

A3-9 Reciprocal Functions

- general form and graph shape
- equation solution methods
- applications

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Summary

Reciprocal functions have the general form $y = \frac{c}{x}$. Their graphs are rectangular hyperbolas. They can be used to represent situations in which the variables are inversely proportional.

Learn

General form

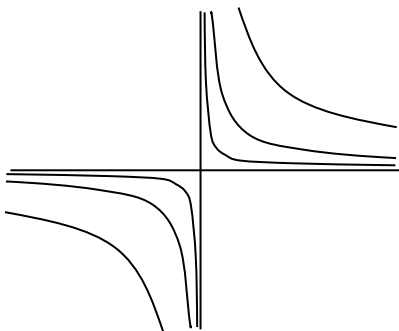
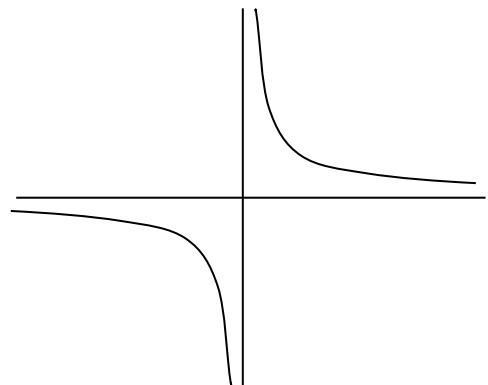
Reciprocal functions have the general form $y = \frac{c}{x}$. c is the only parameter.

Graph Shape

The graph of a reciprocal function looks like this.

This shape is called a hyperbola, a rectangular hyperbola to be specific.

Considering the positive part (in the top right quadrant), it can be seen in the graph to the right that, as x gets bigger, y gets smaller and vice versa.

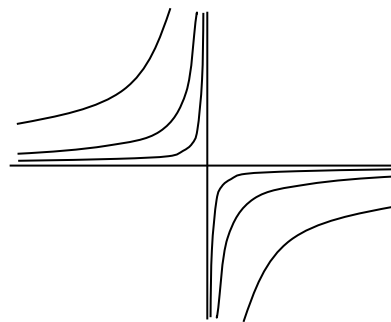


The effect of making c larger is just to move the graph further out from the origin. The graph to the left shows three reciprocal functions with different values for c . The outermost one has the highest value for c .

The above assumes that c is positive. If c is negative, the graphs will look like this.

When using reciprocal functions, however, we nearly always have c positive.

When sketching the graph of a reciprocal function, just find one point on the graph where the x - and y -coordinates are similar by picking an x -value, subbing it into the formula and getting the y -value. Then plot the graph through that point.



Solving Equations

If we substitute for the dependent variable in a reciprocal function like $y = \frac{6}{x}$, we get something like $4 = \frac{6}{x}$. You know how to solve this type of equation by doing the same thing to both sides:

$$\begin{aligned}4 &= \frac{6}{x} \\ \times x & \quad \times x \\ 4x &= 6 \\ \div 4 & \quad \div 4 \\ x &= 1.5\end{aligned}$$

Practice

Q1 (a) Solve $10 = \frac{2}{x}$

(b) If $p = \frac{6}{h}$ find the value of h when $p = 15$.

Applications of reciprocal functions

Reciprocal functions represent inverse proportion. On a journey, the time taken, t , is inversely proportional to the speed, s . The greater the speed, the smaller the time and vice versa.

The relation between speed and time can be written as $t = \frac{c}{s}$ where c is the parameter. In this case, c is actually the distance travelled.

Note that in any reciprocal function the variables can be swapped and the relation is still a reciprocal function.

So $t = \frac{c}{s}$ can be written $s = \frac{c}{t}$.

Practice

- Q2 A race track is 200 m long. Write the relation between speed in m/s and time for one lap in seconds as
- (a) a formula (b) a sample table (c) a graph
- (d) Find the time taken if the speed is 4.2 m/s.

Solve

- Q51 Nearby stars seem to move back and forth against more distant stars as the Earth orbits the Sun. This is an example of parallax. The angle through which the star seems to move is inversely proportional to the distance of the star. A star at 3.1 light years moves by 1 second of arc ($\frac{1}{3600}$ of a degree).
- (a) Write the relation between apparent movement in seconds and distance in light years as a formula.
- (b) Use your formula to find the angle moved by Alpha Centauri which is 4.1 light years away
- (c) Use your formula to find the angle moved by a star 500 light years away.
- (d) Find the distance to star whose parallax is 0.000 05 seconds.

Revise

Revision Set 1

- Q61 (a) What is the general form of a reciprocal relation?
- (b) Sketch the shape of the graph of a reciprocal relation.
- (c) What happens to the graph as the value of the parameter c increases?
- (d) Solve $\frac{4}{t} = 7$
- Q62 $t = \frac{120}{p}$ where t is the number of hours taken to clean a fleet of buses and p is the number of people working on it.
- (a) How long will it take for 8 people to do the job?
- (b) How many people will be needed to get the job done in 5 hours?

Q63 The time it takes to clean a stadium is inversely proportional to the number of people employed to clean it. It takes 3 people 8 hours.

- (a) Write the relation between number of people and time as a formula.
 (b) Use the formula to find the time it would take 5 people.

Answers

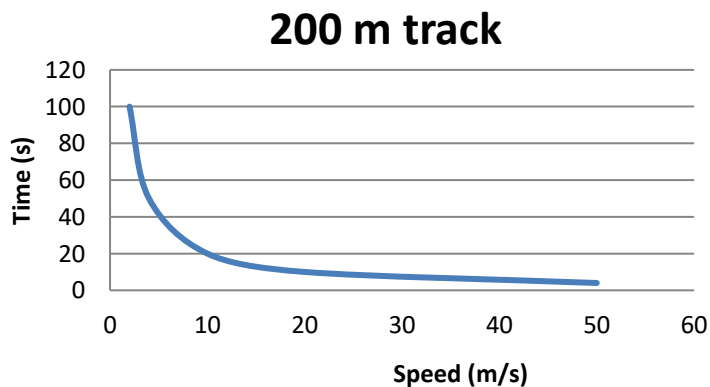
Q1 (a) 0.2 (b) 0.4

Q2 (a) $time = 200 \div speed$ or $speed = 200 \div time$

(b)

Speed (m/s)	2	4	10	20	50
Time (s)	100	50	20	10	4

(c)



(d) 47.6 s

Q51 (a) $distance = 3.1 \div angle$ (b) 0.756 seconds (c) 0.0062 seconds (d) 62 000 light years

Q61 (a) $y = \frac{c}{x}$ (b)  (c) it move away from the origin (d) $\frac{4}{7}$

Q62 (a) 15 h (b) 24

Q63 (a) $number\ of\ people = 24 \div time$ or $time = 24 \div number\ of\ people$ (b) 4.8 h