M1 Maths

A5-3 Power Functions

- general form and graph shape
- equation solution methods
- applications

Summary Learn Solve Revise Answers

Summary

The general form of power functions is $y = ax^n$, where the parameters a and n are real numbers and x is a positive real number. Graphs pass through the point $(1, a)$; if $n > 0$, the gradient is positive, if $n < 0$, the gradient is negative.

Power equations are solved by raising both sides to the power of $^1/_n$. Power functions have applications where the dependent variable is proportional to some power of the independent variable, including with area and volume and in physics.

Learn

General Form

Power functions have the general form $y = ax^n$, where the parameters a and n are real numbers and x is a positive real number.

Examples of power functions are: $y = x^3$, $y = 3x^{1.25}$, $y = x^{0.5}$ ($y = \sqrt{x}$), $y = x^{-1}$ ($y = \frac{1}{x}$), $y = 5x^{-2}$ ($y = \frac{5}{x^2}$), $y = x^{-0.6}$, $y = 3x^0$ ($y = 3$).

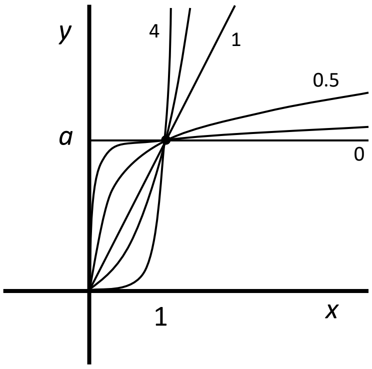
As you can see there is some overlap between power functions and polynomial functions. Also, reciprocal functions are power functions with $n = -1$.

Because some fractional powers of negative numbers are not real, we will concern ourselves only with positive values for the independent variable.

Shapes of the Graphs

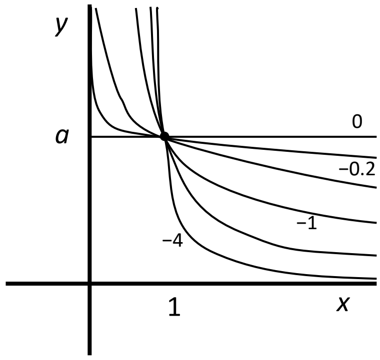
With any function of the form $y = ax^n$, when $x = 1$, $y = a$. Therefore, all such functions pass through the point $(1, a)$.

If n is 0 or positive, the graphs vary as shown to the left.



If n is 0 or negative, the graphs vary as shown to the right.

The value of n is shown for some of the graphs



Practice

Q1 Without using a graphics calculator, sketch rough graphs for each of the following power functions. You may draw them all on the same set of axes, but show the scales on the axes.

(a) $y = x^4$ (b) $y = 3x^4$ (c) $y = ½x^5$ (d) $y = x^{1.5}$

(e) $y = 2\sqrt{x}$ (f) $y = \dfrac{1.5}{x^2}$ (g) $y = \dfrac{2}{\sqrt[5]{x}}$

Check your sketches with the graphics calculator.

Solution of Equations from Power Functions

Solving power function equations is easy. We use the method of doing the same thing to both sides, along with Index Law 3 ($(a^m)^n = a^{mn}$).

Suppose we have $h = 5t^3$ and we want the value of t when $h = 200$. Substituting, we get

$$200 = 5t^3$$

The operations performed on t are cubing, then multiplying by 5.

First we divide by 5 $\quad 40 = t^3$

The we raise both sides to the power of $^1/_3$ $\quad 40^{1/3} = (t^3)^{1/3}$

We chose $^1/_3$ because it is 1 over 3,

and by the third index law, $(t^3)^{1/3} = t$, so $\quad 40^{1/3} = t$

Then we simply enter that into the calculator to get $\quad 3.42 = t$

Practice

Q2 Use the method above to solve:

(a) $55 = 2x^4$ (b) $3x^3 = 10$ (c) $\frac{1}{2}x^5 = 237$ (d) $18 = x^{1.5}$

(e) $0.03 = 5x^{10}$ (f) $x^8 = 44.29$ (g) $0.4x^{3.5} = 12$ (h) $7 = x^{1.72}$

Q3 Use the same method to solve these, but express surds and fractions as simple powers first, e.g. write $\sqrt{x}$ as $x^{0.5}$ and $\frac{1}{x^3}$ as x^{-3}.

