M1 Maths

A4-4 Inequalities

- solve single-unknown inequalities and graph the solutions on a number line

Summary Learn Solve Revise Answers

Summary

An inequality (or inequation) is a statement containing a <, >, ≤ or ≥ sign instead of an = sign.

Inequalities can be solved in the same way as equations except that, when both sides are multiplied or divided by a negative number, the inequality sign is reversed.

Learn

Suppose there was a 100 m race for 5 year-olds in 3 years time. One might wonder how old a child would have to be now to have a prospect of entering.

Most people could answer this simply in their head: they would have to be 2. But we could also solve it with an equation: Let the child's age now be a; then the requirement is that $a + 3 = 5$. We can solve this by subtracting 3 from both sides to get $a = 2$.

Now suppose the children had to be 5 years or less at the time of the race. Then the equation would become $a + 3 \leq 5$. Because we have a ≤ sign rather than a = sign, we call this an inequation or, more commonly, an inequality. It would also be an inequality if it had a ≥, <, or > sign.

Solving inequalities is very much like solving equations. We just get the unknown on its own on one side by performing inverse operations. So to solve $a + 3 \leq 5$, we would subtract 3 from both sides to get $a \leq 2$. This tells us that the child must be 2 years old or less now if we want to enter it into the race.

A more complex inequality might be $2(x - 5) + 4 > 12$. We would solve this as follows.

$$\begin{aligned} 2(x-5)+14 &> 12 \\ -14 \quad & \quad -14 \\ 2(x-5) &> -2 \\ \div 2 \quad & \quad \div 2 \\ x-5 &> -1 \\ +5 \quad & \quad +5 \\ x &> 4 \end{aligned}$$

Of course, you don't need to include the operation lines.

There is just one difference from solving equations: if we multiply or divide both sides by a negative number, then we flip the inequality sign round the other way. > becomes <; < becomes >; ≥ becomes ≤; ≤ becomes ≥.

To see why, think about this: If $-x < -5$, then $x > 5$.

So to solve $12 - 2x < 3$, we could proceed as follows.

$$12 - 2x < 3$$
$$-2x < -9$$
$$x > 4.5$$

Practice

Q1 Solve the following inequalities. Show working as laid out above.

(a) $x + 7 < 20$

(b) $2a - 4 \geq 42$

(c) $3(t + 5) - 12 > 9$

(d) $\frac{2x-5}{4} \leq 3$

(e) $-5w - 4 > 16$

(f) $3(8 - 3s) + 12 \leq 3$

(g) $4x + 5 < 3$

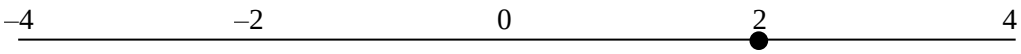
(h) $2h + 1 \geq -8$

(i) $3(r - 5) + 2r \geq 8 - r$

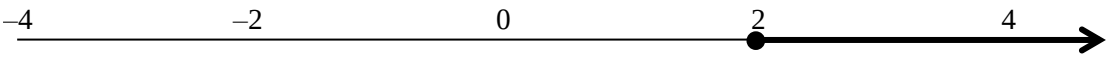
(j) $\frac{4}{(12-2c)} > 9$

Graphing Solutions

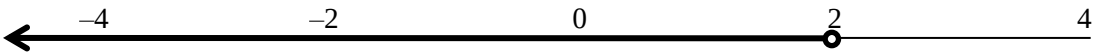
The equation $4x + 5 = 13$ has solution $x = 2$. This can be plotted (graphed) on a number line like this:



The inequality $4x + 5 \geq 13$ has solution $x \geq 2$. This can be plotted (graphed) on a number line like this:



The thick line covers the possible values for x. The arrow indicates that the line continues past the end of the diagram to infinity. The solid dot at 2 indicates that the number 2 is included. If the number 2 wasn't included, as with the inequality $4x + 5 < 13$ which has solution $x < 2$, we would use an empty dot like this:



Practice

Q2 Graph the solutions to the inequalities in P1, each on a suitable number line.

Solve

Q51 Solve these inequalities and graph the solutions.

(a) $|x| < 5$ (b) $|x - 3| \geq 1$ (c) $|8 - 2x| \leq 4$

[Note that $|x|$ means the absolute value of x, i.e. its value with any negative sign removed. So $|2| = 2$ and $|-2| = 2$.]

