understand for $(x+y)^3$ and $(x+y)^4$ than for the earlier ones. In general,

$$(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y^1 + \binom{n}{2}x^{n-2}y^2 + \ldots + \binom{n}{n-2}x^2y^{n-2} + \binom{n}{n-1}x^1y^{n-1} + \binom{n}{n}y^n$$

This is the **binomial expansion** or the **binomial theorem**. It is a short-cut for expanding $(x+y)^n$.

Practice

Q35 Expand the following using the binomial theorem. Show the step with the combinations as working.

(a) $(x+y)^3$	(b) $(x+y)^5$	(c) $(a+b)^6$	(d) $(a+b)^2$
(e) $(x+1)^3$	(f) $(a+1)^4$	(g) $(x+3)^3$	(h) $(c+2)^5$
(i) $(x-2)^3$	(j) $(x-1)^4$	(k) $(x-a)^3$	(l) $(x-10)^4$

Pascal's Triangle

Combinations can be evaluated using the formula ${}^nC_r = \binom{n}{r} = \frac{n!}{(n-r)!r!}$.

Or they can found on a graphics calculator.

But there is a third way, using Pascal's Triangle.

Pascal's Triangle is constructed by putting a 1 at the top, then numbers diagonally beneath it which are sums of the numbers above, then repeating to get this:

```
            1
          1   1
        1   2   1
      1   3   3   1
    1   4   6   4   1
  1   5  10  10   5   1
1   6  15  20  15   6   1
```

and so on.

Pascal's Triangle has a lot of interesting properties, but the one relevant here is that in any row, n (taking the top 1 as row 0), the numbers are the coefficients of the binomial expansion of $(x+y)^n$.

They are therefore also the combinations $\binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \binom{n}{3}, \binom{n}{4}$ etc.

To expand say $(x+y)^5$, it can sometimes be easier to write out lines 0 to 5 of Pascal's triangle than to work out or look up the combinations involved.

The coefficients are 1, 5, 10, 10, 5, 1.

So $(x + y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

Practice

Q36 Draw up Pascal's Triangle to Row 10.

Q37 Use your triangle to expand the following:

(a) $(x + y)^5$ (b) $(x + y)^{10}$ (c) $(a + 2)^7$ (d) $(a - 1)^6$

Q38 Add the numbers in each row of your triangle and hence predict the sum of the numbers in Row 20.

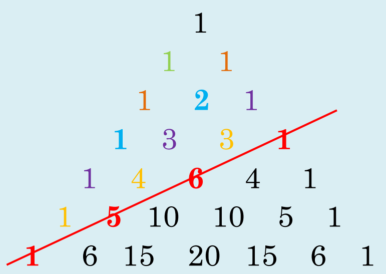
Other Patterns in Pascal's Triangle

Looking at the rows of Pascal's Triangle, you might find the powers of 11: 11^0, 11^1, 11^2 and so on. This should make sense as 11^n can be seen as $(10 + 1)^n$.

The third diagonal (1, 3, 6 etc.) contains the triangular numbers and pairs of successive numbers from this diagonal add to make the square numbers.

The first diagonal is a constant sequence defined by $t_n = 1$ (where t_n is the nth term in the sequence); the second diagonal is a linear sequence defined by $t_n = n$; the third diagonal is a quadratic sequence defined by $t_n = ½n^2 + ½n$; the fourth diagonal is a cubic sequence and so on.

In more gently sloping diagonals, the numbers add to produce the Fibonacci sequence. E.g.



The odd numbers form the Sierpinski Gasket. You might like to Google that.

There are other patterns which you could investigate too.

Solve

Q51 In a lottery, you choose 8 numbers between 1 and 46. 6 numbers are then drawn. If you have all those 6 among your 8, you win. What is the probability of winning?

Q52 If you are dealt 5 cards from a pack of 52, what is the probability of getting four aces?

Q53 If you are dealt 5 cards from a pack of 52, what is the probability of getting four of a kind (4 of the same number e.g. four 9s or four kings)?

