M1 Maths

A4-2 Quadratic Functions

- general form and graph shape
- equation solution methods
- applications
- writing and solving quadratic equations

Summary Learn Solve Revise Answers

Summary

A quadratic function has the general form $y = ax^2 + bx + c$. This is sometimes called the expanded form. It can also be expressed in factorised form and vertex form.

The graph of a quadratic function is a parabola with a vertical axis of symmetry.

To sketch the graph, we can use any or all of these facts: the parameter c gives the y-intercept; b gives the gradient at the y-intercept; a determines the width and way up of the parabola; the vertex form or the expression $\frac{-b}{2a}$ can be used to obtain the vertex coordinates; and solving $y = 0$ gives the x-intercepts.

Equations from quadratic functions (quadratic equations) can be solved by factorising, by completing the square, by use of the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, or by graphing. Quadratic equations can have two different roots (solutions), two equal roots or no roots.

Quadratic functions have many applications including area problems and the relation between height and time for a projectile. Quadratic equations can be written and solved to solve problems in these contexts.

Learn

General Form

A quadratic function is like a linear function, but with an x^2 term added. So the **general form** of a quadratic function is $y = ax^2 + bx + c$. [Note that we use b instead of m for the coefficient of x.] The parameters a, b and c are usually called the coefficient of x^2, the coefficient of x and the constant term.

There are three forms commonly used for a quadratic function. The general form,

$y = ax^2 + bx + c$, is sometimes called the **expanded form**.

The second form is the **factorised form**. The factorised form of $y = x^2 – 5x + 6$ is $y = (x – 2)(x – 3)$. You met this in Module A4-1. You can convert between expanded form and factorised form by expanding or factorising.

The third form is the **vertex form**. The vertex form of $y = x^2 – 5x + 6$ is $y = (x – 2½)^2 – ¼$. It consists of the square of a linear term plus a constant.

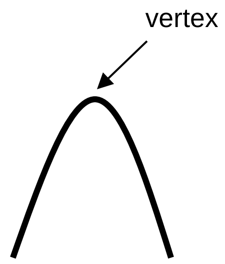
Changing to Vertex Form

To convert the expanded form to the vertex form from, we change the first two terms. To convert $y = x^2 – 5x + 6$, we change $x^2 – 5x$, to $(x – 2½)^2$. (We halve the coefficient of x and use this as the constant in the squared bracket.)

But this expands to $x^2 – 5x + 6¼$ rather than $x^2 – 5x$, so we have to subtract the 6¼ giving us $(x – 2½)^2 – 6¼$. (We subtract the square of the constant term in the bracket.)

The whole expression is then $y = (x – 2½)^2 – 6¼ + 6$, which is $y = (x – 2½)^2 – ¼$.

This form is called the vertex form because the numbers in it give us the coordinates of the vertex. In this case, the vertex is at (2½, –¼). Note that the negative of the number inside the bracket gives the x-coordinate. [This is because the bracket squared will always be positive and the expression will be a minimum when the bracket is zero, in this case when $x = 2½$.] The number outside the bracket is the y-coordinate. [This is because that will be the value of the expression when the bracket is zero.]



If the coefficient of x^2, a is not 1, we first take out a factor of a.
If we have $12x^2 + 2x – 6$, we proceed like this:

$$
\begin{aligned}
&12x^2 + 2x - 6 \\
&= 12[x^2 + {}^1/_6x - ½] \\
&= 12[(x - {}^1/_{12})^2 - {}^1/_{144} - ½] \\
&= 12[(x - {}^1/_{12})^2 - {}^{73}/_{144}] \\
&= 12(x - {}^1/_{12})^2 - {}^{73}/_{12}
\end{aligned}
$$

The vertex is then at $({}^1/_{12}, -{}^{73}/_{12})$.

The 12 doesn't affect the position of the vertex, just the narrowness of the graph.

The process of converting to vertex form is often called '**completing the square**'.

To convert factorised form to vertex for, we expand first, then proceed as above.

To convert vertex form back to expanded form, we just expand and simplify.

Practice

Q1 For each of these functions, complete the square to get the vertex form and the coordinates of the vertex. [For the ones in factorised form, you will need to expand first.]