Revise

Revision Set 1

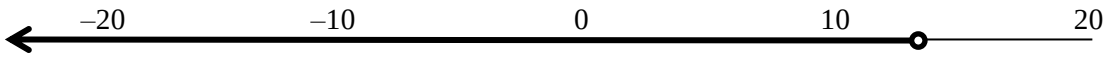
Q61 Solve the following inequalities and graph the solution on a number line.

(a) $4h + 9 < 1$ (b) $5(1 - x) - 2x \geq 11 - x$

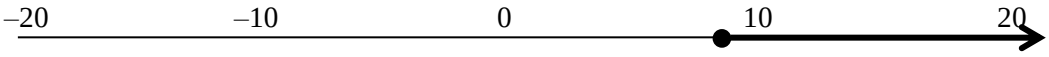
Answers

Q1
(a) $x < 13$
(b) $a \geq 8.5$
(c) $t > 2$
(d) $x \leq 16.5$
(e) $w < -4$
(f) $s \geq 11/3$
(g) $x < -½$
(h) $h \geq -4.5$
(i) $r \geq 23/6$
(j) $c > -52/9$

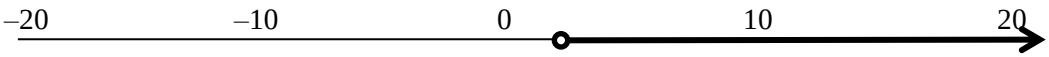
Q2 (a)



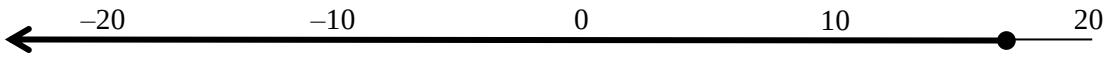
(b)



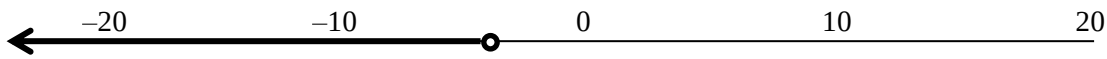
(c)



(d)



(e)



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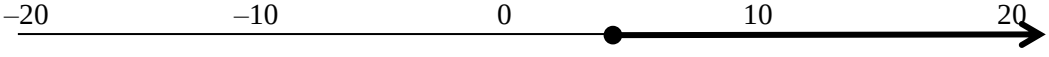
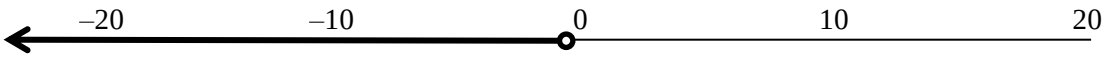
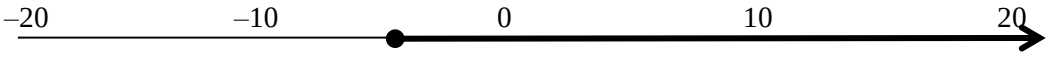


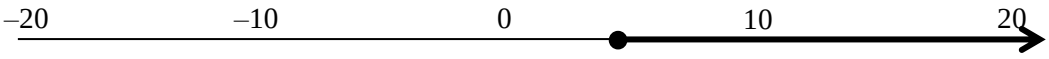
(g)



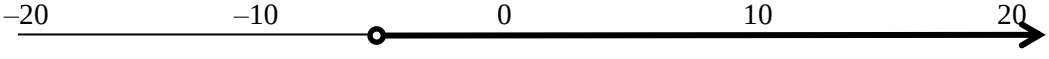
(h)



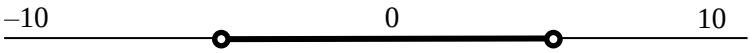
(i)



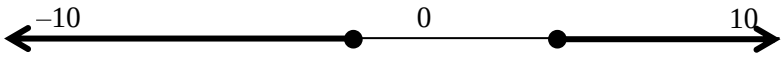
(j)



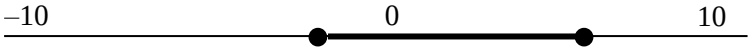
Q51 (a) $-5 < x < 5$



(b) $x \leq 2$ or $x \geq 4$



(c) $2 \leq x \leq 6$



Q61 (a) $h < -2$

(b) $x \leq -1$