Q54 If you are dealt 5 cards from a pack of 52, what is the probability of getting three of a kind?

Q55 If you are dealt 5 cards from a pack of 52, what is the probability of getting a full house (3 of one number and 2 of another, e.g. 99K9K)?

Q56 6 people have 6 seats in a row in a cinema. Josh and Ruby insist on sitting together. Otherwise, they are seated at random. What is the probability that Alan sits next to Sophie?

Q57 Use the first two terms of a binomial expansion to give an approximate value for 1.001^{12}.

Revise

Revision Set 1

Q61 Casey has 4 skirts and 5 tops. If she wears one of each, how many ways can she dress?

Q62 How many sequences of four different digits can be made?

Q63 Find (a) 7! (b) ${}^{9}P_4$ (c) ${}^{11}C_6$

Q64 6 people are to sit on 6 seats. In how many ways can they arrange themselves, one to a seat?

Q65 Archie has 14 applicants for 6 jobs. He chooses the best 6. What is the probability that he chooses the 6 whose names come first in the alphabet?

Q66 What is the probability of getting exactly 3 sixes if you roll 7 dice?

Q67 Use the binomial theorem to expand
(a) $(x + y)^5$ (b) $(x + 3)^3$ (c) $(x - 1)^4$

Q68 Use Pascal's Triangle to expand
(a) $(a + 2)^7$ (b) $(a - 2)^6$

Revision Set 2

Coming soon

Revision Set 3

Coming soon

Answers

Q1 $2 \times 5 \times 4 = 40$
Q2 $2 \times 2 \times 3 \times 7 = 84$
Q3 $5 \times 5 = 25$
Q4 $26 \times 26 \times 10 \times 10 = 67\ 600$
Q5 $26 \times 26 \times 26 = 17\ 576$
Q6 $10 \times 9 \times 8 = 720$
Q7 $3 \times 2 \times 1 = 6$
Q8 $5 \times 4 \times 3 \times 2 \times 1 = 120$
Q9 $10 \times 9 \times 8 = 720$
Q10 $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$
Q11 $6 \times 6 \times 6 \times 6 \times 6 \times 6 = 46\ 656$
Q12 (a) 24 (b) 40 320 (c) 479 001 600 (d) About 4×10^{26} (e) 1 (f) 1
Q13 (a) 6 (b) 40 320 (c) 336 (d) 11 880 (e) About 6.7×10^{11} (f) 2
Q14 ${}^9P_4 = 3024$
Q15 (a) ${}^6P_1 = 6$ (b) ${}^6P_2 = 30$ (c) 120 (d) 360 (e) 720 (f) 720
Q16 ${}^5P_5 = 120$
Q17 ${}^{12}P_3 = 1320$
Q18 ${}^4P_4 = 24$
Q19 ${}^{10}P_{10} = 3\ 628\ 800$
Q20 ${}^{10}P_3 = 720$
Q21 $10 \times 10 \times 10 = 1000$
Q22 ${}^{13}P_5 = 154\ 440$
Q23 $13^5 = 371\ 293$
Q24 (a) 1 (b) 1 (c) 56 (d) 495 (e) 184 756 (f) 2
(g) 2520 (h) 21 (i) 5040 (j) 1 (k) 7 (l) 7
Q25 ${}^7C_4 = 35$
Q26 ${}^{12}C_5 = 792$
Q27 ${}^{10}C_5 = 252$
Q28 $^1/_{252}$
Q29 ${}^{52}C_5 = 2\ 598\ 960$
Q30 ${}^{46}C_6 = 9\ 366\ 819$
Q31 $^1/_{9\ 366\ 819}$