(a) $y = x^2 + 4x + 11$ (b) $y = x^2 - 2x - 4$

(c) $y = x^2 + 3x - 1$ (d) $y = 2x^2 + 10x + 3$

(e) $a = 2c^2 - 4c + 5$ (f) $y = x^2 + 4x$

(g) $s = x^2 + 0.2x + 2$ (h) $y = -2 + 3x - 3x^2$

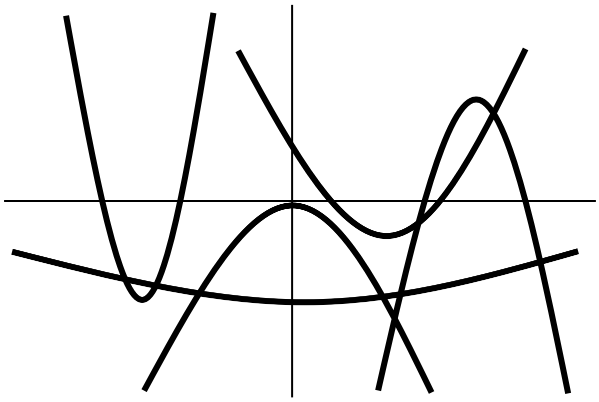
(i) $h = (t + 2)(t - 3)$ (j) $p = 2(3s - 5)(s + 1)$

Q2 Convert each of the following to the expanded form and the factorised form

(a) $y = (x - 2)^2 - 3$ (b) $y = (x + 4\frac{1}{2})^2 + 8$

Graph Shape

The graph of a quadratic function is a parabola with an axis of symmetry parallel to the y-axis. The parameters, a, b and c, determine the location, the width and which way up it is. Some example are shown in the diagram below.



The top or bottom of the curve is called the vertex.

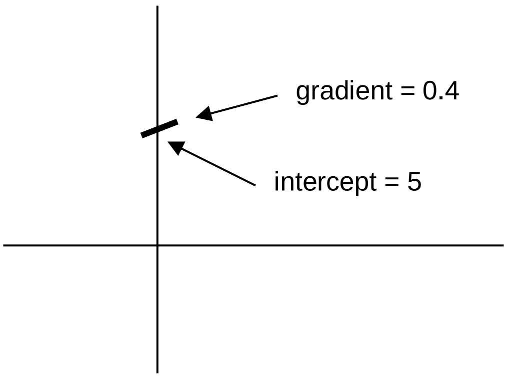
There are a number of techniques that can be used to tie down the graph, given the formula. They are listed below using the sample function $y = 0.2x^2 + 0.4x + 5$. They are listed in order from easiest to use to hardest to use. Which ones you use will depend on how accurate you want your graph to be.

1. *y-intercept*

The parameter c in the expanded form gives you the y-intercept. This should be obvious because, at the y-axis, $x = 0$, so $y = c$.

2. *Gradient at the y-axis*

When x is close to zero (between –1 and 1), the value of x^2 is less than the value of x. For example, when $x = 0.01$, $x^2 = 0.0001$.



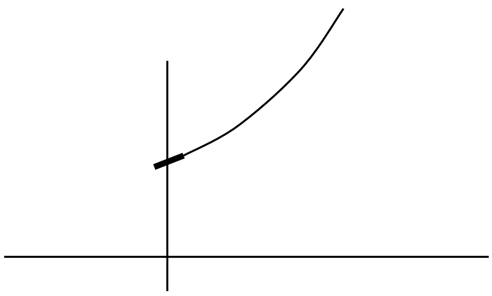
So near the y-axis, the graph of a quadratic function $y = ax^2 + bx + c$ is like the graph of the linear function $y = bx + c$ with the ax^2 term not significant.

$y = 0.2x^2 + 0.4x + 5$ looks like $y = 0.4x + 5$ near the y-axis. So its gradient is 0.4.

So b gives the gradient at the y-axis.

3. *Curvature*

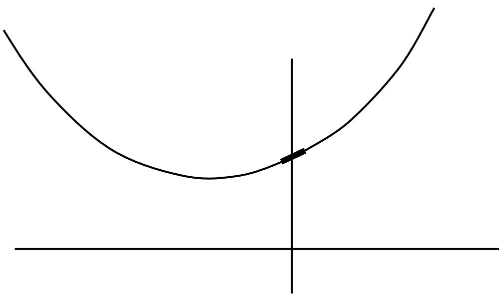
However, as we go further to the right in the example above, $y = 0.2x^2 + 0.4x + 5$, the $0.2x^2$ term becomes significant and makes the value of y greater than it would be if it was just $y = 0.4x + 5$.



The greater x is, the greater the effect of the $0.2x^2$ term.

The graph then curves as shown to the right.

