M1 Maths

A4-1 Factorising

- factorising with circles
- expanding and factorising quadratics

Summary Learn Solve Revise Answers

Summary

Factorising is the reverse of expanding.

One way to factorise a set of terms is to break each term down into its simplest factors, then circle a factor which is present in all the terms, then circle another uncircled factor which is present in all the terms, and so on until there are none left. Then we put a bracket around the expression and put all the circled factors from any one of the terms outside the bracket and leave the uncircled factors inside. When you have had some experience with this you might be able to do the process in your head for simple expressions.

We expand products of binomials, like $(x + 4)(x - 2)$, by multiplying each term in one bracket by each term in the other bracket. The crab claw and foil methods can be used to help you through the process.

The result of such an expansion will be a trinomial expression like $x^2 + 2x - 8$. You need to be able to factorise trinomials. You write two brackets, both with a x in, like this: $(x \quad)(x \quad)$. Then you use guess and check to work out the numbers that go in with the xs. There are short cuts to this process, such as knowing that the product of the two numbers is the constant term in the trinomial and the sum of the two numbers is the coefficient of x in the trinomial. When the coefficient of x^2 is not 1, then the process boils down to more basic guess and check.

Learn

Factorising with Circles

What is factorising?

Factorising is the opposite of expanding. It is also called factorisation, or factoring if you're American.

If we expand $3(a + 2)$, we get $3a + 6$.

If we factorise $3a + 6$, we get $3(a + 2)$.

Factorising is called factorising because the result is a product of factors rather than a sum of terms. Factors are things that are multiplied together to make a product. Terms are things that are added together to make a sum.

In $5c + 10 = 5(c + 2)$, $5c$ and 10 are terms; 5 and $(c + 2)$ are factors.

Why learn to factorise?

Being able to factorise is important for a number of reasons, some of which will become apparent later. But one thing we can use it for straight away is for problems like this:

Make a the subject of $p = ax^2 + 3ax$

If you like, try to do this before you read on.

The problem is that a is in there more than once.

We can do it, however, by factorising to $p = a(x^2 + 3x)$, then we can divide by $(x^2 + 3x)$ to get $a = \frac{p}{x^2+3x}$

How to factorise

We can factorise simple and complex expression using the circle method. But before we go on to that, we need to make sure we remember how to express numbers as products of prime factors. This was explained in Modules N1-1 and N1-7. Go back and have another look if you need to. As an example of this, 30 = 2×3×5.

Practice

Q1 Write these numbers as products of primes:

(a) 30	(b) 6	(c) 12	(d) 14	(e) 20	(f) 8
(g) 7	(h) 48	(i) 120	(j) 100	(k) 72	(l) 78

Now the circle method. Suppose we need to factorise $30a^4c + 6a^2bc - 12a^3c^3$.

First we break all the numbers down to prime factors and all the powers down into simple factors.

$2\ 3\ 5\ a\ a\ a\ a\ c + 2\ 3\ a\ a\ b - 2\ 2\ 3\ a\ a\ a\ c\ c\ c$

Then we look at the first factor in the first term, which is a 2, and we see if there is a 2 in <u>every</u> other term. If there is, we circle one 2 in each term.

②3 5 $a\ a\ a\ a\ c$ +②3 $a\ a\ b$ –②2 3 $a\ a\ a\ c\ c\ c$

Then we look at the next factor in the first term, which is a 3, and we see if there is a 3 in every other term. There is, so we circle one 3 in each term.

$$\textcircled{2}\textcircled{3}\,5\,a\,a\,a\,a\,c + \textcircled{2}\textcircled{3}\,a\,a\,b - \textcircled{2}\,2\,\textcircled{3}\,a\,a\,a\,c\,c\,c$$

Then we go on to the next factor in the first term, a 5, and we see if there is a 5 in *every* other term. There isn't so we go on to the next factor in the first term, an a, and we see if there is an a in *every* other term. If there is, we circle one a in each term.

$$\textcircled{2}\textcircled{3}\,5\,\textcircled{a}\,a\,a\,a\,c + \textcircled{2}\textcircled{3}\,\textcircled{a}\,a\,b - \textcircled{2}\,2\,\textcircled{3}\,\textcircled{a}\,a\,a\,c\,c\,c$$

Then we go on to the next factor in the first term, another a. We see if there is another a (that is an uncircled a) in *every* other term. There is, so we circle one more a in each term.