Q32 ${}^{5}C_{2} \times 0.5^{5} = 0.3125$

Q33 ${}^{10}C_{2} \times 0.1^{2} \times 0.9^{8} = 0.194$

Q34 ${}^{10}C_{3} \times (1/6)^{3} \times (5/6)^{7} = 0.155$

Q35 (a) $(x + y)^{3} = x^{3} + 3x^{2}y + 3xy^{2} + y^{3}$

(b) $(x + y)^{5} = x^{5} + 5x^{4}y + 10x^{3}y^{2} + 10x^{2}y^{3} + 5xy^{4} + y^{5}$

(c) $(a + b)^{6} = a^{6} + 6a^{5}b + 15a^{4}b^{2} + 20a^{3}b^{3} + 15a^{2}b^{4} + 6ab^{5} + b^{6}$

(d) $(a + b)^{2} = a^{2} + 2ab + b^{2}$

(e) $(x + 1)^{3} = x^{3} + 3x^{2} + 3x + 1$

(f) $(a + 1)^{4} = a^{4} + 4a^{3} + 6a^{2} + 4a + 1$

(g) $(x + 3)^{3} = x^{3} + 9x^{2} + 27x + 27$

(h) $(c + 2)^{5} = c^{5} + 10c^{4} + 40c^{3} + 80c^{2} + 80c + 32$

(i) $(x - 2)^{3} = x^{3} - 6x^{2} + 12x - 8$

(j) $(x - 1)^{4} = x^{4} - 4x^{3} + 6x^{2} - 4x + 1$

(k) $(x - a)^{3} = x^{3} - 3x^{2}a + 3xa^{2} - a^{3}$

(l) $(x - 10)^{4} = x^{4} - 40x^{3} + 600x^{2} - 4000x + 10000$

Q36

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

Q37 (a) $(x + y)^{5} = x^{5} + 5x^{4}y + 10x^{3}y^{2} + 10x^{2}y^{3} + 5xy^{4} + y^{5}$

(b) $(x + y)^{10} = x^{10} + 10x^{9}y + 45x^{8}y^{2} + 120x^{7}y^{3} + 210x^{6}y^{4} + 252\,x^{5}y^{5} + 210x^{4}y^{6} + 120x^{3}y^{7} + 45x^{2}y^{8} + 10xy^{9} + y^{10}$

(c) $(a + 2)^{7} = a^{7} + 14a^{6} + 84a^{5} + 280a^{4} + 560a^{3} + 672a^{2} + 448a + 256$

(d) $(a - 1)^{6} = a^{6} - 6a^{5} + 15a^{4} - 20a^{3} + 15a^{2} - 6a + 1$

Q38 $2^{20} = 1\,048\,576$

Q51 ${}^{8}C_{6} \div {}^{46}C_{6} = 0.000\,002\,989$

Q52 $48 \div {}^{52}C_{5} = 0.000\,0185$

Q53 $13 \times 48 \div {}^{52}C_{5} = 0.000\,24$

Q54 $13 \times 49 \times 48 \div {}^{52}C_{5} = 0.011\,8$

Q55 ${}^{4}C_{3} \times {}^{4}C_{2} \times 13 \times 12 \div 52C_{5} = 0.001\,44$

Q56 $({}^{4}P_{4} \times 2) \div ({}^{5}P_{5} \times 2) = 0.2$

Q57 1.012

Q61 $4 \times 5 = 20$

Q62 $10 \times 9 \times 8 \times 7 = 5040$

Q63 5040

Q64 720

Q65 $1 \div {}^{14}C_6 = 0.000\ 333$

Q66 ${}^{7}C_3 \times (1/6)^3 \times (5/6)^4 = 0.078$

Q67 (a) $(x + y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

(b) $(x + 3)^3 = x^3 + 9x^2 + 27x + 27$

(c) $(x - 1)^4 = x^4 - 4x^3 + 6x^2 - 4x + 1$

Q68 (a) $(a + 2)^7 = a^7 + 14a^6 + 84a^5 + 280a^4 + 560a^3 + 672a^2 + 448a + 256$

(b) $(a - 2)^6 = a^6 - 12a^5 + 60a^4 - 160a^3 + 240a^2 - 192a + 128$