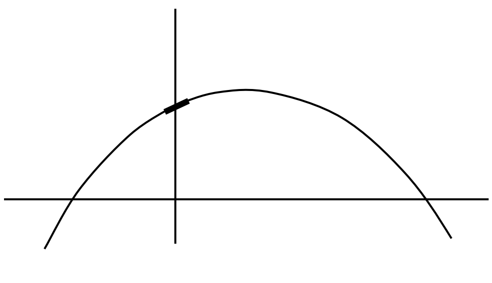
For negative values of x, $0.2x^2$ is positive, so y is greater than it would be for $y = 0.4x + 5$ to the left of the axis as well.



So the graph looks like the picture to the right.

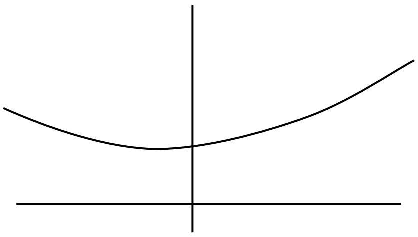
The more negative x is the greater the effect of the $0.2x^2$ term.

If the parameter a is negative, then the x^2 term will cause the y-values to be lower and so the graph will curve downwards like this.

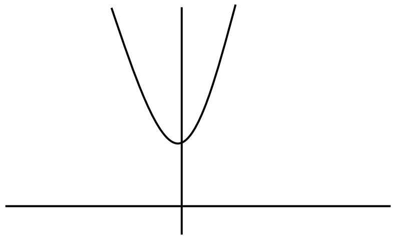


So the graph will be a curve – a parabola – opening upwards of a is positive, opening downwards if a is negative.

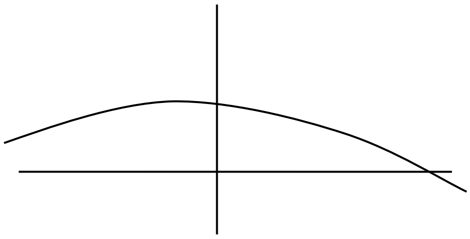
Furthermore, the greater the value of a (positive of negative), the greater the curvature will be and so the narrower the graph will be.



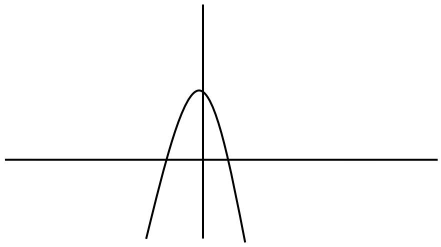
$y = 5 + 0.5x + 0.2x^2$



$y = 5 + 0.5x + 3x^2$



$y = 5 + 0.5x - 0.2x^2$



$y = 5 + 0.5x - 3x^2$

4. *Vertex Location*

The vertex coordinates can be obtained in two ways.

The first is by completing the square to get the formula into vertex form as explained above.

The second is to use the fact that the x-coordinate of the vertex is $\frac{-b}{2a}$, where a and b are the parameters. For example the x-coordinate of $y = 0.2x^2 + 0.4x + 5$ is $\frac{-0.4}{2\times0.2} = -1$.

We then sub the x-value into the formula to get the y-coordinate, in this case $y = 0.2 \times (-1)^2 + 0.4 \times (-1) + 5 = 4.8$. So the vertex is at $(-1, 4.8)$.

Practice

Q3 Sketch the following quadratic functions using the techniques above. After doing each one, graph it on a graphics calculator to check that you got it about right.

(a) $y = x^2 + 2x - 3$

(b) $y = 4x^2 - 2x + 1$

(c) $k = -t^2 + 4t - 1$

(d) $g = \frac{x^2}{5} + 3x + 2$

(e) $a = -3c^2 - 2c$

(f) $y = 3x^2 + 4$

(g) $s = x^2 + 0.2x + 2$

(h) $y = -2 + 3x - x^2$

Solving Quadratic Equations

If you substitute for the dependent variable in a quadratic function, you get a quadratic equation.

For example, if $y = 3x^2 + 2x - 7$ and you sub $y = 5$, you get $3x^2 + 2x - 7 = 5$. This is a quadratic equation.

Quadratic equations are characterised by having (in general) an x^2 term and an x term. This makes them impossible to solve by undoing. But we often need to solve quadratic equations.

There are three commonly used techniques: factorising, completing the square and using the quadratic formula.

Solving by Factorising

Let's say we write down two numbers. Then we multiply them and that gives us 0.

From that, we know that either the first number we wrote down is zero, or the second number is zero (or both are zero). This is called the null factor theorem and is the basis for solving quadratic equations by factorising.

If $ab = 0$, then either $a = 0$ or $b = 0$

In the same way, if $(r + 5)(r - 2) = 0$, then either $(r + 5) = 0$ or $(r - 2) = 0$.