$$\textcircled{2}\textcircled{3}\,5\,\textcircled{a}\textcircled{a}\,a\,a\,c + \textcircled{2}\textcircled{3}\,\textcircled{a}\textcircled{a}\,b - \textcircled{2}\,2\,\textcircled{3}\,\textcircled{a}\textcircled{a}\,a\,c\,c\,c$$

Then we go on to the next factor in the first term, yet another a. We see if there is an uncircled a in *every* other term. There isn't, so we leave that a, then go on to the next factor in the first term that isn't an a. It's a c. So we look to see if there is a c in every other term. If there is we circle it. But there isn't, so we don't circle anything.

$$\textcircled{2}\textcircled{3}\,5\,\textcircled{a}\textcircled{a}\,a\,a\,c + \textcircled{2}\textcircled{3}\,\textcircled{a}\textcircled{a}\,b - \textcircled{2}\,2\,\textcircled{3}\,\textcircled{a}\textcircled{a}\,a\,c\,c\,c$$

Now we have come to the end of the first term, so we have finished circling. We then write all the circled factors in the first term (which should be the same set as in each of the other terms) outside of brackets, and write the uncircled factors of each term inside the brackets.

$$2\;3\;a\;a\;(5\;a\;a\;c + b - 2\;\;a\;c\;c\;c)$$

Then we re-write this putting the prime numbers together and writing the variables as powers. This is our factorised expression. If we want to check if we did it right, we can

$$6a^2(5a^2c + b - 2ac^3)$$

expand it and see if it comes to the expression we started with.

Practice

Q2 Factorise the following.

(a) $4a^2 + 8a$ (b) $5x^3 + 20x^2$ (c) $10 + 2s$

(d) $6t^2 + 4tw + 2t$ (e) $8w^3 + 24w^2 + 14rw^2$ (f) $4gf + 10f$

(g) $16x^2y + 12xy^2$ (h) $4prt + 4hrt - 3rt^2$ (i) $3n^4 + 9n^6$

(j) $4av - 8a^2v^3 + 5av^2$ (k) $4xa - 20xb$ (l) $8r^2s^2 + 12rs^3 - 20r^2s^2$

(m) $12t - 2$ (n) $8x^2 - 20x$ (o) $-4h + 8f$

(p) $-3xz^2 - 9xyz$ (q) $-4p + 8q^2$ (r) $5n + 9mnp$

When factorising fairly simple expressions, we can sometimes do it in our head rather than actually drawing the circles. So we might factorise $12x^2y^2 + 8xyz$ by noticing that both numerals are multiples of 4 and that there is an x and a y factor in both terms. We can then write the expression as $4xy\,(3xy + 2z)$.

If we miss a factor, we can fix this by factorising again until we can see that there are no more common factors. For instance, if we didn't notice the y common factor in the above example we might write

$12x^2y^2 + 8xyz$

$= 4x\,(3xy^2 + 2zy)$ then do a second factorisation to

$= 4xy\,(3xy + 2z)$

Practice

Q3 Factorise these, using the in-your-head method.

(a) $2ab + 12b$ (b) $4x^2 - 20x$ (c) $10a^4 + 2a$

(d) $6t^3 + 2t^2$ (e) $8 + 24w^2$ (f) $4 + 10f$

(g) $3x^2 + 6x + 12$ (h) $4p + 4h + 8k$ (i) $3n + 9n^2$

Expanding Quadratics

$x + 4$ is a **binomial**. bi- means two, -nomial means numbers. It is two numbers added together – or two terms added together.

$(x + 4)(x + 3)$ is a product of two binomials. It has two sets of brackets.

Expanding a product of binomials is re-writing it without the brackets. $(x + 4)(x + 3)$ expands to $x^2 + 7x + 12$. Having three terms, this is called a **trinomial**. We will see how to do this expansion shortly.

$(x + 4)(x + 3)$ and $x^2 + 7x + 12$ are also called **quadratics**. You will learn more about quadratics in Module A4-2.

Why learn to expand products of binomials?

There will be situations where products of binomials need to be written in expanded form. You will need to do this when learning calculus if you do a calculus course later.

But a more immediate reason to learn it is because we need to be able to expand them before we can master the reverse process of factoring, e.g. $x^2 + 7x + 12$ to $(x + 4)(x + 3)$. We need to be able to factorise such expressions to solve quadratic equations which we will meet in Module A4-2.

