M1 Maths

A4-6 Algebraic Proofs

- algebraic proofs

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Summary

We can prove that every number produces the same result when certain operations are performed on it by using algebra, choosing a variable for the number and performing the operations on that variable. We can prove that not every number produces a given result simply by finding one that doesn't.

Learn

Take a whole number and square it. Then multiply the previous number by the following number. Then subtract the product you get from the square you obtained first up.

For example, take 7. Squaring it gives 49. The previous number multiplied by the next number is 6 × 8, which is 48. Then 49 – 48 = 1.

Try other starting numbers. Do you notice anything?

You should find that your answers are generally 1. Will your answer always be 1 or are there some exceptions? To show that it is always 1, you need to prove it algebraically. If it works for a whole number, n, then, as n can be any whole number, it will always work.

You might like to try to prove this yourself before you read on.

Here is a proof.

> Let the first whole number be n.
>
> Its square is n^2.
>
> The previous number is $n - 1$; the following number is $n + 1$
>
> Multiplying these gives $(n - 1)(n + 1)$, which expands to $n^2 - 1$
>
> Subtracting the product from the square gives us $n^2 - (n^2 - 1)$, which is 1.

As n can be any whole number, this proves that the pattern will work for all whole numbers.

To prove that a pattern will not always hold, all you have to do is produce one case where it doesn't.

For example, think of a number. Let's say it's 5. Double it, then add 4. This gives 14. Will the result always be a multiple of 7?

To answer that, try another number, say 12. $12 \times 2 + 4 = 28$, which is also a multiple of 7.

At this point, we can try to prove that we will always get a multiple of 7. Or we can try a few other numbers first.

Let's try 6. $6 \times 2 + 4 = 16$, which is not a multiple of 7. Therefore the pattern does not always hold.

Practice Set 1

Q1 Prove whether or not the following patterns will always hold.

(a) Think of a whole number, add 8, then multiply by 2, then subtract 6, then divide by 2, then subtract the number you started with, then add 5. Do you always end up with 10?

(b) Think of a number, square it, add the number you started with, add one more than the number you started with, take the square root, subtract 1. Do you always end up with the number you started with?

(c) Take a whole number, add 2, multiply it by 6, add 4, divide by 2, add 1. Will the result always be a multiple of 3?

(d) Think of a whole number, square it, then double it, then double it again. Will the result always be a perfect square?

(e) Think of a number, add 7, then multiply by 2, then subtract 7. Will the result always be a multiple of 7?

(f) Take a two digit number with different digits. Then take another two digit number which has the same digits, but round the other way. Add them. Will the result always be a multiple of 11?

Q2 Prove whether or not the following patterns will always hold.

(a) Add two odd numbers. Will the result always be even?

(b) Multiply two odd numbers. Will the result always be odd?

(c) Add two prime numbers. Will the result always be even?

(d) On the number grid below, pick any square of four numbers. (An example square is shown.) Find the product of the top-left and bottom-right numbers in the square. Then find the product of the top-right and bottom-left numbers in the square. Will the difference between the products always be the same?

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

(e) Take a 3-digit number. Take another 3-digit number which has the same digits, but reversed. Add them. Will the result always be a palindrome (i.e. a number which is the same if you reverse it)?

Solve

Q51 Think of a number. Multiply the number 3 more than it by the number 2 less than it. Then subtract the square of the number you started with, then add 7. Will you always get the number you started with? If not, modify one step of the instructions so that you do.

Q52 This is a hard one. Take a 3-digit number with no zeros and all digits different. Take another 3-digit number which has the same digits, but reversed. Subtract the smaller number from the bigger to get a new 3-digit number. Then take another 3-digit number which has the same digits as this number, but reversed. Then add the two numbers. Is the result always 1089? Prove it.

Revise

Revision Set 1

Q61 Prove whether or not the following patterns will always hold.

(a) Think of a number, multiply it by 3, then add 7, then double it, then subtract 8. Will the result always be a multiple of 6?

(b) On the calendar below, select any rectangle of numbers. (One possible such rectangle is shown.) Start with the number in the top left corner, subtract the number in the bottom left corner, then add the number in the bottom right corner, then subtract the number in the top right corner. Will you always get the same answer?

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

(c) Think of a number. Multiply the number 2 more than it by the number 1 less than it. Then subtract the square of the number you started with, then add the number you started with. Will you always get the same result?

Answers

All answers are proofs, but the following indicates whether the patterns will hold.

Q1	(a) Yes	(b) Yes	(c) Yes	(d) Yes	(e) No	(f) Yes
Q2	(a) Yes	(b) Yes	(c) No	(d) Yes	(e) No	
Q51	No	Q52 Yes				
Q61	(a) Yes	(b) Yes	(c) No			