If $(r + 5) = 0$, then $r = -5$. If $(r - 2) = 0$, then $r = 2$

So there are two solutions to the equation $(r + 5)(r - 2) = 0$.

They are $r = -5$ and $r = 2$.

The equation will be true when $r = -5$ and when $r = 2$. Sub each value in to check.

So . . . if we write a quadratic equation as a product of two factors equal to 0, then we can solve it.

Suppose we have the quadratic function $y = x^2 - 6x + 12$ and suppose we know $y = 4$.

Then we have the equation $x^2 - 6x + 12 = 4$

If we then subtract 4 from both sides, we have $x^2 - 6x + 8 = 0$. This is called the standard form of the equation: all terms on the left, zero on the right.

We can then factorise to get $(x - 4)(x - 2) = 0$

Then we use the null factor theorem to conclude that $x - 4 = 0$ or $x - 2 = 0$. And from this we conclude that $x = 4$ or $x = 2$.

So that's how you solve a quadratic equation by factorising.

- Start with the equation $x^2 - 6x + 12 = 4$
- Rearrange it to get one side equal to 0 $x^2 - 6x + 8 = 0$
- Factorise $(x - 4)(x - 2) = 0$
- Put each factor equal to 0 $x - 4 = 0$ or $x - 2 = 0$
- Solve the two linear equations $x = 4$ or $x = 2$

Practice

Q4 Solve the following quadratic equations by factorising:

(a) $x^2 + 5x + 12 = 6$ (b) $x^2 - 6x + 10 = 2$ (c) $x^2 - 10x + 4 = -5$

(d) $x^2 - x = 30$ (e) $p^2 - 10 = -1$ (f) $n^2 - 7\text{n} + 25 = 13$

(g) $a^2 + 13a + 40 = 0$ (h) $c^2 + 4c = 12$ (i) $s^2 = 24 - 2s$

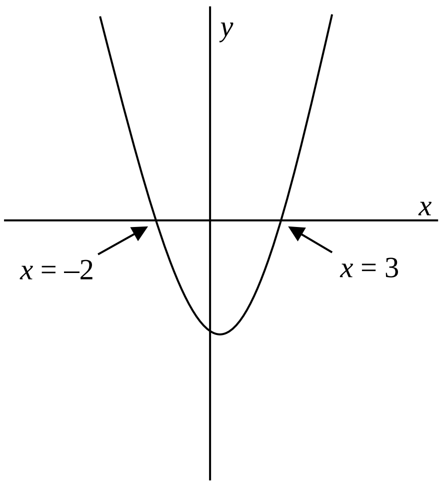
(j) $n^2 - 4n + 20 = 8 + 3n$ (k) $x^2 - 10 = x + 10$ (l) $a^2 + 3a = 4$

(m) $6x^2 = 7x + 3$ (n) $4 = 10x^2 - 9x + 6$ (o) $6x^2 - 5x = 1$

(p) $9 + 4x^2 = -12x$ (q) $10x^2 + 20x + 6 = 2x^2 + 2x - 3$

Understanding Solutions of Quadratic Equations in Terms of Graphs

Suppose we have the equation $x^2 - x - 6 = 0$. Its solutions are $x = -2$ and $x = 3$.



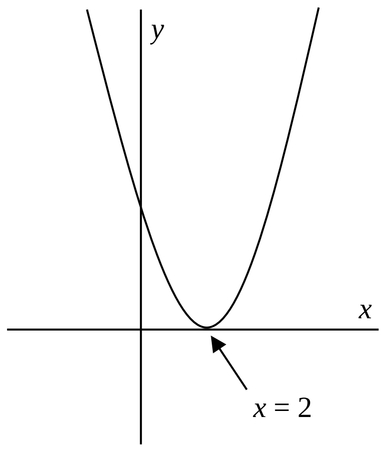
We could graph the function $y = x^2 - x - 6$. The solutions are the points on the graph where the function $x^2 - x - 6$ equals 0, i.e. where $y = 0$. In other words they are the x-intercepts, the points where the graph crosses the x-axis.

Make sure this makes sense before you go on. It isn't just true for quadratics. It is true for any equation of the form *some expression equals zero*.

Two distinct roots

The solutions of a quadratic equation are sometimes called its roots. As you can see from the graph, this quadratic equation has two distinct (different) roots, $x = -2$ and $x = 3$.