The Crab Claw Method

To expand $(x + 4)(x + 3)$ we multiply every term in the first bracket by every term in the second bracket.

First we multiply the first x by the second x.	$(x + 4)(x + 3)$	x^2
Then we multiply the first x by the 3.	$(x + 4)(x + 3)$	$x^2 + 3x$
Then we multiply the 4 by the second x.	$(x + 4)(x + 3)$	$x^2 + 3x + 4x$
Then we multiply the 4 by the 3.	$(x + 4)(x + 3)$	$x^2 + 3x + 4x + 12$
Then we collect terms		$x^2 + 7x + 12$

Drawing these lines on as you go, gives you a way of keeping track and making sure you don't miss any steps. You'll know you've finished when you have what looks like a crab claw. Hence the name 'Crab Claw Method'.

The FOIL Method

Another way of looking at the same process is called the FOIL Method.

$(\mathbf{x} + 4)(\mathbf{x} + 3)$	$x \times x$	the **F**irst number in each bracket
$(\mathbf{x} + 4)(x + \mathbf{3})$	$x \times 3$	the **O**utside numbers
$(x + \mathbf{4})(\mathbf{x} + 3)$	$4 \times x$	the **I**nside numbers
$(x + \mathbf{4})(x + \mathbf{3})$	4×3	the **L**ast number in each bracket

Answer = $x^2 + 4x + 3x + 12$ = $x^2 + 7x + 12$

The acronym **FOIL** provides a system for doing the multiplications so we don't forget any. Think about it whichever way suits you best.

Practice

Q4 Expand the following.

(a) $(x + 2)(x + 3)$ (b) $(a + 5)(a + 1)$ (c) $(t + 4)(t + 3)$

(d) $(h + 5)(h + 5)$ (e) $(r + 5)(r - 2)$ (f) $(x + 4)(x - 3)$

(g) $(b + 1)(b - 3)$ (h) $(c - 3)(c + 5)$ (i) $(x - 2)(x - 3)$

(j) $(x - 3)(x + 3)$ (k) $(2x + 3)(x + 2)$ (l) $(x + 5)(3x + 1)$

(m) $(2x + 3)(x - 5)$ (n) $(3x + 2)(2x + 1)$ (o) $(2x - 4)(2x - 3)$

(p) $(x - 1)(6x - 3)$ (q) $(2x - 7)(3x - 2)$ (r) $(4x - 3)(x + 4)$

(s) $(3 - x)(x + 2)$ (t) $(4 - 3x)(2 - x)$ (u) $(x + 2)(2 - 3x)$

Bigger Expansions

If you need to expand a product of 3 or more binomials, the easiest way to do it is to multiply the first two, then multiply the result by the next one and so on. When multiplying brackets with more than two terms, we just multiply every term in one by every term in the other.

For example to expand $(x + 1)(x + 2)(x - 1)$ we do this:

$$\begin{aligned} & (x + 1)(x + 2)(x - 1) \\ &= (x^2 + 2x + x + 2)(x - 1) \\ &= (x^2 + 3x + 2)(x - 1) \\ &= x^3 - x^2 + 3x^2 - 3x + 2x - 2 \\ &= x^3 + 2x^2 - x - 2 \end{aligned}$$

Practice

Q5 Expand the following.

(a) $(x + 2)(x - 3)(x + 1)$ (b) $(a + 5)(2a + 1)(a - 1)(a - 1)$

Factorising Quadratics

An expanded quadratic generally has three terms – an **x^2 term**, an **x term** and a **constant term**. The x^2 and x terms have **coefficients** – numbers that indicate how many x^2s and how many xs. In the trinomial $x^2 + 7x + 12$, the coefficient of x^2 is 1, the coefficient of x is 7 and the constant is 12.

Factorising trinomials is the reverse of expanding products of binomials.

Expanding $(x + 4)(x + 3)$ gives $x^2 + 7x + 12$. Factorising $x^2 + 7x + 12$ gives $(x + 4)(x + 3)$.

Factorising trinomials is done by guess and check. Guess the factorised version, then expand it to see if your guess was right. If not, change your guess.

But there is a technique that can make your guess and check quite quick.