(a) $12 = 2\sqrt{x}$ (b) $\frac{1.5}{x^2} = 4$ (c) $\frac{2}{\sqrt[5]{x}} = 21$ (d) $\sqrt[3]{x^4} = 50$

(e) $0.0046 = \sqrt{x} \div 12$ (f) $\frac{1}{x^2} = 3.8$ (g) $\frac{5}{\sqrt[4]{x}} = 1$ (h) $\sqrt[5]{x^2} = 12$

Some Applications of Power Functions

The formulae for the area of a square and the volume of a cube are simple power functions: $A = s^2$ and $V = s^3$. Similarly the area of a circle and the volume of a sphere: they are $A = {}^{\pi}/_{4}d^2$ and $V = {}^{\pi}/_{6}d^3$.

The heat radiated by a body at absolute temperature T is given by the formula $H = kT^4$, where H is the rate of heat loss and k is a constant. The rate of convective heat loss from a body is given by $H = kT^{1.25}$, where T is the temperature difference between the body and the air.

The electrostatic or gravitational force between two objects with separation d is given by $F = kd^{-2}$.

Square root relations are common. The relation between the side length and area of a square is $s = A^{0.5}$. The relation between the record amount of rain (anywhere in the world), R and the time period, t, over which is was measured approximates $R = kt^{0.5}$.

Solve

Q51 As mentioned, the highest recorded rainfall in a given time interval, t, is roughly proportional to the square root of t. The highest rainfall recorded in a 15-minute interval is 198 mm at Plumb Point, Jamaica on 12 May 1916. Roughly what would be the highest rainfall recorded in a one-hour period, in a 24-hour period and in a 365-day period?

Revise

Revision Set 1

Q61 Give the general form of power functions.

Q62 On the same axes, sketch the graphs of $y = x$, $y = x^2$, $y = x^{1/2}$, $y = 2x^3$
Check your sketches with the graphics calculator.

Q63 Solve:

(a) $x^3 = 10$ (b) $3x^5 = 189$ (c) $6 \times \sqrt[3]{x} = 4.3$ (d) $\frac{1}{x^4} = 0.026$

Q64 The edge length of a cube is equal to the cube root of the volume. Find the edge length if the volume is 200 cm^3.

Revision Set 2

Q71 Give the general form of power functions

Q72 On the same axes, sketch the graphs of $y = x$, $y = x^3$, $y = x^{1/2}$, $y = 2x^2$
Check your sketches with the graphics calculator.

Q73 Solve:

(a) $x^4 = 10$ (b) $3x^6 = 201$ (c) $4 \times \sqrt[3]{x} = 4.5$ (d) $\frac{1}{x^3} = 0.026$

Q74 The volume of a sphere is given by $\frac{\pi}{6}d^3$, where d is the diameter. Find the diameter of a sphere with volume 660 cm^3.

Revision Set 3

Q81 Give the general form of

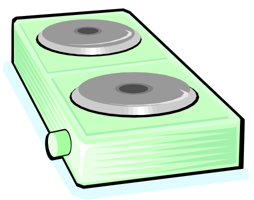
(a) power functions (b) exponential functions

Q82 On the same axes, sketch the graphs of $y = x$, $y = x^2$, $y = x^{1/2}$, $y = 2x^2$
Check your sketches with the graphics calculator.

Q83 Solve:

(a) $x^{7.5} = 12$ (b) $2x^3 = 48$ (c) $5 \times \sqrt[4]{x} = 120$ (d) $\frac{1}{x^8} = 45$

Q84 The rate of loss of energy by radiation from a hot body is given by $E = 0.000\,062\,T^4$, where E is in Watts and T is the absolute temperature in Kelvins. At what temperature will the body radiate at 60 kW?



Answers

Q2	(a) 2.29	(b) 1.49	(c) 3.43	(d) 6.87
	(e) 0.60	(f) 1.61	(g) 2.64	(h) 3.10
Q3	(a) 36	(b) 0.61	(c) .00000784	(d) 18.8
	(e) 0.00305	(f) 0.513	(g) 625	(h) 499

Q51 The calculated answers are: 396 mm in 1 hour, 1939 mm in 24 hours and 37 m in 365 days. The actual records are: 401 mm in 1 hour at Sahngdu, China on 3 July 1975, 1825 mm in 24 hours at Foc Foc, La Reunion Island on 7-8 January 1966 and 26 m at Cherranpunji, India in August 1860 to July 1861.

Q61	(a) $y = ax^n$			
Q63	(a) 2.15	(b) 2.29	(c) 0.368	(d) 2.49
Q64	5.85 cm			
Q71	(a) $y = ax^n$			
Q73	(a) 1.78	(b) 2.02	(c) 1.42	(d) 3.38
Q74	10.8 cm			
Q81	(a) $y = ax^n$			
Q83	(a) 1.39	(b) 2.88	(c) 331 776	(d) 0.621
Q84	176 K			