Two equal roots



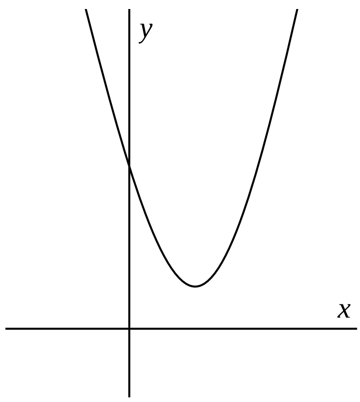
Now let's look at the equation $x^2 - 4x + 4 = 0$.

It factorises to $(x - 2)(x - 2) = 0$. So its solutions are $x = 2$ and $x = 2$. It could be said that it has just one root, but usually we say it has two equal roots.

Let's have a look at the graph of $y = x^2 - 4x + 4$. It looks like this. You can look at this as its two x-intercepts having come together at the point $x = 2$.

No roots

Now let's look at $x^2 - 4x + 5 = 0$.



The graph of $y = x^2 - 4x + 5$ looks like this.

As you can see, $x^2 - 4x + 5$ does not equal 0 for any value of x.

So the equation $x^2 - 4x + 5 = 0$ has no solutions or no roots.

If you try to factorise $x^2 - 4x + 5$, you will find that it cannot be factorised. If it could, there would be solutions to $x^2 - 4x + 5 = 0$. Try to factorise it if you like.

Practice

Q5 Use a graphing calculator to decide whether the following have two distinct (different) roots, two equal roots or no roots.

(a) $x^2 + 5x + 2 = 0$ (b) $x^2 - 3x + 10 = 0$ (c) $x^2 - 6x + 9 = 0$

(d) $4x^2 - 2x + 1= 0$ (e) $-2p^2 - 3p - 5 = 0$ (f) $-3n^2 - 6n - 3 = 0$

Solving by Completing the Square

Take the equation: $x^2 + 6x - 27 = 0$

If we complete the square we get $(x + 3)^2 - 9 - 27 = 0$

$(x + 3)^2 - 36 = 0$

We can then proceed like this: $(x + 3)^2 = 36$

$x + 3 = 6$ or $x + 3 = -6$

$x = 3$ or $x = -9$

This can be a bit quicker than factorising. But it has another advantage too. Not all expressions factorise to give factors with whole numbers. For instance, if we try to factorise $x^2 + 4x - 7 = 0$, we find that there are no two whole numbers which multiply to give –7 and add to give 4. The factorised version is actually something like $(x - 1.32)(x + 5.32)$.

We can, however, get this by completing the square:

$$x^2 + 4x - 7 = 0$$
$$(x + 2)^2 - 11 = 0$$
$$(x + 2)^2 = 11$$
$$x + 2 = \sqrt{11} \quad \text{or} \quad x + 2 = -\sqrt{11}$$
$$x = -2 + \sqrt{11} \quad \text{or} \quad x = -2 - \sqrt{11}$$
$$\text{i.e.} \quad x = -2 \pm \sqrt{11}$$

When solving by completing the square, when you get to the $(x + 2)^2 = 11$ stage, if the number on the right is positive, then there will be two distinct roots; if it is 0, then there will be two equal roots; if it is negative, then there are no roots.

Practice

Q6 Solve the following equations by completing the square:

(a) $x^2 + 6x - 7 = 0$ (b) $x^2 - 6x + 8 = 0$ (c) $x^2 - 3x + 4 = 0$

(d) $2x^2 - 5x - 3 = 0$ (e) $12x^2 + x - 12 = 0$ (f) $3n^2 + 18n = 21$

(g) $x^2 + 10x + 17 = 0$ (h) $x^2 + 4x - 2 = 0$ (i) $2x^2 - 4x - 7 = 0$

(j) $h^2 - 4h + 20 = 13 + 3h$ (k) $2x^2 - 4x - 4 = 0$ (l) $3r^2 - 2r - 5 = 4r + r^2 - 4$

The **zeros** of a quadratic function are the values of x for which $y = 0$. The zeros of $y = x^2 + 2x - 8$ are the same as the solutions (or roots) of $x^2 + 2x - 8 = 0$. Finding the zeros gives us the x-intercepts and this can be useful in sketching graphs.

Solving Using the Quadratic Formula

A third way to solve a quadratic is to use the **quadratic formula**. As with completing the square, the formula will work even if the solutions aren't simple whole numbers.

Completing the square and using the formula take longer than factorising for quadratics that factorise easily, but can be quicker if the quadratic is hard to factorise. Most people, therefore, when solving a quadratic, will look at it to see if it will factorise easily. If it doesn't, then they will use one of the other methods.