If you expand $(x + 4)(x + 3)$ you get $x^2 + 7x + 12$. Notice that the constant, 12, is the product of the constants in the binomials $(12 = 4 \times 3)$ and the coefficient of x is the sum of the two constants $(7 = 4 + 3)$.

When we factorise $x^2 + 7x + 12$, we write $(x + \quad)(x + \quad)$

Then we look for two numbers to go in the brackets which multiply to make 12 and which add to make 7. They can only be 3 and 4. So we put 3 and 4 in the brackets to get $(x + 3)(x + 4)$ or $(x + 4)(x + 3)$ – both expand to the same thing, so both are correct.

It takes a lot of practice to get good at factorising trinomials. But you will need to be good at it to solve quadratic equations in the next module. So it's worth doing all the practice exercises.

Practice

Q6 Factorise the following.

(a) $x^2 + 5x + 6$	(b) $x^2 + 7x + 10$	(c) $x^2 + 8x + 15$
(d) $a^2 + 6a + 5$	(e) $t^2 + 6t + 8$	(f) $c^2 + 10c + 25$
(g) $x^2 + 5x + 4$	(h) $x^2 + 2x + 1$	(i) $x^2 + 9x + 18$
(j) $x^2 + 4x + 3$	(k) $x^2 + 10x + 24$	(l) $x^2 + 9x + 14$
(m) $x^2 + 15x + 56$	(n) $x^2 + 7x + 6$	(o) $x^2 + 8x + 12$
(p) $x^2 + 13x + 30$	(q) $x^2 + 11x + 30$	(r) $x^2 + 10x + 21$

It gets a little harder if some of the coefficients are negative. Suppose we need to factorise $x^2 - 4x - 12$. We have to find two numbers that multiply to make -12 and which add to make -4. For them to multiply to make -12, clearly one needs to be positive and one needs to be negative. We might try 3 and -4: their product is -12, but their sum is −1. If we try −3 and 4, that won't work either. So try 6 and −2. But their sum is 4. So try −6 and 2. That works!

So the factorised version is $(x - 6)(x + 2)$.

After a bit of practice, you'll be much more likely to pick the right numbers first time.

Practice

Q7 Factorise the following.

(a) $a^2 - 6a - 7$	(b) $c^2 - 6c + 5$	(c) $s^2 + 3s - 10$
(d) $a^2 - 5a + 6$	(e) $c^2 - 3c + 2$	(f) $s^2 + s - 12$
(g) $x^2 + 10x + 25$	(h) $p^2 + 4p - 21$	(i) $p^2 - 4p - 21$
(j) $x^2 - 9x + 8$	(k) $x^2 - 7x + 6$	(l) $x^2 - 2x - 3$
(m) $x^2 - x - 2$	(n) $x^2 + x - 2$	(o) $x^2 - 4x + 3$
(p) $x^2 - 6x + 9$	(q) $x^2 + 0x - 4$	(r) $x^2 - 9$
(s) $x^2 + x - 12$	(t) $x^2 - 2x - 8$	(u) $x^2 - 2x + 1$
(v) $x^2 - 2x - 15$	(w) $x^2 + 16x + 15$	(x) $x^2 - 10x + 9$

You might have noticed from (q) and (r) in the last exercise that when the coefficient of x is 0, the numbers you need are the square root of the constant and its negative. This can be a handy short-cut when factorising expressions like $x^2 - 25$.

Everybody enjoys factorising trinomials, so here are a few more.

Practice

Q8 Factorise these.

(a) $a^2 - 8a + 7$	(b) $c^2 - 8c + 15$	(c) $s^2 + 9s - 10$
(d) $a^2 - 9a + 14$	(e) $c^2 - 12c + 11$	(f) $s^2 + 10s - 11$
(g) $x^2 + 9x + 20$	(h) $p^2 - 25$	(i) $h^2 - 16$
(j) $x^2 - 1$	(k) $x^2 - 6x + 9$	(l) $x^2 - 2x - 15$
(m) $x^2 + 8x + 16$	(n) $x^2 + x - 12$	(o) $x^2 - 4x + 4$
(p) $x^2 + 6x + 9$	(q) $x^2 - 36$	(r) $x^2 - 49$
(s) $x^2 + 4x - 12$	(t) $x^2 - 2x + 24$	(u) $x^2 - 4x + 3$
(v) $x^2 + 2x - 35$	(w) $x^2 - 100$	(x) $x^2 - 10x + 24$

If the coefficient of x^2 is not 1

So far we have only factorised trinomials where the coefficient of x^2 is 1. If the coefficient of x^2 is not 1, then the job can take a bit longer. Let's do $12x^2 + 14x - 20$.

First, see if the three terms have any common factors. In this case there is a common factor of 2, so we can write the expression as $2(6x^2 + 7x - 10)$. Then we factorise the bracket.

The 6 can factorise to 6×1 or to 3×2, so we have to try
$2(6x \quad)(x \quad)$ and $2(3x \quad)(2x \quad)$.

Usually we try the numbers that are closest together first, in this case $3x$ and $2x$:
$2(3x \quad)(2x \quad)$

Now the product of the constants is –10, so we just try the various possible pairs, i.e. 5, 2 and 10, 1, one of which must be negative. Because the $3x$ and $2x$ are different, it now matters which bracket which number goes into. For instance $2(3x + 5)(2x - 2)$ is different from $2(3x - 2)(2x + 5)$. So there are lots of possible combinations.

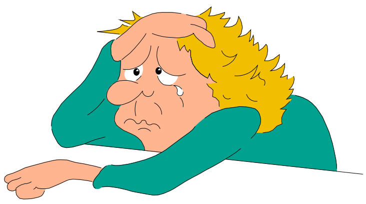
$2(3x + 5)(2x - 2)$	$2(3x - 5)(2x + 2)$
$2(3x + 2)(2x - 5)$	$2(3x - 2)(2x + 5)$
$2(3x + 10)(2x - 1)$	$2(3x - 10)(2x + 1)$
$2(3x + 1)(2x - 10)$	$2(3x - 1)(2x + 10)$

We have to expand each in turn to see if it's right.

E.g. $2(3x + 5)(2x - 2) = 2(6x^2 + 4x - 10)$ No.

$2(3x - 5)(2x + 2) = 2(6x^2 - 4x - 10)$ No.

$2(3x + 2)(2x - 5) = 2(6x^2 - 11x - 10)$ No.



And so on.

In fact, none of these are right, so then we have to try $2(6x \quad)(x \quad)$

$2(6x + 5)(x - 2) = 2(6x^2 - 7x - 10)$ No.

$2(6x - 5)(x + 2) = 2(6x^2 + 7x - 10)$ Hooray!

If the coefficient of x^2 is negative, e.g. in $-x^2 + 5x - 6$, then take out a negative factor, in this case –1 to get $-(x^2 - 5x + 6)$, then factorise to $-(x - 2)(x - 3)$.

As you can see, factorising expressions where the coefficient of x^2 is not 1 can be tedious. Once again, practice does make it easier to spot which combinations are more likely, though.

Practice

Q9 Factorise the following:

(a) $2x^2 - 9x + 7$ (b) $100x^2 + 500x + 600$ (c) $10x^2 + 5x - 15$

(d) $6x^2 - 7x - 3$ (e) $10x^2 - 9x + 2$ (f) $6x^2 - 5x - 1$

The List Method

The list method is an alternative method for factorising quadratics where the coefficient of x^2 isn't 1. It has the advantage of involving less guess and check and therefore often being quicker. It has the disadvantage of being an algorithm with several steps that need to be remembered. If you decide to use it, you will always have the guess-and-check method as a back-up in case you forget the steps.

Suppose we need to factorise $10x^2 - 7x - 12$

$a = 10$, $c = -12$. So $ac = -120$

List the factor pairs for 120: 1, 120 2, 60 3, 40 4, 30 5, 24 6, 20 8, 15 10, 12

Because the constant term is negative, we need a pair of factors whose difference is 7 (the coefficient of x). (If the constant were positive, we would need a pair whose sum is 7.)

The pair is 8, 15 (We don't actually need to list all the factor pairs, just find a pair with a difference of 7.)