Here is the quadratic formula:

$$\text{If } ax^2 + bx + c = 0 \text{, then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To use this, write down your equation in standard form (zero on the right).

$$2x^2 + 3x - 7 = 0$$

Then write down the values of a, b and c.

$$a = 2 \quad b = 3 \quad c = -7$$

Then write the formula, sub these values into it and simplify.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times -7}}{2 \times 2}$$

$$= \frac{-3 \pm \sqrt{65}}{4}$$

$$= -\frac{3}{4} + \frac{\sqrt{65}}{4} \quad \text{or} \quad -\frac{3}{4} - \frac{\sqrt{65}}{4}$$

$$= 1.266\ldots \quad \text{or} \quad -2.766$$

If there are no roots, then the square root in the formula will have a negative number inside it. If the roots are equal, then the square root will have 0 inside it.

We sometimes call the $b^2 - 4ac$ inside the square root the discriminant. The discriminant determines the nature of the roots: positive – two distinct roots; 0 – two equal roots; negative – no roots.

Practice

Q7 Use the formula to solve the following equations. If there are no roots, say so.

(a) $x^2 + 5x + 2 = 0$ (b) $x^2 - 3x + 10 = 0$ (c) $x^2 - 6x + 9 = 0$

(d) $4x^2 - 2x + 1 = 0$ (e) $-2p^2 - 3p + 5 = 0$ (f) $-3n^2 - 6n - 3 = 0$

(g) $x^2 - 4x = 3$ (h) $5x^2 - 4 = 3x$ (i) $10x^2 - 40x - 20 = 0$

(j) $-6x^2 - 2x + 7 = x + 2$ (k) $-2t^2 + 3t + {}^5/_2 = 0$ (l) $3r^2 = 6r - 2$

The quadratic formula is one of the few things in maths that you need to learn parrot fashion. It can be derived when needed, but it takes a long time and you probably don't know how to do it.

Below are a couple of techniques which you might like to know about, but which probably aren't essential.

Note: Using *x*-intercepts when sketching quadratics

Earlier in this module, we looked at techniques for sketching graphs of quadratic functions. We looked at four techniques: y-intercept, gradient at the y-axis, curvature and vertex location. Now that we know how to solve quadratic equations, we have a fifth technique – x-intercepts. The roots of a quadratic tell us the x-intercepts.

However, unless we are solving the equation anyway, this technique is more laborious than the others and is generally only used if we need the graph to be quite accurate.

Note: Using Solutions to get Factors

If we find the factors of a quadratic expression $ax^2 + bx + c$, we can use these to get solution to $ax^2 + bx + c = 0$, as we have done above.

But, of course it is also possible to go the other way: if we find the solution to $ax^2 + bx + c = 0$, then we have the factors of $ax^2 + bx + c$.

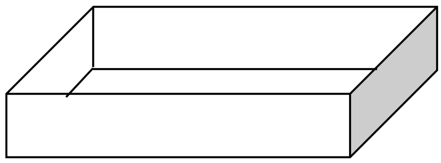
For instance, once we find the solution of say $x^2 + 5x + 2 = 0$ is $x = -0.44$ or $x = -4.56$, then we know that $x^2 + 5x + 2 = (x + 0.44)(x + 4.56)$

However, as the main reason for factorising a quadratic is to solve an equation, we are unlikely to want to factorise it if we have already solved it by some other means.

Some Applications of Quadratic Functions

Areas often involve quadratic functions. This is because areas are produced by multiplying two lengths and this often leads to a square term.

For example, suppose a 30 cm by 20 cm sheet of paper is to have x cm folded up around all the sides to make a tray of depth x cm.

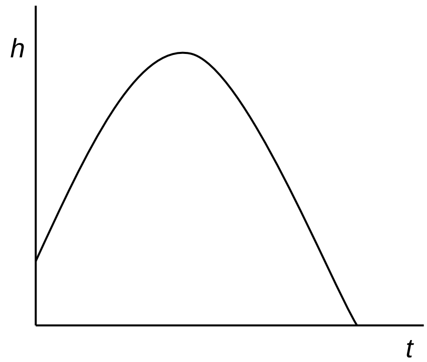


The area of the base would be $(30 - 2x) \times (20 - 2x)$.

This can be written as $600 - 100x + 4x^2$

The height of an object falling under gravity is a quadratic function. If an object is dropped from a cliff, the distance fallen is a quadratic function of time. To be precise, it is $d = 5t^2$. This is a quadratic function with b and c equal to 0.

If an object is thrown upwards from a height of 15 m with an initial velocity of 40 m/s, its height, h at time t seconds after launching will be given by the function $h = 20 + 40t - 5t^2$. [Note that if there was no gravity the formula would be $h = 20 + 40t$. The $-5t^2$ is a correction for gravity.] This situation is shown in the graph to the right.



Writing and Solving Quadratic Equations

In Modules A2-1 to A3-3, you learnt to solve problems by writing and solving linear equations. You have also used the skill in Trigonometry and other places.

You also wrote and solved equations from reciprocal functions in Trigonometry.

Quite a range of problems can be solved by writing and solving quadratic equations. You should be able to use what you have learnt to solve the following problems.

Practice

Q8 Rudolf thought of a number, multiplied it by itself, then added 3 times the original number. If he ended up with 54, what two numbers could he have started with?

Q9 The height of a cannon ball t seconds after firing is given by $h = 30t - 5t^2$.

(a) At what time does it hit the ground?

(b) At what time does it reach its greatest height? (Change to vertex form.)

(c) At what times is it 25 m above the ground?

Q10 The length of a rectangular pool is 16 m. Its diagonal is 4 m less than twice its width. What is the width?

Q11 Bronwyn dug a square hole for a fish pond. When it was finished, she decided to enlarge it so that it was 4 m longer and 3 m wider. The new area would then be 72 m^2. Write and solve an equation to find its size before the enlargement.

Q12 In a right-angle triangle, one of the two shorter sides is 5 cm longer than the other. The area of the triangle is 88 cm^2. Write and solve an equation to find the length of the shortest side.

Solve

Q51 Solve by factorising $x^2 + 3\frac{1}{2}x + 3 = 0$. Check your solution by completing the square.

Q52 Find the formula for the quadratic function which just touches the x-axis (doesn't cross it) at $x = 6$ and passes through (4, 8)

Q53 Find the value of b for which $x^2 + bx + 5 = 0$ has equal roots.

Q54 Using the quadratic formula or otherwise, for the equation $ax^2 + bx + c = 0$, find, in simplest form:
(a) the sum of the roots
(b) the product of the roots

Q55 Find the values of a, b and c, given that the function $y = ax^2 + bx + c$ has a y-intercept of 4 and that the sum of the roots of 6 and the product of the roots is 12.

Q56 If $y = 2x^2 + bx + c$ has x-intercepts of 2 and 7, find the values of b and c.

Q57 By completing the square, show that the solutions of $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Revise

Revision Set 1

Q61 (a) For quadratic functions
- give the general form
- describe the shape of the graph
- explain the effect on the graph of changing each of the parameters
- list two situations that can be modelled with this type of function

(b) Without a graphics calculator, sketch the graph of $y = x^2 + 2x - 8$.
Then check your sketch with a graphics calculator.

(c) Solve $s^2 - 6s + 12 = 4$ by factorising.

(d) Solve $6x^2 + 7x - 3 = 0$ by completing the square.

(e) Solve $3x^2 + x - 7 = 0$ using the formula.

(f) Solve $2x^2 + 3x - 8 = 0$ by graphing.

(g) Wendy's vegie patch is square. Charlotte's vegie patch is 2 m longer than Wendy's and 1 m wider. Charlotte's patch is 56 m^2. Write and solve an equation to find the size of Wendy's patch.

Revision Set 2

Q71 (a) For quadratic functions
- give the general form
- describe the shape of the graph
- explain the effect on the graph of changing each of the parameters
- list two situations that can be modelled with this type of function

(b) Without a graphics calculator, sketch the graph of $y = 2x^2 - 3x - 1$.

(c) Then check your sketch with a graphics calculator

(d) Solve $6x^2 - 4x = 12 - 5x$ by factorising

(e) Solve by completing the square $x^2 + 5x = 7$

(f) Solve $3a - a^2 = 5$ using the formula

(g) Solve $x^2 + x = 6$ by graphing

(h) A ladder is standing on horizontal ground, leaning against a vertical wall. The top of the ladder is 3 m further from the base of the wall than the bottom of the ladder is. The area of the triangle formed by the ladder, the ground and the wall is 35 m^2. Write and solve an equation to find out how far the base of the ladder is from the wall.

Revision Set 3

Q81 (a) For quadratic functions
- give the general form
- describe the shape of the graph
- explain the effect on the graph of changing each of the parameters
- list two situations that can be modelled with this type of function

(b) Without a graphics calculator, sketch the graph of $y = x^2 - x + 5$.

(c) Then check your sketch with a graphics calculator

(d) Solve $x^2 + 4x = 5$ by factorising

(e) Solve $w^2 - 2w + 1 = 0$ using the formula

(f) Solve $x^2 + 6x - 21 = 0$

(g) Solve $x^2 - 2x + 7 = 0$ by graphing

(h) A right-angle triangle has its second-longest side 4 cm longer than its shortest side. Its area is 54 cm². Write and solve an equation for find the perimeter of the triangle.