We then write the quadratic as $10x^2 - 15x + 8x - 12$

Then factorise the first two terms and the last two terms $5x(2x - 3) + 4(2x - 3)$

Then factorise to $(5x + 4)(2x - 3)$

Practice

Q10 Factorise the following using the guess and check method or the list method, whichever you prefer::

(a) $40x^2 + 120x + 90$ (b) $12x^2 + 19x - 10$ (c) $-4x^2 + 17x - 4$

(d) $15x^2 + 26x + 8$ (e) $-12x^2 + 3x + 54$ (f) $30x^2 - 31x - 44$

(g) $10x^2 + 44x + 16$ (h) $12x^2 + 13x - 4$ (i) $36x^2 + 12x - 3$

(j) $4x^2 - 9$ (k) $5x^2 - 80$ (l) $9 - x^2$

(m) $5x^2 - 20x$ (n) $6x - 2x^2$ (o) $24x^2 + 2x - 5$

(p) $36x^2 - 46x - 52$ (q) $29x - 10x^2 - 10$ (r) $12x^2 + 17x - 14$

Solve

Q51 Expand $(x + 1)^5$

Q52 Research Pascal's Triangle. Look at the row that begins 1 5, and compare the numbers in the row with the coefficients in your expansion from Q51.

Q53 Use Pascal's Triangle to predict the expansion of $(x + 1)^8$ without performing the expansion.

Revise

Revision Set 1

Q61 Factorise the following completely.

(a) $2a + 8$ (b) $3sf + 9f$ (c) $4n^2 + an$

Q62 Expand the following:

(a) $(x + 3)(x – 1)$ (b) $(2h – 3)(3h – 4)$ (c) $(x + 2)(x – 2)(x + 1)$

Q63 Factorise the following:

(a) $a^2 – 7a + 10$ (b) $x^2 – 16$ (c) $10a^2 – 12a + 2$

Answers

Q1 (a) $2a + 6$ (b) $3x – 6$ (c) $10h + 20$
(d) $–4t – 8$ (e) $–40 – 120w$ (f) $4g + 40f$
(g) $3x^2 + 3$ (h) $–4p + 8$ (i) $3n + 18$

Q2 (a) $4a(a + 2)$ (b) $5x^2(x + 4)$ (c) $2(5 + s)$
(d) $2t(3t + 2w + 1)$ (e) $2w^2(4w + 12 + 7r)$ (f) $2f(2g + 5)$
(g) $4xy(4x + 3y)$ (h) $rt(4p + 4h – 3t)$ (i) $3n^4(1 + 3n^2)$
(j) $av(4 – 8av^2 +5v)$ (k) $4x(\mathrm{a} – 5b)$ (l) $4rs^2(2r + 3s – 10r)$
(m) $2(6t – 1)$ (n) $4x(2x – 5)$ (o) $–4(h + 2f)$
(p) $–3xz(z + 3y)$ (q) $–4(p − 2q^2)$ (r) $n(5 + 9mp)$

Q3 (a) $2b(a + 6)$ (b) $4x(x – 5)$ (c) $2a(5a^3 + 1)$
(d) $2t^2(3t + 1)$ (e) $8(1 + 3w^2)$ (f) $2(2 + 5f)$
(g) $3(x^2 + 2x + 4)$ (h) $4(p + h + 2k)$ (i) $3n(1 + 3n)$

Q4 (a) $x^2 + 5x + 6$ (b) $a^2 + 6a + 5$ (c) $t^2 + 7t + 12$
(d) $h^2 + 10h + 25$ (e) $r^2 + 3r – 10$ (f) $x^2 + x – 12$
(g) $b^2 – 2b – 3$ (h) $c^2 + 2c – 15$ (i) $x^2 – 5x + 6$
(j) $x^2 – 9$ (k) $2x^2 + 7x + 6$ (l) $3x^2 + 16x + 1$
(m) $2x^2 – 7x – 15$ (n) $6x^2 + 7x + 2$ (o) $4x^2 – 14x + 12$
(p) $6x^2 – 9x + 3$ (q) $6x^2 – 25x + 14$ (r) $4x^2 + 13x – 12$
(s) $–x^2 + x + 6$ (t) $6x^2 – 10x + 8$ (u) $–3x^2 – 4x + 4$

Q5 (a) $x^3 – 7x – 6$ (b) $2a^4 + 7a^3 –15a^2 + a + 5$

Q6 (a) $(x + 2)(x + 3)$ (b) $(x + 5)(x + 2)$ (c) $(x + 5)(x + 3)$

	(d) $(a + 5)(a + 1)$	(e) $(t + 2)(t + 4)$	(f) $(c + 5)(c + 5)$
	(g) $(x + 1)(x + 4)$	(h) $(x + 1)(x + 1)$	(i) $(x + 3)(x + 6)$
	(j) $(x + 1)(x + 3)$	(k) $(x + 6)(x + 4)$	(l) $(x + 7)(x + 2)$
	(m) $(x + 7)(x + 8)$	(n) $(x + 6)(x + 1)$	(o) $(x + 2)(x + 6)$
	(p) $(x + 10)(x + 3)$	(q) $(x + 5)(x + 6)$	(r) $(x + 7)(x + 3)$
Q7	(a) $(a - 7)(a + 1)$	(b) $(c - 5)(c - 1)$	(c) $(s - 5)(s + 2)$
	(d) $(a - 3)(a - 2)$	(e) $(c - 2)(c - 1)$	(f) $(s + 4)(s - 3)$
	(g) $(x + 5)(x + 5)$	(h) $(p + 7)(p - 3)$	(i) $(p + 3)(p - 7)$
	(j) $(x - 1)(x - 8)$	(k) $(x - 6)(x - 1)$	(l) $(x - 3)(x + 1)$
	(m) $(x - 2)(x + 1)$	(n) $(x - 1)(x + 2)$	(o) $(x - 3)(x - 1)$
	(p) $(x - 3)(x - 3)$	(q) $(x + 2)(x - 2)$	(r) $(x - 3)(x + 3)$
	(s) $(x - 3)(x + 4)$	(t) $(x - 4)(x + 2)$	(u) $(x - 1)(x - 1)$
	(v) $(x - 5)(x + 3)$	(w) $(x + 15)(x + 1)$	(x) $(x - 9)(x - 1)$
Q8	(a) $(a - 7)(a - 1)$	(b) $(c - 5)(c - 3)$	(c) $(s - 10)(s + 1)$
	(d) $(a - 7)(a - 2)$	(e) $(c - 11)(c - 1)$	(f) $(s + 11)(s - 1)$
	(g) $(x + 5)(x + 4)$	(h) $(p + 5)(p - 5)$	(i) $(h + 4)(h - 4)$
	(j) $(x - 1)(x + 1)$	(k) $(x - 3)(x - 3)$	(l) $(x - 5)(x + 3)$
	(m) $(x + 4)(x + 4)$	(n) $(x - 3)(x + 4)$	(o) $(x - 2)(x - 2)$
	(p) $(x + 3)(x + 3)$	(q) $(x + 6)(x - 6)$	(r) $(x - 7)(x + 7)$
	(s) $(x - 2)(x + 6)$	(t) $(x - 6)(x + 4)$	(u) $(x - 3)(x - 1)$
	(v) $(x - 5)(x + 7)$	(w) $(x + 10)(x - 10)$	(x) $(x - 6)(x - 4)$
Q9	(a) $(2x - 7)(x - 1)$	(b) $100(x + 2)(x + 3)$	(c) $5(x - 1)(2x + 1)$
	(d) $(2x - 3)(3x + 1)$	(e) $(5x + 5)(x - 2)$	(f) $(6x + 1)(x - 1)$
Q10	(a) $10(2x + 3)(2x + 3)$	(b) $(12x - 5)(x + 2)$	(c) $-(4x - 4)(x - 4)$
	(d) $(5x + 2)(3x + 4)$	(e) $-3(4x - 9)(x + 2)$	(f) $(6x - 11)(5x + 4)$
	(g) $2(5x + 2)(x + 4)$	(h) $(4x - 1)(3x + 4)$	(i) $3(2x + 1)(6x - 1)$
	(j) $(2x + 3)(2x - 3)$	(k) $5(x + 4)(x - 4)$	(l) $-(x - 3)(x + 3)$
	(m) $5x(x - 4)$	(n) $- 2x(x - 3)$	(o) $(12x - 5)(2x + 1)$
	(p) $2(18x + 13)(x - 2)$	(q) $-(2x - 5)(5x - 2)$	(r) $(12x - 7)(x + 2)$
Q51	$x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$		
Q61	(a) $2(a + 4)$	(b) $3f(s + 3)$	(c) $n(4n + a)$
Q62	(a) $x^2 + 2x - 3$	(b) $6h^2 - 17h + 12$	(c) $x^3 + x^2 - 4x - 4$
Q63	(a) $(a - 5)(a - 2)$	(b) $(x + 4)(x - 4)$	(c) $(5a - 1)(2a - 2)$