Answers

Q1
(a) $y = (x + 2)^2 + 7$ (b) $y = (x - 1)^2 - 5$
(c) $y = (x + 1.5)^2 - 3.25$ (d) $y = 2(x + 2.5)^2 - 9.5$
(e) $y = 2(c - 1)^2 + 3$ (f) $y = (x + 2)^2 - 4$
(g) $y = (x + 0.1)^2 + 1.99$ (h) $y = - 3(x - 0.5)^2 - 1.25$

Q2
(a) $y = x^2 - 4x + 1$ (b) $y = x^2 + 9x + 30.25$

Q4
(a) $x = -2$ or $x = -3$ (b) $x = 2$ or $x = 4$ (c) $x = 1$ or $x = 9$
(d) $x = 6$ or $x = -5$ (e) $p = 3$ or $p = -3$ (f) $n = 3$ or $n = 4$
(g) $a = -5$ or $a = -8$ (h) $c = 2$ or $c = -6$ (i) $s = 4$ or $s = -6$
(j) $n = 3$ or $n = 4$ (k) $x = 5$ or $x = -4$ (l) $a = 1$ or $a = -4$
(m) $x = 1\frac{1}{2}$ or $x = \frac{1}{3}$ (n) $x = \frac{1}{2}$ or $x = \frac{2}{5}$ (o) $x = 1$ or $x = -\frac{1}{6}$
(p) $x = -1\frac{1}{2}$ (q) $x = -\frac{3}{4}$ or $x = -1\frac{1}{2}$

Q5
(a) different (b) none (c) different
(d) equal (e) none (f) different

Q6
(a) $x = 1$ or $x = -7$ (b) $x = 2$ or $x = 4$ (c) $x = -1$ or $x = 4$
(d) $x = 3$ or $x = -\frac{1}{2}$ (e) $x = \frac{2}{3}$ or $p = -\frac{3}{4}$ (f) $n = 1$ or $n = -7$
(g) $x = -5 \pm \sqrt{8}$ (h) $x = -2 \pm \sqrt{2}$ (i) $x = 1 \pm \sqrt{9/2}$
(j) $h = 3\frac{1}{2} \pm \sqrt{21/4}$ (k) $r = \frac{1}{10} \pm \sqrt{161/100}$ (l) $r = 1\frac{1}{2} \pm \sqrt{11/4}$

Q7
(a) $x = -\frac{5}{2} \pm \frac{\sqrt{17}}{2}$ (b) no roots (c) $x = 3$
(d) no roots (e) $p = 1$ or $p = -2\frac{1}{2}$ (f) $n = -1$
(g) $x = 2 \pm \frac{\sqrt{28}}{2}$ (h) $x = \frac{3}{10} \pm \frac{\sqrt{89}}{10}$ (i) $x = 2 \pm \frac{\sqrt{24}}{2}$
(j) $x = -\frac{1}{4} \pm \frac{\sqrt{129}}{12}$ (k) $x = \frac{3}{4} \pm \frac{\sqrt{29}}{4}$ (l) $x = 1 \pm \frac{\sqrt{12}}{6}$

Q8 6 or −9

Q9 (a) $t = 6$ (b) $t = 3$ (c) $t = 1$ and $t = 5$

Q10 12 m

Q11 5 m by 5 m

Q12 11 cm

Q51 $x = -2$ or $x = -1\frac{1}{2}$

Q52 $y = 2(x - 6)^2$

Q53 $\sqrt{20}$

Q54 (a) $-b/a$ (b) c/a

Q55 $a = \frac{1}{3}$, $b = -2$, $c = 4$

Q56 $b = -18$, $c = 28$

Q61 (a) See text
(d) $s = 2$ or $s = 4$
(e) $x = {}^1/_3$ or $x = -{}^3/_2$
(f) $x = -1.70$ or $x = 1.37$
(g) $x = 1.39$ or $x = -2.89$
(h) 6 m by 6 m

Q71 (a) See text
(d) $x = {}^4/_3$ or $x = -{}^3/_2$
(e) $x = -6.14$ or $x = 1.14$
(f) no roots
(g) $x = 2$ or $x = -3$
(h) 7 m

Q81 (a) See text
(d) $x = 1$ or $x = -5$
(e) $w = 1$
(f) $x = -8.48$ or $x = 2.48$
(g) no roots
(h) 36 cm